Neutron's Spin and Radius

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Abstract The neutron's Spin is a negligible fraction of $\hbar$.

We present a ring model for the neutron to approximate its spin, and its radius

We assume that the Subneutrons, the quarks, move along a circle of radius r_{n} at light speed c, $\frac{c}{2\pi r_{\text{n}}} = \nu_{\text{n}}$ times per second. This associates with the neutron a wave of length $\lambda_{\text{n}} = 2\pi r_{\text{n}}$.

The neutron's frequency, mass, and energy are inversely proportional to its radius.

$$\nu_{\text{n}} = \frac{c}{2\pi r_{\text{n}}},$$

$$m_{\text{n}} \sim (1 + \tfrac{5}{3\pi})\tfrac{1}{9}10^{-7} e^2 \frac{1}{r_{\text{n}}},$$

$$m_{\text{n}}c^2 \sim (1 + \tfrac{5}{3\pi})\tfrac{1}{9}10^{-7} e^2 c^2 \frac{1}{r_{\text{n}}}.$$

The approximate neutron's radius is

$$r_{\text{n}} \sim (1 + \tfrac{5}{3\pi})\tfrac{1}{9}10^{-7} \frac{e^2}{m_{\text{n}}} \sim 2.6 \times 10^{-19}\,\text{m}.$$

The neutron's Radius-Energy Relation suggests that a heavier positively charged Baryon made of three quarks, is a neutron with a smaller radius.

The quarks' harmonic motion explains neutron's diffraction.

The Neutron's Spin is $\sim 10^{-5}\hbar$.

While $m_{\text{n}} \approx m_{\text{p}}$, $r_{\text{n}} \sim 0.87 r_{\text{p}}$

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Contents

Neutron's Spin is not $\frac{1}{2}\hbar$

The postulate that the neutron's spin is $\frac{1}{2}\hbar$, appears in textbooks, as a fact well-established by a theory that no one knows its details, and well-confirmed in experiments that never took place.

Some authors believe that Dirac's Equation for the neutron's wave function ψ implies that the neutron spin is $\frac{1}{2}\hbar$. But any $\text{constant} \times \psi$ solves the Dirac equation, and the normalization means, $\int |\text{constant} \times \psi| = 1$.

Many authors believe that it follows from Quantum Field Theory. But that theory uses the Atomic system units where $\hbar = 1$. Hence it may at most indicate orientation of revolution.

In fact, monographs about quarks such as [Efimov], [Ripka], [Klinkhamer], and [Kokkedee] avoid nucleon's spin, while monographs such as [Roberts], and [Troshin] that mention spin, avoid its value.

Moreover, this postulate violates special Relativity.

0.1 *Postulating the Neutron's Spin to be* $\frac{1}{2}\hbar$ *requires revolution at speeds greater than light speed, violating Special Relativity.*

Proof: A rigid Spherical neutron with mass m_{n} and radius r_{n}, that spins at speed v_s has moment of inertia $\frac{2}{5}m_{\text{n}}r_{\text{n}}^2$, and Spin Angular Momentum

$$I\omega = (\tfrac{2}{5}m_{\text{n}}r_{\text{n}}^2)(\frac{v_s}{r_{\text{n}}}) = \tfrac{2}{5}m_{\text{n}}r_{\text{n}}v_s,$$

Postulating that the Spin is $\frac{1}{2}\hbar$,

$$\tfrac{2}{5}m_{\text{n}}r_{\text{n}}v_s = \tfrac{1}{2}\hbar,$$

$$v_s = \tfrac{5}{4}\frac{\hbar}{m_{\text{n}}r_{\text{n}}}$$

$$= \tfrac{5}{4}\frac{m_{\text{e}}v_B r_B}{m_{\text{n}}r_{\text{n}}},$$

$$= \tfrac{5}{4}\frac{m_{\text{e}}}{m_{\text{p}}}\frac{\alpha c r_B}{r_{\text{p}}}, \text{ where } \alpha \approx \tfrac{1}{137}, \ \tfrac{m_{\text{e}}}{m_{\text{n}}} \approx \tfrac{1}{1839}$$

Using $r_B \sim 5\cdot 10^{-11}\text{m}$, and $r_{\text{p}} \sim 3\times 10^{-19}\text{m}$, (obtained here)

$$v_{\text{spin}} \sim \tfrac{5}{4}\tfrac{1}{137}\tfrac{1}{1839}\tfrac{5\cdot 10^{-11}}{3\times 10^{-19}}c$$

$$\sim 828c \gg c.\square$$

The arbitrary postulate that the neutron has spin $\frac{1}{2}\hbar$ has to avoided.

We will approximate the neutron's Spin from the orbital angular momentum of the subneutrons.

1.

Evidence for Subneutrons

In the 1960's, high energy experiments indicated that the nucleons are composite particles. The Subparticles were proposed under the names Partons, Quarks, Aces,… to guarantee the exclusive rights of the proposer.

The theory created to establish the existence of Subneutrons uses mathematical symbols, but its inaccuracies, and inconsistencies, prevent us from any serious critique of it.

In particular, monographs about the Subneutrons do not suggest an elementary model for the structure of the neutron, and without such model we cannot approximate the spin, and the radius of the neutron.

The evidence for Subneutrons had to lead to a planetary model for the neutron. Just to balance the electric forces on them, the Subneutrons must be moving, and since they are not going anywhere, the motion is in a closed orbit.

2.

Diffraction and De Broglie Wave

Diffraction of neutrons had to suggest harmonic motion of Subneutrons within the neutron. That harmonic motion manifests itself in a physical wave.

Without the subneutrons circulating within the boundaries of the neutron, the diffraction of neutrons remains a mystery.

De Broglie wave is based on the speculation that like the photon ϕ which is a particle with speed c, and wavelength

$$\lambda_\phi = \frac{c}{\nu_\phi} = \frac{hc}{h\nu_\phi} = \frac{hc}{m_\phi c^2} = \frac{h}{m_\phi c},$$

any particle p with speed v_p, has an associated longitudinal wavelength

$$\lambda_p = \frac{h}{m_p v_p}.$$

The diffraction of neutrons, that must be due to harmonic motion of their subneutrons, was attributed to a wiggling neutron.

Since it is impossible to visualize an neutron wiggling along it path, a property called wave particle duality was invented.

Even De Broglie realized that his wave represents the uncertainty in the particle location, [Dan1], [de Broglie].

Thus, the denial of an elementary neutron model, eliminated the wave that underlies the harmonic motion of the subneutrons, and the neutron remained a puzzle as to whether it is a wave or a particle.

3.

Subneutrons' Motion

The Spin suggests a harmonic circular motion of the subneutrons. Then, the centripetal forces of repulsion will balance the Lorentz magnetic and electric forces of attraction, to yield a stable structure.

3.1 Closed Orbit

To stay within the neutron boundaries,

the subneutrons should have a closed orbit.

3.2 Central Force

By [Routh, p. 274], a closed orbit results from a central force that is proportional to the inverse square of the distance,(such as the Coulomb electric force) or directly to the distance(such as the centripetal force).

Subneutrons charges supply

the electromagnetic force to close the orbit.

3.3 Orbit Stability

By [Routh, p.280] Central Force orbits are stable. That is, they are bounded in a ring between two circles. The stability of the neutron indicates such orbits.

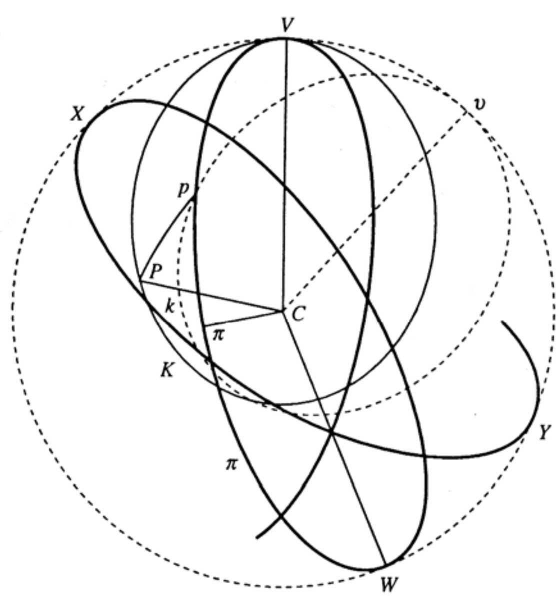
3.4 Planar Motion

Since the electric force is inverse squared law force,

Subneutrons' orbits will be in the same plane,

and not on a sphere

The plane of motion of the particle turns around to generate a sphere <u>only under a non-inverse squared law force.</u>



[Chandrasekhar, p. 195].

3.5 Subneutrons' Speed

In [Dan4], we assumed that the subelectrons circulated the electron's center at light speed. Since the neutron and the electron mirror each other, we'll assume that

the Subneutrons move along a circle of radius r_{n}*, at light speed* c*,*

$$\nu_{\mathrm{n}} = \frac{c}{2\pi r_{\mathrm{n}}} \text{ times per second.}$$

The Subneutrons tangential speed in their circular path is c

Thus, the neutron has an associated wave of length $\lambda_{\mathrm{n}} = 2\pi r_{\mathrm{n}}$, which explains neutrons' diffraction.

3.6 Only one Energy State; No assumption of Gluons

Moving at light speed, the Subprotons are charged radiation particles. We will assume that the proton has only one energy state, the quarks have specific orbits, in which they do not radiate. The quarks do not exchange gluons. The QFT assumption of gluons has no experimental basis, and is not necessary in our Current-Ring Model for the proton.

4.

The Neutron Structure

4.1 The Charges and Masses of the Subneutrons

The u quark has $-\frac{2}{3}e$ charge and mass m_{u} between

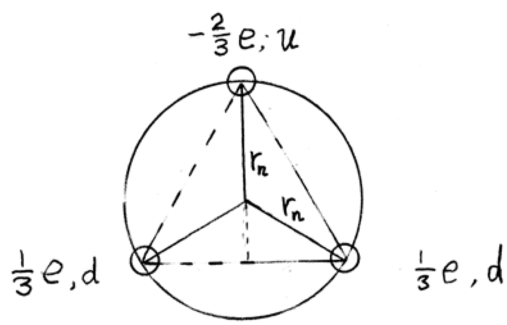
$$0.35 \quad \text{and} \quad 0.6 \quad m_d,$$ an average of $0.475 \quad m_d$, [PDG]

Each of the d quarks has $\frac{1}{3}e$ charge and mass m_{d} between

$$4.1 \quad \text{and} \quad 5.8 \ \text{MeV},$$ an average of 4.95 MeV, [PDG]

4.2 The Location of the Subneutrons

Locating the quarks at the vertices of an equilateral triangle will result in greater attraction that will distort the symmetry.

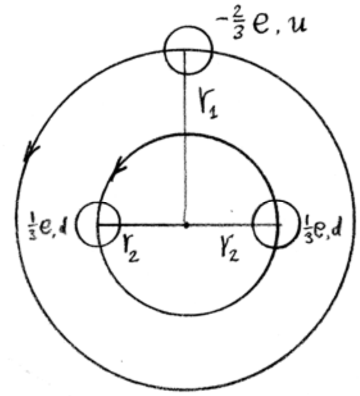


The planetary model will not allow all three subneutrons to be on the same circle.

But to simplify the discussion, we will let the two d subneutrons share the same orbit.

To temper the effect of the attraction between the d and each of the u quarks, the distance between them has to be larger then the distance between the d quarks.

Therefore, the orbit of the u Subneutron will have a larger radius. This means two current rings. One with larger radius r_1, and one with smaller radius r_2,



Since the correct model is made of at least two current rings, the neutron has no radius. What we mean by the neutron radius, r_n, is a number between the two ring radii,

$$r_1 < r_n < r_2,$$

the order of the size of the two rings.

In fact, each Subneutron has its own radius approximated by r_n.

5.

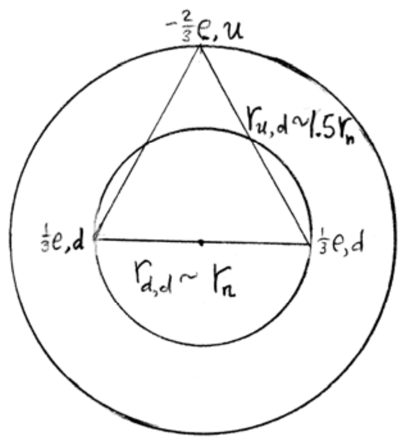
Binding Electric Energy

5.1 The Neutron's Binding Electric Energy

$$U_{electric} \sim -\frac{1}{36\pi\varepsilon_0}\frac{e^2}{r_{\mathrm{n}}}$$

$$= -\frac{1}{9}10^{-7}c^2\,\frac{e^2}{r_{\mathrm{n}}}$$

Proof: $$U_{electric} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{(\frac{1}{3}e)(\frac{1}{3}e)}{r_{d,d}} + 2\frac{(\frac{1}{3}e)(-\frac{2}{3}e)}{r_{u,d}}\right\}$$



Approximating

$$r_{u,d} \sim \tfrac{3}{2}r_{\mathrm{n}},$$

$$r_{d,d} \sim r_{\mathrm{n}},$$

we have

$$U_{electric} \sim \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_{\mathrm{n}}}\left\{\frac{\frac{1}{9}}{1} - 2\frac{\frac{2}{9}}{\frac{3}{2}}\right\}$$

$$\sim -\frac{5}{108\pi\varepsilon_0}\frac{e^2}{r_{\mathrm{n}}}$$

$$= -\frac{5}{27}10^{-7}c^2\,\frac{e^2}{r_{\mathrm{n}}}.$$

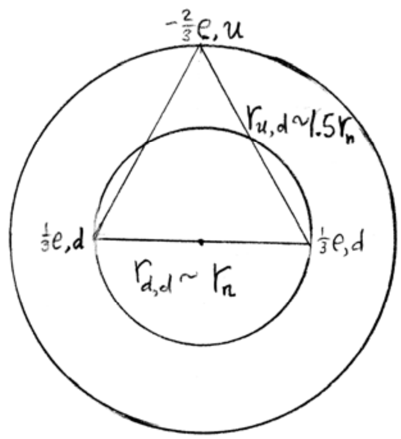
6.

Binding Magnetic Energy

6.1 Repulsion Magnetic Energy between the d quarks

$$\frac{1}{9\pi}\frac{\mu_0}{4\pi}\frac{e^2}{r_n}c^2 = \frac{1}{9\pi}\frac{1}{10^7}\frac{e^2}{r_n}c^2$$

<u>*Proof*</u>:



The d quark with $\frac{1}{3}e$ charge generates the current

$$\tfrac{1}{3}e\nu_n = \tfrac{1}{3}e\frac{c}{2\pi r_n} = \frac{ec}{6\pi r_n},$$

which at distance r_n, has the magnetic field

$$\mu_0\frac{1}{2\pi r_n}\left(\frac{ec}{6\pi r_n}\right) = \frac{1}{3\pi}\frac{\mu_0}{4\pi}\frac{ec}{r_n^{\ 2}}.$$

That field applies to the other d quark, the Lorentz force,

$$(\tfrac{1}{3}e)c(\frac{1}{3\pi}\frac{\mu_0}{4\pi}\frac{ec}{r_{\mathrm{n}}{}^2}) = \frac{1}{9\pi}\frac{\mu_0}{4\pi}\frac{e^2}{r_{\mathrm{n}}{}^2}c^2.$$

Multiplying the force by r_{n}, the magnetic repulsion energy is approximately

$$\frac{1}{9\pi}\frac{\mu_0}{4\pi}\frac{e^2}{r_{\mathrm{n}}}c^2 = \frac{1}{9\pi}\frac{1}{10^7}\frac{e^2}{r_{\mathrm{n}}}c^2.$$

6.2 Attractive Magnetic Energy between u and d quarks

$$-\frac{8}{27\pi}\frac{1}{10^7}\frac{e^2}{r_{\mathrm{n}}}c^2$$

<u>*Proof*</u>: The u quark generates the current

$$-\tfrac{2}{3}e\nu_{\mathrm{n}} = -\tfrac{2}{3}e\frac{c}{2\pi r_{\mathrm{n}}} = -\frac{ec}{3\pi r_{\mathrm{n}}},$$

which at distance $1.5r_{\mathrm{n}}$, has the magnetic field

$$\mu_0\frac{1}{2\pi(\tfrac{3}{2}r_{\mathrm{n}})}(-\frac{ec}{3\pi r_{\mathrm{n}}}) = -\frac{4}{9\pi}\frac{\mu_0}{4\pi}\frac{ec}{r_{\mathrm{n}}^2}.$$

That field applies to the d quark charge, the Lorentz force,

$$(\tfrac{1}{3}e)c(-\frac{4}{9\pi}\frac{\mu_0}{4\pi}\frac{ec}{r_{\mathrm{n}}^2}) = -\frac{4}{27\pi}\frac{\mu_0}{4\pi}\frac{e^2}{r_{\mathrm{n}}{}^2}c^2.$$

Multiplying the force by r_{n}, the magnetic attraction energy between the u quark, and each of the d quarks is approximately

$$-\frac{4}{27\pi}\frac{\mu_0}{4\pi}\frac{e^2}{r_{\mathrm{n}}}c^2 = -\frac{4}{27\pi}\frac{1}{10^7}\frac{e^2}{r_{\mathrm{n}}}c^2.$$

Thus, the magnetic attraction energy is approximately

$$-\frac{8}{27\pi}\frac{1}{10^7}\frac{e^2}{r_{\mathrm{n}}}c^2$$

6.3 The Neutron's Binding Magnetic Energy

$$U_{magnetic} \sim -\frac{5}{27\pi}\frac{1}{10^7}\frac{e^2}{r_{\mathrm{n}}}c^2$$

Proof: The sum of 6.1, and 6.2.

7.

Neutron's Rotation Energy

7.1 The Neutron's Rotation Energy

$$U_{\text{rotational}} = (m_{\text{d}} + \tfrac{1}{2}m_{\text{u}})c^2 \sim \frac{1}{160}m_{\text{n}}c^2$$

Proof: The d quarks with $\frac{1}{3}e$ and mass m_{d} have rotation energy

$$2(\tfrac{1}{2}m_{\text{d}}{r_{\text{p}}}^2{\omega_{\text{p}}}^2) = m_{\text{d}}c^2.$$

The u quark with $-\frac{2}{3}e$ and mass m_{u} has rotation energy

$$\tfrac{1}{2}m_{\text{u}}{r_{\text{p}}}^2{\omega_{\text{p}}}^2 = \tfrac{1}{2}m_{\text{u}}c^2.$$

The rotation energy of the neutron is

$$(m_{\text{d}} + \tfrac{1}{2}m_{\text{u}})c^2 \sim (\frac{1}{200} + \frac{1}{800})m_{\text{n}}c^2 = \frac{1}{160}m_{\text{n}}c^2$$

8.

Neutron's Energy and radius

8.1 The Neutron's Energy

$$\boxed{m_{\mathrm{n}}c^2 \sim (1+\tfrac{5}{3\pi})\tfrac{1}{9}10^{-7}e^2\frac{1}{r_{\mathrm{n}}}c^2}$$

Proof:

$$m_{\mathrm{n}}c^2 = \underbrace{U_{\mathrm{electric}}}_{\sim\frac{1}{9}10^{-7}c^2\frac{e^2}{r_{\mathrm{n}}}} + \underbrace{U_{\mathrm{magnetic}}}_{\sim\frac{5}{3\pi}\frac{1}{9}10^{-7}c^2\frac{1}{r_{\mathrm{n}}}e^2} + \underbrace{U_{\mathrm{rotational}}}_{\sim\frac{1}{160}m_{\mathrm{n}}c^2}$$

8.2 The Neutron's Mass

$$\boxed{m_{\mathrm{n}} \sim (1+\tfrac{5}{3\pi})\tfrac{1}{9}10^{-7}e^2\frac{1}{r_{\mathrm{n}}}}$$

8.3 The Neutron's Radius

$$\boxed{r_{\mathrm{n}} \sim (1+\tfrac{5}{3\pi})\tfrac{1}{9}10^{-7}e^2\frac{1}{m_{\mathrm{n}}}}$$

Substituting

$$e = -1.60217733\times 10^{-19}\mathrm{C},$$

$$m_{\mathrm{n}} = 1.6749286\times 10^{-27}\mathrm{Kg},$$

$$\boxed{r_{\mathrm{n}} \sim 2.60627576\times 10^{-19}\mathrm{m}}.$$

8.4 $$r_{\mathrm{p}} \sim 1.15 r_{\mathrm{n}}$$

$$r_{\mathrm{n}} \sim 0.87 r_{\mathrm{p}}$$

Proof:

$$\frac{r_{\mathrm{p}}}{r_{\mathrm{n}}} \sim \frac{(1+\frac{1}{\pi})\frac{4}{27}10^{-7}e^2\,\dfrac{1}{m_{\mathrm{p}}}}{(1+\frac{5}{3\pi})\frac{1}{9}10^{-7}e^2\,\dfrac{1}{m_{\mathrm{n}}}}$$

$$= \frac{(1+\frac{1}{\pi})4}{(1+\frac{5}{3\pi})3}\frac{m_{\mathrm{n}}}{m_{\mathrm{p}}}$$

$$= (1.148466247)(1.001378404)$$

$$= 1.150049298$$

9.

Neutron Radius-Energy Relation

9.1 *Neutron's Frequency, Mass, Energy are proportional to* $\frac{1}{r_{\text{n}}}$

$$\nu_{\text{n}} = \frac{c}{2\pi}\frac{1}{r_{\text{n}}},$$

$$m_{\text{n}} \sim (1 + \tfrac{5}{3\pi})\tfrac{1}{9}10^{-7}e^2\frac{1}{r_{\text{n}}}.$$

$$\boxed{m_{\text{n}}c^2 \sim (1 + \tfrac{5}{3\pi})\tfrac{1}{9}\frac{1}{10^7}e^2c^2\frac{1}{r_{\text{n}}}}$$, *the neutron's Radius-Energy Relation.*

The Neutron Radius-Energy Relation suggests that a heavier positively charged Baryon made of three quarks, is a neutron with a smaller radius.

10.

Neutron Spin

10.1 Neutron Spin by Quarks' Orbital Angular Momentum

$$2m_d cr_n + m_u cr_n \sim 1.551618488 \times 10^{-5}\hbar$$

Proof: $2m_d cr_n + m_u cr_n = (2m_d + m_u)cr_n$

$$= (2\tfrac{1}{200}m_n + \tfrac{1}{400}m_n)cr_n$$

$$\sim \tfrac{1}{80}m_n cr_n$$

Substituting $m_n r_n \sim (1 + \frac{5}{3\pi})\frac{1}{9}\frac{\mu_0}{4\pi}e^2$,

$$\sim \tfrac{1}{80}(1 + \tfrac{5}{3\pi})\tfrac{1}{9}\tfrac{\mu_0}{4\pi}e^2 c$$

Substituting $e^2\mu_0 c = 2h\alpha$, where $\alpha \approx \frac{1}{137}$,

$$= \tfrac{1}{80}(1 + \tfrac{5}{3\pi})\tfrac{1}{9}\tfrac{1}{4\pi}2\alpha h$$

$$= \tfrac{1}{80}(1 + \tfrac{5}{3\pi})\tfrac{1}{9}\alpha\hbar$$

$$\approx \tfrac{1}{80}(1 + \tfrac{5}{3\pi})\tfrac{1}{9}\tfrac{1}{137}\hbar$$

$$\approx 1.551618488 \times 10^{-5}\hbar$$

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