Correction to
Rationals Countability and Cantor's Proof

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Professor Boyko Banchev pointed to my error in this paper, in an email that says

"…your statement that Cantor

> '…uses a mapping that is not one-one. Thus, the countability of the rationals was not proved by Cantor'

is wrong. Cantor's function is an immediate application of the triangular number formula (counting the sum of the lengths of the first $\mu+\nu-2$ diagonals), and it is certainly a valid bijection from $\mathbb{N}\times\mathbb{N}\to\mathbb{N}$.

Your hand calculation on pp.2-3 is incorrect with respect to the values of λ."

Indeed, Cantor wrote,

"*…the element* (μ,ν) *comes at the* λ *th place, where*

$$(9)\qquad \lambda=\mu+\frac{(\mu+\nu-1)(\mu+\nu-2)}{2}.$$

The variable λ *takes every numerical value* $1,2,3,\ldots$, *once. Consequently, by means of (9), a reciprocally univocal relation subsists between the aggregates* $\{\nu\}$ *and* $\{(\mu,\nu)\}$."

and we have

$$\mu_1 = 1, \quad \nu_1 = 1, \quad \lambda_1 = 1, \quad \text{and} \quad (1,1) \to 1$$

$$\mu_2 = 1, \quad \nu_2 = 2, \quad \lambda_2 = 2, \quad \text{and} \quad (1,2) \to 2$$

$$\mu_3 = 2, \quad \nu_3 = 1, \quad \lambda_3 = 3, \quad \text{and} \quad (2,1) \to 3$$

$$\mu_4 = 1, \quad \nu_4 = 3, \quad \lambda_4 = 4, \quad \text{and} \quad (1,3) \to 4$$

$$\mu_5 = 2, \quad \nu_5 = 2, \quad \lambda_5 = 5, \quad \text{and} \quad (2,2) \to 5$$

$$\mu_6 = 3, \quad \nu_6 = 1, \quad \lambda_6 = 6, \quad \text{and} \quad (3,1) \to 6$$

$$\mu_7 = 1, \quad \nu_7 = 4, \quad \lambda_7 = 7, \quad \text{and} \quad (1,4) \to 7$$

$$\mu_8 = 2, \quad \nu_8 = 3, \quad \lambda_8 = 8, \quad \text{and} \quad (2,3) \to 8$$

$$\mu_9 = 3, \quad \nu_9 = 2, \quad \lambda_9 = 9, \quad \text{and} \quad (3,2) \to 9$$

$$\mu_{10} = 4, \quad \nu_{10} = 1, \quad \lambda_{10} = 10, \quad \text{and} \quad (4,1) \to 10$$

The mapping is one-one because if $\lambda_j = \lambda_k$ then,

$$\mu_j + \frac{(\mu_j + \nu_j - 1)(\mu_j + \nu_j - 2)}{2} = \mu_k + \frac{(\mu_k + \nu_k - 1)(\mu_k + \nu_k - 2)}{2},$$

$$(\mu_j - \mu_k)(\mu_j + \mu_k - 1) + (\nu_j - \nu_k)(\nu_j + \nu_k - 3) + 2(\mu_j\nu_j - \mu_k\nu_k) = 0.$$

Since each of the three terms is independent of the other two terms, each is zero. Then,

$$(\mu_j - \mu_k)(\mu_j + \mu_k - 1) = 0 \Rightarrow \mu_j = \mu_k,$$

since $(\mu_j + \mu_k - 1) \geq 1$, for any j, and k.

$$\mu_j\nu_j - \mu_k\nu_k = 0 \Rightarrow \mu_j\nu_j = \mu_k\nu_k \Rightarrow \nu_j = \nu_k,$$

since $\mu_j = \mu_k$, for any j, and k.□

The mapping that we propose in the paper “Rationals Countability and Cantor’s Proof is one-one as well.