GAUGE-INSTITUTE

Variational Calculus

Infinitesimal Variational Calculus   H. Vic Dannon

Abstract: We define Variations as Infinitesimal Functions, and investigate their properties.

Variational Calculus is based on Euler's differential equation, Hence on differentials, and as such it is infinitesimal. But it is derived in an integral context, which imposes unnecessary global restrictions, requires unnecessary arguments, and is ill-equipped to resolve its over hundred-years-old problems.

Heuristic conception of Infinitesimals resulted in Ill-defined Variations. Texts of Variational Calculus apply variations inefficiently, or avoids them altogether.

We develop Variational Calculus in Infinitesimal context to

eliminate unnecessary arguments,

to eliminate restrictive global assumptions

and to give meaning to the poorly understood, and thus, ill-defined variation δy, and variational derivative

$$\frac{\delta F}{\delta y}.$$

We supply a correct proof to the unresolved to date Weierstrass Sufficient condition for a Minimum, and prove its equivalence to Legendre Sufficient Condition for a Minimum. In non-infinitesimal context, the Legendre Sufficient Condition is misunderstood as being insufficient.

Infinitesimal Elasticity, Variational Principles, and Castigliano Theorems   H. Vic Dannon   and   Vadim Komkov

Abstract   The Equilibrium Equations of Elasticity are never derived as Euler's Variational Equations. A Hamiltonian is not identified in Elasticity, and Hamilton's Equations are absent from Elasticity Theory.

Elasticity Theory includes Variational Principles known as Castigliano Theorems. Do the Castigliano Theorems of Elasticity Theory stem from Variational Principles that are unknown in Mechanics?

We show that Castigliano's Theorems are equivalent to Euler's Variational Equations, and to Hamilton's Variational Equations.

To that end we had to set up the theory of elasticity with infinitesimals, and give proofs to fundamental theorems of elasticity we found unproven.

The incomplete analysis that left Elasticity Theorems unproven, was not helped by concern about tensors in several Elasticity texts. For the sake of clarity, we avoided proofs with tensor indices here.