Electron’s Spin, Diffraction, and Radius

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Abstract The electron’s Spin is a negligible fraction of $\hbar$.

We present a ring model for the electron to approximate its spin, explain its diffraction, and approximate its radius.

We assume that an electron $e$ is composed of charged subelectrons moving along a circle of radius $r_e$ at light speed $c$, $\frac{c}{2\pi r_e} = \nu_e$ times per second. This associates with the electron a wave of length $\lambda_e = 2\pi r_e$.

The electron’s frequency, mass, and energy are inversely proportional to its radius.

$$\nu_e = \frac{c}{2\pi r_e},$$

$$m_e \sim (1 + \frac{1}{\pi}) \frac{4}{27} 10^{-7} e^2 \frac{1}{r_e},$$

$$m_e c^2 \sim (1 + \frac{1}{\pi}) \frac{4}{27} 10^{-7} e^2 c^2 \frac{1}{r_e}.$$ 

The approximate electron’s radius is

$$r_e \sim (1 + \frac{1}{\pi}) \frac{4}{27} 10^{-7} \frac{e^2}{m_e} = 5.5035842 \times 10^{-16} \text{m}.$$
It is $\frac{1}{5}$ the “Classical electron radius”, which is derived from an unlikely estimate.

The electron’s Radius-Energy Relation suggests that the muon $\mu$, and the taon $\tau$ are electrons with smaller radii.

$$r_{\mu} = \frac{m_e}{m_{\mu}} r_e \approx \frac{1}{207} r_e \sim 2.7 \times 10^{-18},$$

$$r_{\tau} = \frac{m_e}{m_{\tau}} r_e \approx \frac{1}{3477} r_e \sim 1.6 \times 10^{-19}.$$

The subelectrons harmonic motion explains electron’s diffraction.

The Electron Spin is of the order of $\sim 10^{-6} \hbar$


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Contents

Electron’s Spin is not $\frac{1}{2} \hbar$, or $-\frac{1}{2} \hbar$

1. The Abraham Lorentz Spherical Electron
2. Evidence for Subelectrons
3. Diffraction and De Broglie Wave
4. Electron Spin
5. Subelectrons’ Motion
6. The Electron Structure
7. Subelectron’s Mass
8. Electric Binding Energy
9. Magnetic Binding Energy
10. Electron’s Rotation Energy
11. The Electron Energy, and Radius
12. Electron Radius-Energy Relation
13. Electron’s Spin Angular Momentum

References
Electron’s Spin is not $\frac{1}{2}\hbar$, or $-\frac{1}{2}\hbar$

The postulate that the electron’s spin is $\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$, appears in textbooks, as a fact established by a theory that no one knows its details, and confirmed in experiments that never took place.

Some authors believe that Uhlenbeck and Goudsmit established that in theory.

Some authors believe that it was confirmed by experiments.

Some authors believe that it follows from Schrödinger Equation: That equation gives rise to the quantum numbers that help build the electronic shell model of the atom. But Schrödinger’s wave function does not lead to quantitative results.

Some believe that Dirac’s Equation for the electron’s wave function $\psi$ implies that the electron spin is $\frac{1}{2}\hbar$. But any constant $\times \psi$ solves the Dirac equation, and the normalization means, $\int |\text{constant} \times \psi| = 1$.

None of the believers seem to wonder how the electron’s spin angular momentum can be of the order of the electron’s orbital angular momentum in Bohr’s ground orbit,

$$m_env_Br_B = \hbar.$$

Moreover, this postulate violates special Relativity.
0.1 Postulating the Electron’s Spin to be $\frac{1}{2} \hbar$ requires revolution at speeds greater than light speed, violating Special Relativity.

Proof: A rigid Spherical electron with mass $m_e$ and radius $r_e$, that spins at speed $v_s$ has moment of inertia $\frac{2}{5} m_e r_e^2$, and Spin Angular Momentum

$$I\omega = \left(\frac{2}{5} m_e r_e^2\right)\left(\frac{v_s}{r_e}\right) = \frac{2}{5} m_e r_e v_s,$$

Postulating that the Spin is $\frac{1}{2} \hbar$,

$$\frac{2}{5} m_e r_e v_s = \frac{1}{2} \hbar,$$

$$v_s = \frac{5}{4} \frac{\hbar}{m_e r_e}$$

$$= \frac{5}{4} \frac{m_e v_B r_B}{m_e r_e},$$

$$= \frac{5}{4} \frac{v_B r_B}{r_e},$$

$$= \frac{5}{4} \frac{\alpha r_B}{r_e}, \quad \text{where } \alpha \approx \frac{1}{137},$$

Using $r_B \sim 5 \cdot 10^{-11} \text{m}$, and $r_e \sim 5 \cdot 10^{-16} \text{m}$, (obtained here)

$$v_{\text{spin}} = \frac{5}{4} \frac{1}{137} \frac{5 \cdot 10^{-11}}{5 \cdot 10^{-16}} c$$

$$\sim 912c \gg c.$$
1.

The Abraham-Lorentz Spherical Electron

In 1902-3, there were two competing theories of the electron: Lorentz’ theory of the electron, and Max Abraham’s theory. Both assumed that the electron is uniformly electrified sphere. Then, repulsion within the electron, between the electron charge uniformly spread on the sphere, will distort the sphere, to a shape where the electrical forces will be balanced by mechanical forces. Lorentz assumed that the repulsion contracts the electron in the direction of its motion by the factor $\gamma = \sqrt{1 - \beta^2}$, Abraham believed that the mechanical energy of the electron is negligible, the electron energy is purely electromagnetic, and there are no mechanical forces on the electron. Thus, Abraham assumed that the electron is a rigid sphere as it moves. The model’s purpose was to examine kinematics at speeds close to light speed, but both Abraham’s rigid sphere, and Lorentz contractible sphere are impossible models for the electron, and exhibit total ignorance of the fundamentals of electric charge.
Even the metallic sphere of a van de graph generator can be charged with repelling charges, only till the voltage produces a spark.

Without any material to adhere to, the same sign charges will repel each other to infinity.

Thus, the spherical electron of Abraham, Lorentz, and Poincare does not exist in our physical world.

Based on the Abraham Electron, The electron was endowed with Spin that was set as $\frac{\sqrt{3}}{2} \hbar$.

We will see that the Spin Angular momentum of the Abraham electron is a negligible fraction of $\frac{\sqrt{3}}{2} \hbar$. 
2.

Evidence for Subelectrons

In 1928, J. J. Thomson presented in [Thomson, p. 33], experimental evidence that the electron is a composite particle.

“...the properties of the electron recently discovered lead to the view that the electron...has itself a structure, being made up of smaller parts which carry charges of electricity.”

Later, Millikan presented in [Millikan, p. 161] evidence from his experiments for the existence of Subelectrons.

“...Ehrenhaft and Zerner even analyze our report on oil droplets and find that these also show in certain instances indications of sub-electrons, for they yield in these observers’ hands too low values of e, whether computed from the Brownian movement or from the law of fall.”

These statements were made long before it has been established that the proton, and the neutron are composed of subparticles with fractional charges.

The evidence for subelectrons had to lead to a planetary model for the electron. Just to balance the electric forces on them, the Subelectrons must be moving, and since they are not going anywhere, the motion is in a closed orbit.
The existence of fractional charges, and consequently, subelectrons, is now well-established. For instance, [Wagoner, pp. 541-5].
3. Diffraction and De Broglie Wave

Diffraction of electrons had to suggest harmonic motion of subelectrons within the electron. That harmonic motion manifests itself in a physical wave.

Without the subelectrons circulating within the boundaries of the electron, the diffraction of electrons remains a mystery.

De Broglie wave is based on the speculation that like the photon $\phi$ which is a particle with speed $c$, and wavelength

$$\lambda_\phi = \frac{c}{\nu_\phi} = \frac{hc}{\hbar\nu_\phi} = \frac{hc}{m_\phi c^2} = \frac{h}{m_\phi c},$$

any particle $p$ with speed $\nu_p$, has an associated longitudinal wavelength

$$\lambda_p = \frac{h}{m_p \nu_p}.$$

The diffraction of electrons, that must be due to harmonic motion of their subelectrons, was attributed to a wiggling electron.

Since it is impossible to visualize an electron wiggling along its path, a property called wave particle duality was invented.

Even De Broglie realized that his wave represents the uncertainty in the particle location, [Dan1], [de Broglie]
Thus, the denial of subelectrons eliminated the wave that underlies their harmonic motion, and the electron remained a puzzle as to whether it is a wave or a particle.
4.

Electron Spin

Currents on the surface of the electron had to be attributed to the motion of subelectrons. The same subelectrons that were proposed by Thomson, and Millikan.

Instead, the electron was endowed with a property called Spin, which means that the electron does not rotate, although it possess rotation...

1\textsuperscript{st} Contradiction,

\textit{there are currents,}

\textit{but no moving charges...}

2\textsuperscript{nd} Contradiction,

\textit{the electron is not rotating.}

\textit{It only has spin...}

3\textsuperscript{rd} Contradiction,

\textit{Spin uses units of Angular Momentum}

\textit{But it is not Spin Angular Momentum}

The creators of Spin claimed that electron’s rotation will violate Special Relativity.

For an electron with speed $v_e$, denote

$$\beta = \frac{v_e}{c}.$$
Then, Abraham Electron transverse mass is [Thomson2, page 258]

\[
m_{\text{Abraham}} = \frac{3}{4\beta^2} m_0 \left[ \frac{1 + \beta^2}{2\beta} \log \frac{1 + \beta}{1 - \beta} - 1 \right].
\]

Hence, according to Abraham,

\[
\frac{e}{m_0} = \frac{3}{4\beta^2} \left[ \frac{1 + \beta^2}{2\beta} \log \frac{1 + \beta}{1 - \beta} - 1 \right] m_{\text{Abraham}}.
\]

The Lorentz Electron mass is

\[
m_{\text{Lorentz}} = \frac{m_0}{\sqrt{1 - \beta^2}}.
\]

Hence, according to Lorentz,

\[
\frac{e}{m_0} = \frac{1}{\sqrt{1 - \beta^2}} \frac{e}{m_{\text{Lorentz}}}
\]

In 1909, Bucherer computed the quantity

\[
\frac{e}{m_0},
\]

with \(m_{\text{Abraham}}\), and with \(m_{\text{Lorentz}}\), [Richardson, p.239-242].

He found that for the Abraham’s electron \(\frac{e}{m_0}\) varied with the electron speed, while for the Lorentz electron \(\frac{e}{m_0}\) remained constant within the limits of the experimental error.

Consequently, Abraham’s electron theory lost to Lorentz electron theory, and in 1925, it was long discarded, together with its
assumption of a rigid spherical electron.
That is when Uhlenbeck and Goudsmit applied Abraham’s theory, [Arabatzis, p.230], to establish their Spin.
In 1925, Uhlenbeck and Goudsmit used Abraham’s theory to convince themselves that currents on the electron surface, will violate Special Relativity.
Clearly, Uhlenbeck and Goudsmit were unaware of Abraham’s incorrect formulas, and failed assumptions.
They did not know that

*Abraham’s Theory contradicts Special Relativity.*

Abraham’s electron transforms under Galilean transformation, while Special Relativity mandates Lorentz Transformations only. Moreover, as we have seen above

*Postulating the Electron’s Spin to be \( \frac{\sqrt{3}}{2} \hbar \)
requires revolution at speeds greater than light speed.

### 4.1 Abraham’s Electron Spin Angular Momentum \( \leq \frac{2}{5 \cdot 137} \hbar \)

*Proof:* A rigid Spherical electron with mass \( m_e \) and radius \( r_e \) that spins at speed \( v_s \) has moment of inertia \( \frac{2}{5} m_e r_e^2 \), and Spin Angular Momentum
\[
\frac{2}{5} m_e r_e^2 \frac{v_s}{r_e} = \frac{2}{5} m_e r_e v_s.
\]

From \( m_e c^2 = \frac{e^2}{4\pi \varepsilon_0 r_e} \), we have \( m_e r_e = \frac{e^2}{4\pi \varepsilon_0 c^2} \), and the Spin Angular Momentum is

\[
= \frac{2}{5} \frac{e^2}{4\pi \varepsilon_0 c^2} v_s.
\]

From \( \alpha = \frac{\mu_0 c e^2}{2\hbar} \), where \( \alpha \approx \frac{1}{137} \) is the fine-structure constant [Dan3], Substitute \( \frac{2\alpha h}{\mu_0 c} = e^2 \)

\[
= \frac{2}{5} \frac{1}{4\pi \varepsilon_0 c^2} \frac{2\alpha h}{\mu_0 c} v_s
\]

\[
= \frac{2}{5} \alpha \frac{v_s h}{c} \frac{h}{2\pi}
\]

\[
\leq \frac{2}{5} \alpha h, \quad \text{since } v_s \leq c.
\]

\[
\approx \frac{2}{5 \cdot 137} \hbar.
\]

4.2 In atomic system units, \( \hbar = 1 \), and the spin indicates only orientation of the revolution. \( \frac{1}{2} \), or \(-\frac{1}{2}\).

It is unclear what photon’s spin 1 may mean. To follow the electron’s example, there should be orientations 1, or −1
5. Subelectrons’ Motion

The Spin suggests a harmonic motion associated with the electron. A spinning sphere of energy is less likely than subelectrons. Then, the centripetal forces of repulsion will balance the Lorentz magnetic and electric forces of attraction, to yield a stable structure.

5.1 Closed Orbit

To stay within the electron boundaries,

*the subelectrons should have a closed orbit.*

5.2 Central Force

By [Routh, p. 274], a closed orbit results from a central force that is proportional to the inverse square of the distance,(such as the Coulomb electric force) or directly to the distance(such as the centripetal force).

*Subelectrons charges supply

*the electromagnetic force to close the orbit.*
5.3 Orbit Stability

By [Routh, p.280] Central Force orbits are stable. That is, they are bounded in a ring between two circles. The stability of the electron indicates such orbits.

5.4 Planar Motion

Since the electric force is inverse squared law force,

Subelectrons’ orbits will be in the same plane,
and not on a sphere

The plane of motion of the particle turns around to generate a sphere only under a non-inverse squared law force.

[Chandrasekhar, p. 195].
6. The Electron’s Structure

We’ll assume that the electron is a circular current vortex of radius $r_e$, composed of three subelectrons, charged particles that move along the circle of radius $r_e$, at light speed $c$, $\nu_e = \frac{c}{2\pi r_e}$ times per second.

Thus, the electron has an associated wave of length $\lambda_e = 2\pi r_e$.

6.1 The Subelectrons tangential speed is $c$

If the subelectrons are the source of subphotons, they have to circulate the ring at light speed, or else, we will be at loss to explain how they acquired that speed when they formed the photon. Thus, the Subelectrons, like the subphotons, are likely to be charged quanta of radiation.

We will assume that

*The Subelectrons tangential speed in their circular path is $c$.*

6.2 The Charges of the Subelectrons

The simplest choice of
Three particles with charge $\frac{1}{3}e$, and mass $\varepsilon$, cannot hold together. The three negative charges will repel each other electrically, as well as be repelled from the center by the centripetal forces on them.

To ensure attraction, and the electron charge $e$, we choose, similarly to the proton structure,

One with charge $-\frac{1}{3}e$, and mass $\varepsilon_1$,

and two with charge $\frac{2}{3}e$, and mass $\varepsilon_2$.

### 6.3 The Location of the Subelectrons

The symmetric location of the subelectrons at the vertices of an equilateral triangle will result in greater repulsion, and no electron.

In fact, the planetary model will not allow all three subelectrons be on the same circle.
To simplify the discussion, we will let the two equally charged subelectrons share the same orbit.

To temper the effect of the repulsion between the two negatively charged subelectrons, the distance between them has to be larger then the distance of either one of them from the third Subelectron. Therefore, the orbit of the third Subelectron will have a smaller radius. This means two current rings. One with radius $r_1$, and one with radius $r_2$,

Since the correct model is made of at least two current rings, the electron has no radius. What we mean by the electron radius, $r_e$, is a number between the two ring radii,

$$r_1 < r_e < r_2,$$

the order of the size of the two rings.

Indeed, each Subelectron has its own radius approximated by $r_e$. 

20
The radius of the proton, and the radius of the neutron have the same meaning.
7. Subelectrons’ Masses

In [Dan2], we approximated the subphotons’ masses from the balance of centripetal repulsion, and electromagnetic attraction. The electron’s structure that we described above, presents the unsolved three body problem, and prohibits that approximation. We shall attempt to extrapolate the values of the subelectrons’ masses from the similar cases of subphotons and Subprotons. In [Dan2], the subphoton mass was between 

\[ 7 \times 10^{-40} \quad \text{and} \quad 3 \times 10^{-39} \text{Kg}, \]

while the photon mass was the order of 

\[ 4 \times 10^{-36} \text{Kg}. \]

Thus, the ratio between a Subphoton’s mass, and a photon’s mass is between 

\[ 10^{-4} \quad \text{and} \quad 10^{-3}. \]

We will assume that 

*The electron’s structure mirrors the proton’s structure,*

and in particular,

*the ratios between the Subprotons and proton masses are*

*the ratios between the subelectrons and the electron masses*
The $d$ subproton has $\frac{1}{3}e$ charge and its mass $m_d$ is between $4.1$ and $5.8$ MeV, an average of $4.95$ MeV, [PDG] which is $\frac{4.95}{938}m_p \approx \frac{1}{189.5}m_p$.

We assume that the $-\frac{1}{3}e$ Subelectron has a mass

$$m_{\epsilon_1} \approx \frac{1}{200}m_e.$$  

Each of the $u$ subprotons has $-\frac{2}{3}e$ charge and mass $m_u$ between $0.35$ and $0.6$ $m_d$, an average of $0.475$ $m_d$, [PDG] which is $\frac{0.475}{938}m_p \approx \frac{1}{399}m_p$.

We assume that each of the $\frac{2}{3}e$ Subelectrons has a mass

$$m_{\epsilon_2} \approx \frac{1}{400}m_e.$$
8.

Binding Electric Energy

8.1 The Electron’s Binding Electric Energy

\[ U_{\text{electric}} \sim -\frac{1}{27\pi\varepsilon_0} \frac{e^2}{r_e} \]

\[ \sim -\frac{4}{27} 10^{-7} \frac{e^2}{r_e} \]

**Proof:**

\[ U_{\text{electric}} = \frac{1}{4\pi\varepsilon_0} \left\{ \frac{(\frac{2}{3}e)(\frac{2}{3}e)}{r_{\varepsilon_2,\varepsilon_2}} + 2 \frac{(-\frac{1}{3}e)(\frac{2}{3}e)}{r_{\varepsilon_2,\varepsilon_1}} \right\} \]

where

\[ r_e < r_{\varepsilon_1,\varepsilon_2} < r_e \sqrt{2} \sim 1.4r_e. \]

Approximating

\[ r_{\varepsilon_1,\varepsilon_2} \sim (1.2)r_e, \]
\[ r_{\varepsilon_2, \varepsilon_2} \sim 2r_e, \]

we have

\[
U_{electric} \sim \frac{1}{4\pi \varepsilon_0} \frac{e^2}{r_e} \left\{ \frac{4}{9} \frac{2}{2} - 2 \frac{2}{9} \frac{1}{(1.2)} \right\}
\]

\[
= -\frac{1}{27\pi \varepsilon_0} \frac{e^2}{r_e}
\]

\[= -\frac{4}{27} 10^{-7} c^2 \frac{e^2}{r_e}.\]
9. Binding Magnetic Energy

9.1 Magnetic Energy of Repulsion between the \( \varepsilon_2 \) Subelectrons

\[
\text{Proof: The charge } \frac{2}{3}e \text{ generates the current }
\]
\[
\frac{2}{3}e \nu_e = \frac{2}{3}e \frac{c}{2\pi r_e} = \frac{ec}{3\pi r_e},
\]
which at distance \( 2r_e \), has the magnetic field

\[
\mu_0 \frac{1}{2\pi(2r_e)} \left( \frac{ec}{3\pi r_e} \right) = \frac{1}{3\pi} \frac{\mu_0 ec}{4\pi r_e^2}.
\]
That field applies to the charge \( \frac{2}{3}e \), the Lorentz force,

\[
\left( \frac{2}{3}e \right)c \left( \frac{1}{3\pi} \frac{\mu_0 ec}{4\pi r_e^2} \right) = \frac{2}{9\pi} \frac{\mu_0 e^2}{4\pi r_e^2} c^2.
\]
Multiplying the force by \( r_e \), the magnetic repulsion energy is approximately

\[
\frac{2 \mu_0 }{9\pi 4\pi r_e} \frac{e^2}{c^2} = \frac{2}{9\pi 10^7} \frac{e^2}{r_e} c^2.
\]
9.2 Magnetic Energy of Attraction between the $\varepsilon_1$ and $\varepsilon_2$

Subelectrons

$$- \frac{10}{27\pi} \frac{1}{10^7} \frac{e^2}{r_c^2}$$

\textbf{Proof:} Each charge $\frac{2}{3}e$ generates the current

$$\frac{2}{3}e\nu_e = \frac{2}{3}e \frac{c}{2\pi r_e} = \frac{ec}{3\pi r_e},$$

which at distance $1.2r_e$, has the magnetic field

$$\mu_0 \frac{1}{2\pi(\frac{6}{5}r_e)} \left( \frac{ec}{3\pi r_e} \right) = \frac{5}{9\pi} \frac{\mu_0 ec}{4\pi r_e^2}.$$ 

That field applies to the charge $-\frac{1}{3}e$, the Lorentz force,

$$(-\frac{1}{3}e)c(\frac{5}{9\pi} \frac{\mu_0 ec}{4\pi r_e^2}) = -\frac{5}{27\pi} \frac{\mu_0 e^2}{4\pi r_e^2} c^2.$$ 

Multiplying the force by $r_e$, the magnetic attraction energy between each Subelectron $\varepsilon_2$, and the Subelectron $\varepsilon_1$ is approximately

$$-\frac{5}{27\pi} \frac{\mu_0 e^2}{4\pi r_e^2} c^2 = -\frac{5}{27\pi} \frac{1}{10^7} \frac{e^2}{r_e} c^2.$$ 

Thus, the magnetic attraction energy is approximately

$$- \frac{10}{27\pi} \frac{1}{10^7} \frac{e^2}{r_e^2} c^2.$$
9.3 The Electron’s Binding Magnetic Energy

\[ U_{\text{magnetic}} \sim -\frac{4}{27\pi} \frac{1}{10^7} \frac{e^2}{r_e} c^2 \]

\textit{Proof}: The sum of 8.1, and 8.2.
10.

Electron’s Rotation Energy

10.1 The Electron’s Rotation Energy

\[ U_{\text{rotational}} = (m_{e_1} + 2m_{e_2})c^2 = \frac{1}{100} m_e c^2 \]

**Proof:** The \(-\frac{1}{3}e\) charge with mass \(m_{e_1}\) has rotation energy

\[ m_{e_1}r_e^2\omega_e^2 = m_{e_1} c^2. \]

The \(\frac{2}{3}e\) charges with masses \(m_{e_2}\) have rotation energy

\[ 2m_{e_2}r_e^2\omega_e^2 = 2m_{e_2} c^2. \]

The rotation energy of the subelectrons is

\[ (m_{e_1} + 2m_{e_2})c^2 \approx \left( \frac{1}{200} + 2 \cdot \frac{1}{400} \right)m_e c^2 = \frac{1}{100} m_e c^2 \]
11. Electron’s Energy and radius

11.1 The Electron’s Energy

\[ m_e c^2 \sim (1 + \frac{1}{\pi}) \frac{4}{27} 10^{-7} e^2 \frac{1}{r_e} \]

**Proof:**

\[ m_e c^2 = U_{\text{electric}} + U_{\text{magnetic}} + U_{\text{rotational}} \]

\[ \sim \frac{4}{27} 10^{-7} e^2 \frac{e^2}{r_e} \sim \frac{4}{\pi 27} 10^{-7} e^2 \frac{1}{r_e} \sim \frac{1}{100} m_e c^2 \]

11.2 The Electron’s Mass

\[ m_e \sim (1 + \frac{1}{\pi}) \frac{4}{27} 10^{-7} e^2 \frac{1}{r_e} \]

11.3 The Electron’s Radius

\[ r_e \sim (1 + \frac{1}{\pi}) \frac{4}{27} 10^{-7} e^2 \frac{1}{m_e} \]

Substituting

\[ e = -1.60217733 \times 10^{-19} \text{C}, \]

\[ m_e = 9.1093897 \times 10^{-31} \text{Kg}, \]

\[ r_e \sim 5.5035842 \times 10^{-16} \text{m}. \]
11.4 the *Classical Electron Radius is five times larger*

\[ 10^{-7} \frac{e^2}{m_e} \approx 2.8179091 \times 10^{-15} \text{ m} \sim 5r_e \]

This follows from the unlikely assumption that the electron’s energy, \( m_e c^2 \), equals the electric binding energy, \( \frac{e^2}{4\pi\varepsilon_0 r_e} \).

Then, the classical electron radius is

\[ \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = \frac{\mu_0 e^2}{4\pi m_e} = 10^{-7} \frac{e^2}{m_e} \approx 2.81794091 \times 10^{-15} \text{ m}. \]

11.5 *Scattering of electrons indicates* \( r_e \sim 0.5 \times 10^{-16} \)

For instance, [Beiser, p.229]
12.

Electron Radius-Energy Relation

12.1 Electron’s Frequency, Mass, Energy are proportional to \( \frac{1}{r_e} \)

\[
\nu_e = \frac{c}{2\pi r_e},
\]

\[
m_e \sim (1 + \frac{1}{\pi}) \frac{4}{27} 10^{-7} e^2 \frac{1}{r_e}.
\]

\[
m_e c^2 \sim (1 + \frac{1}{\pi}) \frac{4}{27} \frac{1}{10^7} e^2 c^2 \frac{1}{r_e},
\]

the electron’s Radius-Energy Relation.

The Electron Radius-Energy Relation suggests that \( \mu \), and \( \tau \)
are electrons with smaller radius.

12.2

\[
r_\mu \sim \frac{1}{207} r_e \sim 2.7 \times 10^{-18},
\]

\[
r_\tau \approx \frac{1}{3477} r_e \sim 1.6 \times 10^{-19}
\]

Proof: \( r_\mu = \frac{m_e}{m_\mu} r_e \approx \frac{1}{207} r_e \sim \frac{1}{207} 5.5035842 \times 10^{-16} m \approx 2.65 \times 10^{-18} m \)

\[
r_\tau = \frac{m_e}{m_\tau} r_e \approx \frac{1}{3477} r_e \sim \frac{1}{3477} 5.5035842 \times 10^{-16} m \approx 1.58 \times 10^{-19} m
\]
13.

Electron Spin Angular Momentum

13.1 Electron Spin by Subelectrons’ Angular Momentum

\[ m_{e_1}c r_e + 2m_{e_2}c r_e \sim 10^{-6}\hbar \]

**Proof:**

\[ m_{e_1}c r_e + 2m_{e_2}c r_e = (m_{e_1} + 2m_{e_2})c r_e \]

\[ \sim \frac{1}{100} m_e c r_e \]

Since \( m_e \sim (1 + \frac{1}{\pi}) \frac{4}{27} \frac{\mu_0}{4\pi} e^2 \frac{1}{r_e} \),

\[ \sim \frac{1}{100} (1 + \frac{1}{\pi}) \frac{4}{27} \frac{\mu_0}{4\pi} e^2 c \]

Substituting \( e^2 \mu_0 c = 2\hbar \alpha \), where \( \alpha \approx \frac{1}{137} \),

\[ = \frac{1}{100} (1 + \frac{1}{\pi}) \frac{4}{27} \frac{1}{4\pi} \alpha \hbar \]

\[ \approx \frac{1}{100} (1 + \frac{1}{\pi}) \frac{1}{27\pi} \frac{1}{137} \hbar \]

\[ \approx 1.13 \times 10^{-6} \hbar \]
References


[PDG] Particle Data Group, at LBNL, and CERN

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