

The Composite Photon

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ABSTRACT: The wave properties of a photon, contradict its being a particle, and were never explained.

We propose a material structure for the photon that explains interference, diffraction and polarization.

Sub-photons: The energy of a photon ϕ , $E_\phi = h\nu_\phi$ suggests that the photon is a current ring made of subphotons.

Assuming so, we approximate the spin of the photon, explain diffraction without the enigmatic wave-particle, and approximate its radius.

The model indicates that photon colors are different energy states of one single photon.

We assume that a photon ϕ is composed of two subphotons: masses of opposite charges, moving in the plane, along a circle of radius r_ϕ , at light speed c ,

$$\nu_\phi = \frac{c}{2\pi r_\phi} \text{ times per second.}$$

We assume two subphotons: δ_1 subphoton, and δ_2 subphoton

Longitudinal Wave: This harmonic motion associates with the photon a longitudinal wave of length

$$\lambda_{\phi} = 2\pi r_{\phi}.$$

The longitudinal wave is the key to photon's interference, diffraction, and polarization.

The Photon's Radius: The radius, $r_{\phi} = \frac{c}{2\pi\nu_{\phi}}$

determines the color of the photon. Each photon's radius r_{ϕ} determines a different color ν_{ϕ} , and appears as a different photon.

But it is the same subphotons circulating at a different radius, and different angular speed.

The radius r_{ϕ} distinguishes our model from a packet of radiation that has no structure.

$$\text{For Green, } r_{\phi} \approx \frac{0.5\mu\text{m}}{2\pi} \approx 0.08\mu\text{m}.$$

$$\text{For Red, } r_{\phi} = \frac{0.7\mu\text{m}}{2\pi} \approx 0.11\mu\text{m}.$$

The photon's frequency, mass, and energy are proportional to $\frac{1}{r_{\phi}}$

$$\nu_{\phi} = \frac{c}{2\pi r_{\phi}},$$

$$m_{\phi} = \frac{h}{2\pi r_{\phi} c} + \frac{3}{2} \frac{\frac{1}{2} + \frac{1}{\pi}}{18 \cdot 10^7} \frac{e^2}{r_{\phi}} \approx \frac{h}{2\pi r_{\phi} c}$$

$$h\nu_{\phi} = \underbrace{\frac{3}{2} \frac{\frac{1}{2} + \frac{1}{\pi}}{18 \cdot 10^7} \frac{e^2}{r_{\phi}} c^2}_{\text{Rotation and Binding}} + \underbrace{m_{\phi} c^2}_{\text{Translation}} \approx m_{\phi} c^2.$$

Subphotons Masses: The mass of a δ_1 Subphoton is

$$m_{\delta_1} = \frac{1}{10^7} \frac{(\frac{1}{3}e)^2}{\lambda_{\phi}} \left(\frac{\pi}{4} + \frac{1}{2}\right)$$

The mass of the δ_2 subphoton is

$$m_{\delta_2} = 4m_{\delta_1}$$

For a Green photon, $\lambda_{\phi} = 5 \cdot 10^{-7} \text{m}$,

$$m_{\delta_1} \approx (7.3)10^{-40} \text{Kg},$$

$$m_{\delta_2} \approx (2.9)10^{-39} \text{Kg}.$$

Spin of a Photon: For a δ_1 photon, $2m_{\delta_1} cr_{\phi} \approx 4 \times 10^{-21} \hbar$,

A negligible fraction of \hbar .

The hypothesis that \hbar is the spin of a photon is false

$$\hbar \text{ is NOT the Spin of any photon}$$

Oscillating Electric Dipole: The subphotons have oscillating electric and magnetic fields which could be generated by an

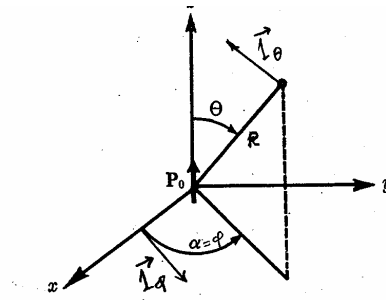
electric dipole with charges

$$\frac{1}{3}e, \text{ and } -\frac{1}{3}e,$$

separated by distance

$$z_\phi,$$

at the center of the photon ring, and at the center of a sphere of radius R .



If over a period, $\frac{1}{\nu_\phi}$, the dipole radiates $h\nu_\phi$ through the sphere, then,

$$z_\phi \approx 24r_\phi$$

For the Green photon, $r_\phi \approx 0.08\mu\text{m} \Rightarrow z_\phi \approx 2\mu\text{m}$

For the Red photon, $r_\phi \approx 0.11\mu\text{m} \Rightarrow z_\phi \approx (2.67)\mu\text{m}$

If the subphotons have charges $-\frac{2}{3}e$, and $\frac{2}{3}e$,

$$z_\phi \approx 97r_\phi$$

Slow Light: Our model for the composite photon allows for slowing down of the photon: After appropriate filtering of light,

the center of the photon may move at bicycle speed. But the photons retain the properties of light.

According to our model, the subphotons will keep encircling the center at light speed.

The Binding and Rotation energies will remain $\frac{3}{2} \frac{\frac{1}{2} + \frac{1}{\pi}}{10^7 \cdot 9} \frac{e^2}{r_\phi} c^2$.

The photon's energy will remain $h\nu_\phi \approx m_\phi c^2$, and $m_\phi \approx \frac{h\nu_\phi}{c^2}$.

Keywords: Subatomic, Sub-electron, Photon, Subphoton, photon Radius, Composite Particles, Current vortex, electron, Radiation Energy, Kinetic Energy, Rotation Energy, Electric Energy, Orbital Magnetic Energy, Spin Magnetic Energy, Centripetal Force, Lorentz Force, Electric Charge, Mass, Wave-particle, Radius-Energy, Electric Oscillating Dipole

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0.

$E = h\nu$ Suggests Sub-photons

Encircling at frequency ν

At the end of the Nineteenth century, the formulas derived for emitted electromagnetic radiation did not fit the measurements.

This so called problem of "Black Body Radiation" was resolved by Planck in 1901¹.

Planck offered his Radiation Law:

The average electromagnetic energy density per unit volume at frequency ν cycles/second emitted by a perfect radiator is

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{kT} - 1}$$

To obtain that, the radiation energy had to be assumed to be composed of atomic packets, multiples of

$$h\nu.$$

The assumption of discrete radiation energy, conflicted with Planck's belief in radiation of continuous waves.

¹ H. Vic Dannon, "[*Zero Point Energy: Planck's Radiation Law*](#)", Gauge Institute Journal Vol.1 No 3, August 2005.

He postulated that radiation is a transition between the energy levels of an oscillator. And, ignoring the symmetry between emission and absorption, he maintained that the absorption of radiation energy is continuous.

Later, Bohr in his description of the Hydrogen spectrum, used the same transition enigma.

And, Einstein, who explained the photo-electric effect assuming photons, supported De Broglie wave associated with matter.

And Schrödinger's wave equation was embraced because it was less confusing than Heisenberg-Dirac Operator Quantum Mechanics.

Bohr's Atom, Einstein's Photo-electric effect, and Compton's effect proved that radiation energy is indeed multiples of

$$h\nu.$$

From $E = mc^2$, the photon ϕ with frequency ν has the mass

$$m_{\phi}(\nu) = \frac{h}{c^2} \nu.$$

That is, electro-magnetic radiation is diluted matter.

This alone establishes

$$E = h\nu,$$

along with $E = mc^2$, as the most important facts of radiation theory. And the h , and the k constants, defined and determined

by Planck², amongst the most important constants in physics
Unaware of the meaning of these crucial facts, many texts equate

$$h \text{ to } 1,$$

doing away with measurement, and with Physics.

In Mathematics, that would be equivalent to equating

$$\pi \text{ to } 1,$$

doing away with numbers, and with Mathematics.

Moreover, the meaning of the frequency ν cycles/second eludes all.

ν , that seems to be invisible in the equation to all, means that

Something in the photon is turning, around its center.

And it cannot be its whole mass.

Because masses only turn around other masses.

It means that the photon has at least two masses.

That the photon is a composite.

We will assume that the photon is a pair of sub-photons with equal charges of opposite sign moving at light speed on a circle.

The electric attraction between the pair is balanced by the centripetal forces on each.

We proceed to discuss the existence of subphotons.

² H. Vic Dannon, "[*The Entropy Constant k, The Radiation Constant h, The Quantized 2nd Law, The Quantized Temperature, & The Temperature Quantum*](#)", Gauge Institute Journal of Math and Physics, Vol.15 No 1, February 2019.

1.

Subphotons

In 1928, J. J. Thomson presented in [Thomson,p.33], experimental evidence that the electron is a composite particle.

“...the properties of the electron recently discovered lead to the view that the electron...has itself a structure, being made up of smaller parts which carry charges of electricity.”

Later, Millikan presented in [Millikan, p. 161] evidence from his experiments for the existence of Subelectrons.

*“...Ehrenhaft and Zerner even analyze our report on oil droplets and find that these also show in certain instances indications of **sub-electrons**, for they yield in these observers' hands too low values of e , whether computed from the Brownian movement or from the law of fall.”*

These was long before it has been established that the proton, and the neutron are composed of subparticles with fractional charges.

The evidence for sub-electrons had to lead to a planetary model for the electron. To balance the electric forces on them, the sub-electrons must be moving in a closed orbit.

The existence of fractional charges, and sub-electrons, is now well-established. For instance, [Wagoner, pp. 541-5]

It is believed that a proton is composed of three sub-protons

Two u 's, each with charge $\frac{2}{3}e$, and one d with charge $-\frac{1}{3}e$.

We assume that

an electron is composed of three sub-electrons Two δ_2 's,

each with charge $-\frac{2}{3}e$, and one δ_1 with charge $\frac{1}{3}e$.

Since accelerating electrons emit photons,

we assume that

the photon is composed of sub-electrons.

That is, subphotons, and subelectrons are identical.

The frequency ν in the Radiation Energy

$$h\nu,$$

suggests a harmonic motion associated with the photon.

There must be at least two sub-photons of opposite signs orbiting within the photon.

Then, the centripetal forces of repulsion will balance the electric forces of attraction, to yield a stable structure.

We will assume that

The photon is a current-circle of radius r_ϕ , composed of

two Subphotons that encircle its center at light speed,

Then, the assumed harmonic motion takes place in the photon's interior.

The harmonic motion of the subphotons suggests a circular orbit.

But before assuming a circle, we submit that the orbit

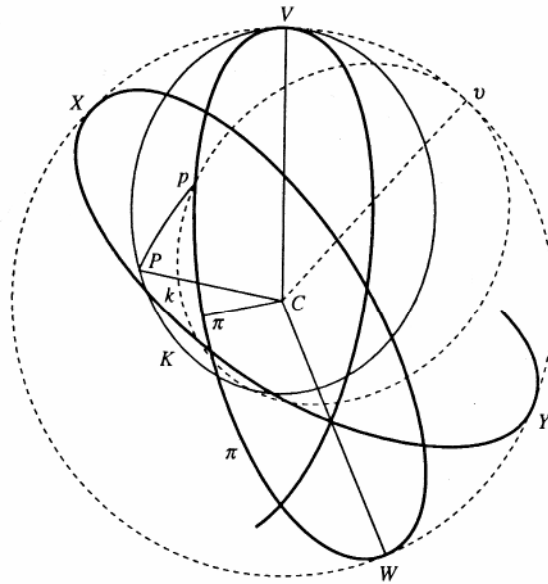
- lies in the plane, Not on a sphere,
- Must be closed,
- is Stable, Bounded between two circles,
- keeps its orientation in space.

2.

The Subphoton Circular Orbit

2.1 The Orbit is in the Plane. Not on a Sphere

Proof: Under non-inverse squared law force, the orbit of the particle turns around, and generates a sphere:



[Chandrasekhar, p.195]

But the inverse-squared electric force, limits the orbit to the plane, similarly to the gravitational force that limits the planets orbits to the ecliptic plane, and limits the stars orbits to the galactic plane.

2.2 The Subphoton's Orbit must be Closed

Proof: The electric force between the subphotons is proportional to the inverse square of the distance. And the centripetal forces on the subphotons are proportional to the distance.

By [Routh, p. 274], such central forces result in a closed orbit.

2.3 The Orbit is Stable, Bounded between Two Circles

Proof: By [Routh, p.280] Central Force orbits are stable.

2.4 Light Polarization Indicates that the Orientation of the Orbit Does Not Change.

2.5 Light Diffraction Indicates Harmonic Motion along a Circle

2.6 The Subphotons Are On The Same Circle

Proof: The subphotons' masses are equal.

2.7 The Subphotons Encircle the Center at Light Speed And the Photon's Center Moves at Light speed.

3.

The De-Broglie Longitudinal Wave of the Photon

De Broglie speculated that like the photon, any matter particle p with speed v_p , has an associated matter longitudinal wave with

$$\lambda_p = \frac{h}{m_p v_p}.$$

Then, diffraction of electrons that is due to uncertainty in their location, was interpreted as support for a wiggling electron.

Eventually, de Broglie realized that his wave represents the uncertainty in the particle location, [Dan 2]³.

In contrast with the Electric, and magnetic field that lie in the plane perpendicular to the propagation of the photon, the de-Broglie wave is longitudinal, and depicts a wiggling electron in the propagation direction, or alternatively, a wiggling photon.

Both reminding of the propagation of water waves, which is rather alien to the motion of matter. In the universe, we observe motion along a line, or in a closed orbit. But no wiggling motion.

³ H. Vic Dannon, "[Wave-Particle Duality: de Broglie Waves and Uncertainty](#)" Gauge Institute Journal Vol. 2 No 4, November 2006

de Broglie matter waves, and the wave-particle fallacy were helped by avoiding the drawing of the wiggling particle



This does not appear in any physics book. Because how many times can we see it before realizing that the photon does not clown around, and the picture does not look like Mechanics.

This raises the question,

what is the meaning of the wave length of a photon?

By introducing a current circle, we replace the wiggling along the propagation direction with harmonic motion of subphotons along a circle, and interpret the wave length of the photon

$$\lambda_{\phi}$$

as the circumference of the circle of radius

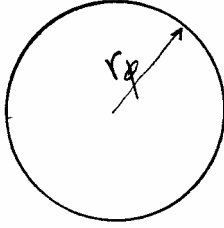
$$r_{\phi}.$$

That is,

$$\lambda_{\phi} = 2\pi r_{\phi}.$$

The longitudinal wave of the photon lies along that circle.

We replace the wave-particle enigma with harmonic motion along a circle of radius r_{ϕ} .



4.

The Longitudinal Wave, and the Transversal Wave of the Subphotons

We assume that the subphotons encircle photon's center at light speed c ,

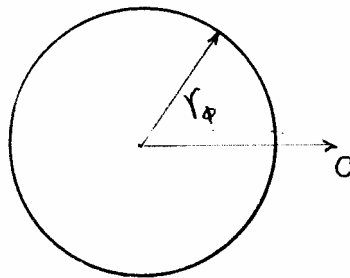
ν_ϕ times per second.

This harmonic motion associates with the photon a wavelength,

$$\lambda_\phi = 2\pi r_\phi.$$

The photon radius is

$$r_\phi = \frac{c}{\omega_\phi} = \frac{c}{2\pi\nu_\phi} = \frac{1}{2\pi}\lambda_\phi$$



The sub-photons circulate the photon center at light speed

$$c = \underbrace{\lambda_\phi}_{2\pi r_\phi} \nu_\phi = r_\phi \underbrace{2\pi\nu_\phi}_{\omega_\phi},$$

$$\nu_\phi = \frac{c}{2\pi r_\phi} \quad \text{times per second}$$

with an associated Longitudinal wavelength

$$\lambda_\phi = \frac{c}{\nu_\phi} = \frac{hc}{h\nu_\phi} = \frac{hc}{m_\phi c^2} = \frac{h}{m_\phi c}.$$

Meanwhile, the circle's center moves along the propagation direction at light speed c .

The diffraction and polarization of light indicate a Transversal wave that is due to that same circular harmonic motion of the subphotons about the photon center.

The Electromagnetic field of a sub-photon is a plane wave transversal to the subphoton direction of motion along the circle.

Thus, The wave associated with the photon is not a virtual De Broglie matter wave.

It is the reason for diffraction, and polarization of the photon.

The subphotons are circulating (at light speed) the photon's center which moves at light speed.

When the speed of the center is slowed down, by filtering, the circular motion is not affected. the photon color does not change, and the associated wave that manifests itself in diffraction, and polarization remains the same.

We proceed with

- the emission of photons in electron-positron annihilation,
- antenna radiation, and
- spectral electrons.

5.

Matter and Radiation

An electron-positron pair may transform into 3 gamma photons.

The electron may be

one Subelectron with $-\frac{1}{3}e$, and two Subelectrons with $\frac{2}{3}e$.

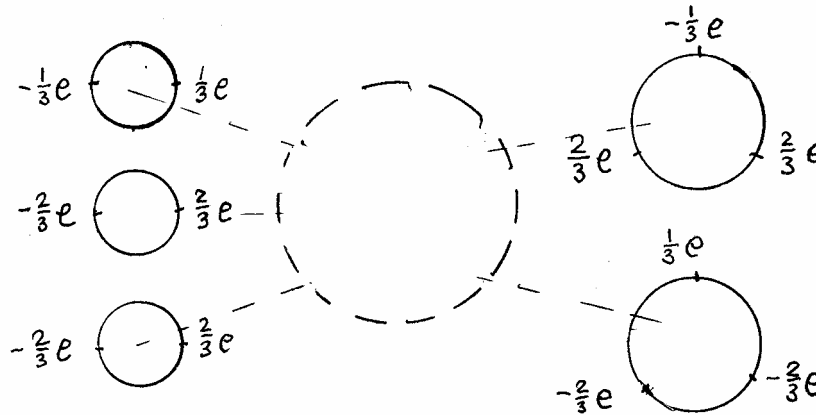
And the positron may be

one Subelectron with $\frac{1}{3}e$, and two Subelectrons with $-\frac{2}{3}e$.

The subelectrons may combine into

- ❖ One photon with $-\frac{1}{3}e$, and $\frac{1}{3}e$
- ❖ Two photons with $-\frac{2}{3}e$, and $\frac{2}{3}e$

Or, conversely, 3 gamma photons may transform into an electron-positron.



6.

Emission from Antenna

When an alternating voltage of frequency ν accelerates electrons along an antenna, the electrons radiate photons of energy $h\nu$.

The vibrations will cause the electrons to release subelectrons, and accelerate them to light speed c ,

Some subelectrons will merge with their anti-particles into a circle, and encircle a center at the vibrations frequency ν to become photons of energy $h\nu$. The frequency of the harmonic motion along the circle should be identical to the frequency of the generating vibrations.

7.

Emission by Spectral Electron

The emission of a photon by a spectral electron remains enigmatic.

Feynman's father asked him⁴

"...If the photon comes out of the atom when it goes from the excited to the lower state,

the photon must have been in the atom, in the excited state...

..How a photon comes out without it having been there in the excited state "

If we assume that the electron is composed from three subelectrons turning around its center, and that the energy of the electron depends on their angular speed,

then the electron's energy difference between the orbits is contained in the excited state, and the photon comes out having been in the higher orbit.

However, we don't know how the energies difference materializes into a pair of subphotons encircling the photon's center.

How the energy most stable form is the composite photon.

The creation of the composite photon from energy eludes us.

⁴ Richard Feynman, The Physics Teacher, September 1969, 319

8.

Subphotons' Mass

8.1 Subphotons' Charges

To have zero charge, the sub-photons must have opposite charges:

$$\frac{1}{3}e, \text{ and } -\frac{1}{3}e, \text{ each with mass } m_{\delta_1}$$

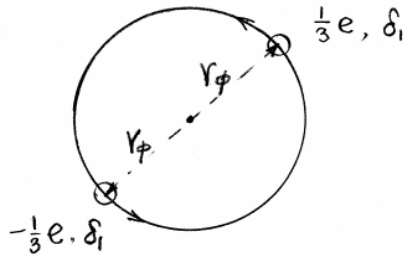
or

$$\frac{2}{3}e, \text{ and } -\frac{2}{3}e, \text{ each with mass } m_{\delta_2}$$

Perhaps, only one of these combinations occurs.

The Mass of δ_1

The photon is composed of the charges $\frac{1}{3}e$, and $-\frac{1}{3}e$.



8.2 The Electric Attraction between the δ_1 Subphotons

$$-\frac{1}{4\pi\epsilon_0} \frac{(\frac{1}{3}e)^2}{(2r_\phi)^2}$$

8.3 The Magnetic Attraction between the δ_1 Subphotons

$$-\frac{2}{\pi} \frac{1}{4\pi\epsilon_0} \frac{(\frac{1}{3}e)^2}{(2r_\phi)^2}.$$

Proof: The charge $\frac{1}{3}e$ generates the current $\frac{1}{3}e\nu$, which at distance $2r_\phi$, has the magnetic field

$$\begin{aligned} \mu_0 \frac{1}{2\pi(2r_\phi)} \left(\frac{1}{3}e\nu_\phi\right) &= \mu_0 \frac{1}{4\pi r_\phi} \frac{1}{3}e \frac{c}{2\pi r_\phi} \\ &= \frac{1}{2\pi} \frac{\mu_0}{\pi} \frac{\frac{1}{3}ec}{(2r_\phi)^2}. \end{aligned}$$

That field applies on the $-\frac{1}{3}e$ charge, the Lorentz force,

$$\begin{aligned} \left(-\frac{1}{3}e\right)c \left(\frac{1}{2\pi} \frac{\mu_0}{\pi} \frac{\frac{1}{3}ec}{(2r_\phi)^2}\right) &= -\frac{2}{\pi} \frac{\mu_0 c^2}{4\pi} \frac{(\frac{1}{3}e)^2}{(2r_\phi)^2} \\ &= -\frac{2}{\pi} \frac{1}{4\pi\epsilon_0} \frac{(\frac{1}{3}e)^2}{(2r_\phi)^2}. \end{aligned}$$

8.4 The Centripetal Repulsion on the δ_1 Subphotons

$$2m_{\delta_1} \frac{c^2}{r_\phi}.$$

8.5 Forces Balance on δ_1 Subphotons

$$2m_{\delta_1} \frac{c^2}{r_\phi} = \frac{1}{4\pi\epsilon_0} \frac{(\frac{1}{3}e)^2}{(2r_\phi)^2} \left(1 + \frac{2}{\pi}\right)$$

8.6

$$m_{\delta_1} = \frac{1}{10^7} \frac{(\frac{1}{3}e)^2}{\lambda_\phi} \left(\frac{\pi}{4} + \frac{1}{2}\right)$$

Proof:

$$2m_{\delta_1} \frac{c^2}{r_\phi} = \frac{1}{4\pi\epsilon_0} \frac{(\frac{1}{3}e)^2}{(2r_\phi)^2} \left(1 + \frac{2}{\pi}\right)$$

$$m_{\delta_1} = \frac{1}{4\pi\epsilon_0 c^2} \frac{1}{2} \frac{(\frac{1}{3}e)^2}{4r_\phi} \left(1 + \frac{2}{\pi}\right)$$

$$= \frac{\mu_0}{4\pi} \frac{1}{2} \frac{(\frac{1}{3}e)^2}{4r_\phi} \left(1 + \frac{2}{\pi}\right)$$

$$= \frac{1}{10^7} \frac{(\frac{1}{3}e)^2}{2\pi r_\phi} \frac{\pi}{4} \left(1 + \frac{2}{\pi}\right)$$

$$= \frac{1}{10^7} \frac{(\frac{1}{3}e)^2}{\lambda_\phi} \left(\frac{\pi}{4} + \frac{1}{2}\right)$$

8.7 Example from the visible spectrum

For

$$\lambda_\phi = 5 \cdot 10^{-7} \text{ m},$$

$$\nu_\phi = \frac{3 \cdot 10^8}{5 \cdot 10^{-7}} = 6 \cdot 10^{14} \text{ Hz}$$

$$m_{\delta_1} = \frac{1}{10^{79} 5 \cdot 10^{-7}} \frac{e^2}{4} \left(\frac{\pi}{4} + \frac{1}{2} \right)$$

$$\approx (7.3)10^{-40} \text{ Kg}$$

The Mass of δ_2

8.8

$$\boxed{m_{\delta_2} = 4m_{\delta_1}}$$

Proof: The photon is composed of charges

$$\frac{2}{3}e, \text{ and } -\frac{2}{3}e, \text{ each with mass } m_{\delta_2}.$$

The electric attraction between δ_2 subphotons

$$-\frac{1}{4\pi\epsilon_0} \frac{\left(\frac{2}{3}e\right)^2}{(2r_\phi)^2}$$

The magnetic attraction between δ_2 subphotons

$$-\frac{2}{\pi} \frac{1}{4\pi\epsilon_0} \frac{\left(\frac{2}{3}e\right)^2}{(2r_\phi)^2}.$$

Proof: The charge $\frac{2}{3}e$ generates the current $\frac{2}{3}e\nu$, which at distance

$2r_\phi$, has the magnetic field

$$\mu_0 \frac{1}{2\pi(2r_\phi)} \left(\frac{2}{3}e\nu_\phi\right) = \mu_0 \frac{1}{4\pi r_\phi} \frac{2}{3}e \frac{c}{2\pi r_\phi}$$

$$= \frac{1}{2\pi} \frac{\mu_0}{\pi} \frac{\frac{2}{3}ec}{(2r_\phi)^2}.$$

That field applies on the $-\frac{2}{3}e$ charge, the Lorentz force,

$$\begin{aligned} \left(-\frac{2}{3}e\right)c\left(\frac{1}{2\pi}\frac{\mu_0}{\pi}\frac{\frac{2}{3}ec}{(2r_\phi)^2}\right) &= -\frac{2}{\pi}\frac{\mu_0c^2}{4\pi}\frac{\left(\frac{2}{3}e\right)^2}{(2r_\phi)^2} \\ &= -\frac{2}{\pi}\frac{1}{4\pi\epsilon_0}\frac{\left(\frac{2}{3}e\right)^2}{(2r_\phi)^2}. \square \end{aligned}$$

The centripetal forces on the δ_2 subphotons are

$$2m_{\delta_2}\frac{c^2}{r_\phi}.$$

The forces balance on the δ_2 subphotons is

$$\begin{aligned} 2m_{\delta_2}\frac{c^2}{r_\phi} &= \frac{1}{4\pi\epsilon_0}\frac{\left(\frac{2}{3}e\right)^2}{(2r_\phi)^2}\left(1 + \frac{2}{\pi}\right) \\ m_{\delta_2} &= \frac{1}{10^7}\frac{\left(\frac{2}{3}e\right)^2}{\lambda_\phi}\left(\frac{\pi}{4} + \frac{1}{2}\right) \\ &= 4m_{\delta_1}. \square \end{aligned}$$

For

$$\lambda_\phi = 5 \cdot 10^{-7} \text{ m},$$

$$m_{\delta_1} \approx (7.3)10^{-40} \text{ Kg},$$

$$m_{\delta_2} \approx (2.9)10^{-39} \text{ Kg}.$$

9.

Binding Electric Energy

9.1 The Electric Energy Binding δ_1

$$U_{electric} = \frac{c^2}{18 \cdot 10^7} \frac{e^2}{r_\phi} = \frac{c\pi\nu_\phi e^2}{10^7 \cdot 9}$$

Proof:

$$\begin{aligned} U_{electric} &= \frac{1}{4\pi\epsilon_0} \frac{(\frac{1}{3}e)^2}{2r_\phi} \\ &= \frac{1}{72\pi\epsilon_0} \frac{e^2}{r_\phi} \\ &= \frac{\mu_0 c^2}{72\pi} \frac{e^2}{r_\phi} \\ &= \frac{c^2}{18 \cdot 10^7} \frac{e^2}{r_\phi} \\ &= \frac{c\pi\nu_\phi e^2}{10^7 \cdot 9}. \square \end{aligned}$$

9.2 The Electric Energy Binding δ_1 is negligible compared with the photon's Radiation Energy

$$\underline{\text{Proof:}} \quad = \frac{c\pi\nu_{\phi}e^2}{10^7 9} = \frac{\pi c e^2}{10^7 9 h} \approx \pi \frac{3 \cdot 10^8 (1.6)^2 10^{-38}}{10^7 9 (6.63) 10^{-34}} \approx 4 \cdot 10^{-4}.$$

9.3 The Electric Energy Binding δ_2

$$U_{electric} = 4 \frac{c^2}{18 \cdot 10^7} \frac{e^2}{r_{\phi}} = 4 \frac{c\pi\nu_{\phi}e^2}{10^7 \cdot 9}$$

9.4 The Electric Energy Binding δ_2 is negligible compared with the photon's Radiation Energy

10.

Binding Magnetic Energy

10.1 The Magnetic Energy Binding δ_1

$$U_{magnetic} = \frac{1}{\pi} \frac{c^2}{9 \cdot 10^7} \frac{e^2}{r_\phi} = \frac{2}{10^7} \frac{1}{9} e^2 \nu_\phi c$$

Proof: The current due to the $-\frac{1}{3}e$ charge that turns ν_ϕ cycles/second is

$$I = -\frac{1}{3}e\nu_\phi.$$

The Magnetic Energy of this current is [Benson, p.486]

$$\frac{1}{2}LI^2.$$

By [Fischer, p.97]

$$L = \mu_0 \frac{\pi(2r_\phi)^2}{2\pi(2r_\phi)} = \mu_0 r_\phi.$$

Thus, the magnetic energy due to the $-\frac{1}{3}e$ charge is

$$\frac{1}{2}\mu_0 r_\phi \left(\frac{1}{3}e\nu_\phi\right)^2.$$

The same magnetic energy is generated by the $\frac{1}{3}e$ charge.

Therefore, the Magnetic Energy of the photon ring is

$$U_{magnetic} = \mu_0 r_\phi \left(\frac{1}{3}e\nu_\phi\right)^2$$

$$\begin{aligned}
&= \frac{4\pi}{10^7} r_\phi \frac{1}{9} e^2 \nu_\phi \frac{c}{2\pi r_\phi} \\
&= \frac{1}{\pi} \frac{1}{10^7} \frac{1}{9} e^2 \underbrace{2\pi r_\phi \nu_\phi}_c \frac{c}{r_\phi} \\
&= \frac{1}{\pi} \frac{1}{10^7} \frac{1}{9} e^2 \frac{1}{r_\phi} c^2 \\
&= \frac{2}{10^7} \frac{1}{9} e^2 \nu_\phi c.
\end{aligned}$$

10.2 The Magnetic Energy Binding δ_1 is negligible compared

with the photon's Radiation Energy

Proof: It is $\frac{2}{\pi}$ of the binding electric energy, which is negligible compared with the photon's radiation energy.

10.3 The Magnetic Energy Binding δ_2

$$U_{magnetic} = 4 \frac{2}{\pi} \frac{c^2}{18 \cdot 10^7} \frac{e^2}{r_\phi} = 4 \frac{2}{\pi} \frac{c \nu_\phi e^2}{10^7 \cdot 9}$$

10.4 The Magnetic Energy Binding δ_2 is negligible compared with the photon's Radiation Energy

11.

The Photon Rotation Energy

11.1 The Rotation Energy of δ_1

$$2m_{\delta_1} \omega_{\phi}^2 r_{\phi}^2 = \frac{c\pi\nu_{\phi} e^2}{10^7 \cdot 9} \left(\frac{1}{2} + \frac{1}{\pi}\right) = \frac{1}{2} \frac{c^2}{10^7 \cdot 9} \frac{e^2}{r_{\phi}} \left(\frac{1}{2} + \frac{1}{\pi}\right)$$

Proof: The rotation energy of the δ_1 is

$$\begin{aligned} 2m_{\delta_1} \omega_{\phi}^2 r_{\phi}^2 &= 2m_{\delta_1} c^2 \\ &= 2 \frac{1}{10^7} \frac{\left(\frac{1}{3}e\right)^2}{\lambda_{\phi}} \left(\frac{\pi}{4} + \frac{1}{2}\right) c^2 \\ &= 2 \frac{1}{10^7} \frac{\left(\frac{1}{3}e\right)^2}{\lambda_{\phi}} \frac{\pi}{2} \left(\frac{1}{2} + \frac{1}{\pi}\right) c^2, \\ &= \frac{c\pi\nu_{\phi} e^2}{10^7 \cdot 9} \left(\frac{1}{2} + \frac{1}{\pi}\right), \end{aligned}$$

11.2 The Rotation Energy of δ_1 is negligible

compared with the photon's energy

Proof: The rotation energy is $\left(\frac{1}{2} + \frac{1}{\pi}\right)$ of the binding electric energy, which is negligible compared with the photon's radiation energy.

11.3 The Rotation Energy of δ_2

$$2m_{\delta_2} \omega_{\phi}^2 r_{\phi}^2 = 4 \frac{c\pi\nu_{\phi} e^2}{10^7 \cdot 9} \left(\frac{1}{2} + \frac{1}{\pi}\right) = 4 \frac{c^2}{18 \cdot 10^7} \frac{e^2}{r_{\phi}} \left(\frac{1}{2} + \frac{1}{\pi}\right)$$

11.4 The Rotation Energy of δ_2 is negligible compared with the photon's energy

12.

The Photon's Energy

12.1 The Photon's Energy

$$h\nu_\phi = \underbrace{\frac{3}{2} \frac{1}{10^7 \cdot 9 r_\phi} + \frac{1}{\pi} \frac{e^2}{c^2}}_{\text{Rotation and Binding}} + m_\phi c^2 \approx m_\phi c^2,$$

$$m_\phi = \frac{h\nu_\phi}{c^2} - \frac{3}{2} \frac{1}{10^7 \cdot 9 r_\phi} + \frac{1}{\pi} \frac{e^2}{c^2} \approx \frac{h\nu_\phi}{c^2} = \frac{h}{2\pi r_\phi c}$$

Proof:
$$h\nu_\phi = 2m_{\delta_1} c^2 + \underbrace{\frac{1}{2} \frac{1}{10^7 \cdot 9 r_\phi} + \frac{1}{\pi} \frac{e^2}{c^2}}_{\text{Binding}} + m_\phi c^2$$

$$= \underbrace{\frac{1}{2} \frac{1}{10^7 \cdot 9 r_\phi} + \frac{1}{\pi} \frac{e^2}{c^2}}_{\text{Rotation}} c^2 + \underbrace{\frac{1}{2} \frac{1}{10^7 \cdot 9 r_\phi} + \frac{1}{\pi} \frac{e^2}{c^2}}_{\text{Binding}} c^2 + m_\phi c^2 \approx m_\phi c^2.$$

$$m_\phi = \frac{h\nu_\phi}{c^2} - \frac{3}{2} \frac{1}{18 \cdot 10^7 r_\phi} + \frac{1}{\pi} \frac{e^2}{c^2} \approx \frac{h\nu_\phi}{c^2},$$

Since the rotation, the electric, and the magnetic binding energies are negligible,

$$h\nu_\phi \approx m_\phi c^2, \text{ and } m_\phi \approx \frac{h\nu_\phi}{c^2}.$$

Similarly, for the δ_2 subphoton. \square

13.

Photon's Radius-Energy Relation

13.1 *Photon's Frequency, mass, and Energy are proportional to $\frac{1}{r_\phi}$,*

$$\nu_\phi = \frac{c}{2\pi r_\phi},$$

$$m_\phi = \frac{h}{2\pi r_\phi c} + \frac{\frac{1}{2} + \frac{1}{\pi}}{18 \cdot 10^7} \frac{e^2}{r_\phi}$$

$$\boxed{h\nu_\phi = \frac{hc}{2\pi r_\phi}}, \text{ the photon Radius-Energy Relation}$$

13.2

$$\frac{r_{\phi_1}}{r_{\phi_2}} = \frac{\nu_{\phi_2}}{\nu_{\phi_1}} = \frac{m_{\phi_2}}{m_{\phi_1}} = \frac{h\nu_{\phi_2}}{h\nu_{\phi_1}}$$

13.3 *A blue photon radius is $\frac{2}{3}$ of a red photon radius, $r_{blue} \approx \frac{2}{3}r_{red}$*

A blue photon energy is $\frac{3}{2}$ of a red photon energy, $h\nu_{blue} \approx \frac{3}{2}h\nu_{red}$

Proof: $\lambda_{blue} \approx 4400\text{\AA}$, $\lambda_{red} \approx 6600\text{\AA}$,

$$\frac{\nu_{red}}{\nu_{blue}} = \frac{r_{blue}}{r_{red}} = \frac{\lambda_{blue}}{\lambda_{red}} \approx \frac{4400\text{\AA}}{6600\text{\AA}} = \frac{2}{3}.$$

The photon ring model suggests that photon's colors are just different energy states of one photon.

In particular, subphotons appear to be former subelectrons, and there is only one electron,

13.4 The One-Photon Hypothesis

Each photon's radius r_ϕ determines a different color ν_ϕ ,

and appears as a different photon.

But it is the same subphotons circulating

at a different radius, and different angular speed

14.

The Photon's Mass

$$m_{\phi} = \frac{h}{2\pi r_{\phi} c} - \frac{3}{2} \frac{\frac{1}{2} + \frac{1}{\pi}}{10^7 \cdot 9} \frac{e^2}{r_{\phi}}$$

$$\approx \frac{h}{c} \frac{1}{2\pi r_{\phi}} = \frac{h\nu_{\phi}}{c^2}$$

The Subphoton mass is much smaller.

This is similar to the u , and d quarks which masses are much smaller than the proton, and neutron masses.

14.1 Example from the visible spectrum

For

$$\lambda_{\phi} = 5 \cdot 10^{-7} \text{ m},$$

$$\nu_{\phi} = \frac{3 \cdot 10^8}{5 \cdot 10^{-7}} = 6 \cdot 10^{14} \text{ Hz},$$

$$m_{\phi} \approx \frac{h\nu_{\phi}}{c^2}$$

$$\approx \frac{1}{9 \cdot 10^{16}} (6.6) 10^{-34} \cdot 6 \cdot 10^{14}$$

$$= (4.4) 10^{-36} \text{ Kg}$$

$$\delta_1 \approx (7.3) 10^{-40} \text{ Kg}$$

15.

The Photon's Spin and Color

It is postulated that the photon has spin \hbar because

$$\begin{aligned} m_\phi cr_\phi &= m_\phi c \frac{c}{2\pi\nu_\phi} \\ &= \frac{1}{2\pi} m_\phi c^2 \frac{1}{\nu_\phi} = \frac{h}{2\pi} = \hbar. \end{aligned}$$

$\approx h\nu_\phi$

But

15.1 \hbar is not a Photon Spin

Proof: $m_\phi cr_\phi$

is the Angular Momentum of a point mass m_ϕ circulating a center at a distance r_ϕ .

But a non-constrained mass will never orbits a massless center.

Thus, $m_\phi cr_\phi = \hbar$ is not the Spin of any photon. \square

15.2 The Orbital Angular Momentum of δ_1 Photon is

$$2m_{\delta_1} cr_\phi = \frac{\alpha}{9c^2} \left(\frac{1}{4} + \frac{1}{2\pi} \right) \hbar \approx 3.7 \times 10^{-21} \hbar,$$

where $\alpha \approx \frac{1}{137}$ is the fine structure constant.

fixed for a photon of any color.

Proof:

$$\begin{aligned}
 2m_{\delta_1} cr_{\phi} &= 2m_{\delta_1} c \frac{c}{2\pi\nu_{\phi}} \\
 &= 2 \frac{1}{10^7} \frac{(\frac{1}{3}e)^2}{\lambda_{\phi}} \left(\frac{\pi}{4} + \frac{1}{2}\right) \frac{1}{2\pi\nu_{\phi}} \\
 &= \frac{1}{c10^7 \cdot 9} e^2 \left(\frac{1}{4} + \frac{1}{2\pi}\right)
 \end{aligned}$$

Substituting $e^2 = \frac{2h\alpha}{\mu_0 c}$, where $\alpha \approx \frac{1}{137}$,

$$\begin{aligned}
 &= \frac{1}{c10^7 \cdot 9} \frac{2h\alpha}{\mu_0 c} \left(\frac{1}{4} + \frac{1}{2\pi}\right) \\
 &= \frac{\alpha}{9c^2} \left(\frac{1}{4} + \frac{1}{2\pi}\right) \hbar \\
 &\approx \frac{\frac{1}{4} + \frac{1}{2\pi}}{9^2 10^{16} 137} \hbar \\
 &\approx 3.7 \times 10^{-21} \hbar . \square
 \end{aligned}$$

16.

The Photon as an Oscillating Electric Dipole

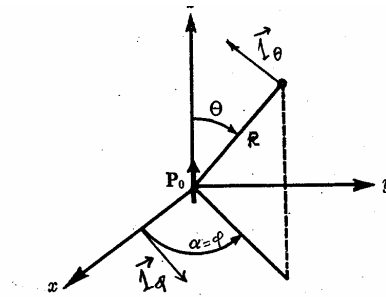
The subphotons have oscillating electric and magnetic fields which could be generated by an electric dipole with charges

$$\frac{1}{3}e, \text{ and } -\frac{1}{3}e,$$

separated by distance

$$z_\phi,$$

at the center of a sphere of radius R .



16.1 *If over a period, $\frac{1}{\nu_\phi}$, the dipole radiates $h\nu_\phi$ through*

the sphere Then, $z_\phi \approx 24r_\phi$

Proof: The fields are on the surface of that sphere on a plane tangent to the sphere and perpendicular to R .

The Electric field along unit vector \vec{l}_θ is [Griffiths, p. 404]

$$\vec{E}_\theta(t) = -\frac{\mu_0}{4\pi} (z_\phi \frac{1}{3} e) \omega_\phi^2 \frac{\sin \theta}{R} \cos \omega_\phi \left(t - \frac{R}{c} \right) \vec{1}_\theta$$

The Magnetic Induction along unit vector $\vec{1}_\phi$ is [Griffiths, p. 405]

$$\vec{B}_\phi(t) = -\frac{1}{c} \frac{\mu_0}{4\pi} (z_\phi \frac{1}{3} e) \omega_\phi^2 \frac{\sin \theta}{R} \cos \omega_\phi \left(t - \frac{R}{c} \right) \vec{1}_\phi$$

The Magnetic field is

$$\vec{H}_\phi(t) = -\frac{1}{c} \frac{1}{4\pi} (z_\phi \frac{1}{3} e) \omega_\phi^2 \frac{\sin \theta}{R} \cos \omega_\phi \left(t - \frac{R}{c} \right) \vec{1}_\phi$$

The power radiated per unit area of the sphere is

$$\vec{E}_\theta(t) \times \vec{H}_\phi(t) = \frac{1}{c} \frac{\mu_0}{16\pi^2} (z_\phi \frac{1}{3} e)^2 \omega_\phi^4 \frac{\sin^2 \theta}{R^2} \cos^2 \omega_\phi \left(t - \frac{R}{c} \right) \vec{1}_R$$

Averaging over time,

$$\begin{aligned} \frac{1}{\omega_\phi T} \int_{\omega_\phi t=0}^{\omega_\phi t=2\pi} \cos^2 \omega_\phi \left(t - \frac{R}{c} \right) d(\omega_\phi t) &= \frac{1}{2\pi} \int_{t=0}^{t=T} \frac{1}{2} [1 - \cos 2\omega_\phi \left(t - \frac{R}{c} \right)] d(\omega_\phi t) \\ &= \frac{1}{2\pi} \frac{1}{2} \underbrace{[\omega_\phi \left(t - \frac{R}{c} \right)]_{t=0}^{t=T}}_{2\pi} - \frac{1}{2\pi} \frac{1}{2} \frac{1}{2} \underbrace{[\sin 2\omega_\phi \left(t - \frac{R}{c} \right)]_{t=0}^{t=T}}_0 = \frac{1}{2} \end{aligned}$$

The time-average of the power per unit area of the sphere is

$$\frac{1}{c} \frac{\mu_0}{16\pi^2} (z_\phi \frac{1}{3} e)^2 \omega_\phi^4 \frac{\sin^2 \theta}{R^2} \frac{1}{2} = \frac{1}{c} \frac{\mu_0}{32\pi^2} (z_\phi \frac{1}{3} e)^2 \omega_\phi^4 \frac{\sin^2 \theta}{R^2}$$

The time-average of the power through the sphere of radius R is

$$\begin{aligned}
& \frac{1}{c} \frac{\mu_0}{32\pi^2} (z_\phi \frac{1}{3} e)^2 \omega_\phi^4 \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\theta=\pi} \frac{\sin^2 \theta}{R^2} (R d\theta) (R \sin \theta d\varphi) = \\
& = \frac{1}{c} \frac{\mu_0}{32\pi^2} (z_\phi \frac{1}{3} e)^2 \omega_\phi^4 2\pi \int_{\theta=0}^{\theta=\pi} \frac{\sin^3 \theta d\theta}{(1-\cos^2 \theta) \underbrace{\sin \theta d\theta}_{-d \cos \theta}} \\
& = \frac{1}{c} \frac{\mu_0}{16\pi} (z_\phi \frac{1}{3} e)^2 \omega_\phi^4 \left\{ \underbrace{\frac{1}{3} [\cos^3 \theta]_{\theta=0}^{\theta=\pi}}_{-2/3} - \underbrace{[\cos \theta]_{\theta=0}^{\theta=\pi}}_{-2} \right\} \\
& = \frac{1}{c} \frac{\mu_0}{16\pi} (z_\phi \frac{1}{3} e)^2 \omega_\phi^4 \frac{4}{3} \\
& = \frac{1}{c} \frac{\mu_0}{12\pi} (z_\phi \frac{1}{3} e)^2 \omega_\phi^4.
\end{aligned}$$

The radiated energy through the sphere of radius R , over time

period $T = \frac{1}{\nu_\phi}$ is

$$\begin{aligned}
\left(\frac{1}{c} \frac{\mu_0}{12\pi} (z_\phi \frac{1}{3} e)^2 \omega_\phi^4 \right) T &= \frac{1}{2\pi r_\phi \nu_\phi} \frac{\mu_0}{12\pi} (z_\phi \frac{1}{3} e)^2 (2\pi \nu_\phi)^4 \frac{1}{\nu_\phi} \\
&= \frac{1}{r_\phi} \frac{\mu_0}{6} (z_\phi \frac{1}{3} e)^2 (2\pi \nu_\phi) \frac{c}{r_\phi} \\
&= \frac{\pi c \mu_0}{3} \left(\frac{z_\phi e}{3r_\phi} \right)^2 \nu_\phi
\end{aligned}$$

We shall assume that this energy is the photon energy $h\nu_\phi$

That is,

$$\frac{\pi c \mu_0}{3} \left(\frac{z_\phi e}{3r_\phi} \right)^2 \nu_\phi = h \nu_\phi$$

$$\frac{\pi c}{3} \frac{4\pi}{10^7} \left(\frac{z_\phi e}{3r_\phi} \right)^2 = h$$

$$\left(2\pi \frac{z_\phi e}{3r_\phi} \right)^2 = h \frac{3 \cdot 10^7}{c} \approx \frac{h}{10}$$

$$z_\phi \approx \left(\frac{h}{10} \right)^{\frac{1}{2}} \frac{3}{2\pi e} r_\phi$$

$$\approx \underbrace{\left(\frac{(6.626)10^{-34}}{10} \right)^{\frac{1}{2}}}_{(0.814002457)10^{-17}} \frac{3}{\underbrace{2\pi \cdot (1.6)10^{-19}}_{(0.298415518)10^{19}}} r_\phi$$

$$\approx (24.29109649)r_\phi$$

For the Green photon, $r_\phi \approx \frac{0.5\mu\text{m}}{2\pi} \approx 0.08\mu\text{m}$

$$z_\phi \approx 2\mu\text{m}$$

For the Red photon, $r_\phi \approx \frac{0.8\mu\text{m}}{2\pi} \approx 0.11\mu\text{m}$

$$z_\phi \approx (2.67)\mu\text{m}$$

16.2 If the subphotons have charges $-\frac{2}{3}e$, and $\frac{2}{3}e$,

$$z_{\phi} \approx 97r_{\phi}$$

17.

Slow Light

Our model for the composite photon allows for slowing down of the photon.

After appropriate filtering of light, the center of the photon may move at bicycle speed.

But the subphotons will keep encircling the center at light speed.

The Binding and Rotation energies will remain the negligible

$$\frac{3 \frac{1}{2} + \frac{1}{\pi} e^2}{2 \cdot 10^7 \cdot 9 r_\phi} c^2.$$

The photon's energy will remain

$$h\nu_\phi \approx m_\phi c^2,$$

and

$$m_\phi \approx \frac{h\nu_\phi}{c^2}.$$

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