American Journal Of Physics claims that $h \to 0$ does not imply Classical Mechanics

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Introduction

In his Quantum Mechanics book, Dirac wrote

"Classical Mechanics may be regarded as the limiting case of Quantum Mechanics when h tends to zero"

In a recent AJP paper, [Klein], the author claims that

"On one hand, Dirac's Dictum…is considered to be true. On the other hand, it cannot be verified."

He claims in his Abstract that since Schrödinger's equation does not supply the proof,

"Classical Mechanics cannot be regarded as emerging from Quantum Mechanics...upon application of the limit $h \to 0$.***"***

We wrote to the Editor of the American Journal Of Physics that $h \to 0$ implies Classical Mechanics, and explained how the Schrödinger's equation evaporates when $h \to 0$.

The AJP editor answered

"... Showing that the Planck Law...reduces to the Rayleigh-Jeans.. formula...would be of no interest to readers of AJP.

Meaning, if it is not interesting they prefer the falsehood.

"...In addition, this calculation does nothing to demonstrate that classical mechanics arises when $h \to 0$*."*

Meaning, The fact that the Rayleigh-Jeans law is Classical Mechanics, is untrue.

In particular, the AJP cannot comprehend that when $h \to 0$, Schrödinger's equation yields only the Zero Wave Function. Then, without Wave Mechanics, we get Classical Mechanics.

The following is what the AJP chose to hide from its readers.

1.

$h \to 0$ implies Classical Mechanics

Proof:

In Blackbody radiation, Planck's energy density at frequency ν is [Woan, p.121]

$$u_\nu(T) = \tfrac{8\pi\nu^2}{c^3} kT \frac{\frac{h\nu}{kT}}{e^{\frac{h\nu}{kT}} - 1}$$

For $h \to 0$, we have $\frac{h\nu}{kT} \to 0$, and the quotient

$$\frac{\frac{h\nu}{kT}}{e^{\frac{h\nu}{kT}} - 1} \to \frac{0}{0}.$$

To find this indeterminate limit, we use L'Hospital rule. Denoting

$$\frac{h\nu}{kT} \equiv x,$$

$$\lim_{x \to 0} \frac{x}{e^x - 1} = \lim_{x \to 0} \frac{D_x x}{D_x(e^x - 1)} = \lim_{x \to 0} \frac{1}{e^x} = \frac{1}{1} = 1.$$

Therefore, $h \to 0$ implies

$$u_\nu(T) \to \frac{8\pi\nu^2}{c^3} kT$$

This last expression is the Rayleigh-Jeans energy density at frequency ν. Namely, the Radiation energy density of Classical Mechanics.

Consequently, $h \to 0$, implies Classical Mechanics.

2.

At $h \to 0$ Schrödinger's equation evaporates

Proof:

For a particle of mass m, in a potential $V(\vec{x}, t)$, Schrodinger's Equation for the wave function $\psi(\vec{x}, t)$ is

$$\frac{1}{2\pi i} h \partial_t \psi - \frac{1}{8\pi^2 m} h^2 \Delta\psi + V(\vec{x}, t)\psi(\vec{x}, t) = 0.$$

For $h \to 0$, the equation becomes

$$V(\vec{x}, t)\psi(\vec{x}, t) = 0.$$

Since $V(\vec{x}, t)$ is non-zero, the wave function must be identically zero. That is, the only solution to the equation is the identically-zero wave function

$$\psi(\vec{x}, t) \equiv 0.$$

Clearly, this does not prevent Newton's laws to hold.

References

[Dan] H. Vic Dannon, "Zero Point Energy, Planck's Radiation Law" Gauge Institute Journal of Math and Physics, Vol.1 No 3, August 2005. posted to www.gauge-institute.org.

[Klein], U. Klein, "*What is the Limit* $h \to 0$ *of Quantum Theory?*" American Journal of Physics, Volume 80, No. 11, November 2012, pp.1009-1016.

[Woan] Graham Woan, The Cambridge handbook of Physics Formulas", Cambridge U. Press, 2000