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Gauge Institute Journal

Riemann Hypothesis

Riemann Hypothesis Proof **H. Vic Dannon.**

Abstract: In his 1859 Zeta paper, Riemann obtained a formula for the count of the primes up to a given number. Riemann's formula has four terms. But only the first and the third terms have non-negligible values. The first is the dominant term, and can be computed precisely. The third is smaller and depends on the provision that all the zeros of the Zeta function in $0 < x < 1$ are on the line $x = 1/2$.

This provision became known as the Riemann Hypothesis, but it was never hypothesized by Riemann, nor was it used by him. Not seeing an easy proof for it, Riemann used only the first term of his formula and obtained an approximation far superior to Gauss for the count of the primes.

In 1935, and 1936, Titchmarsh and Comrie [Hasel, p. xii] confirmed by computations that the first 1042 zeros of Zeta in $0 < x < 1$ with

$$0 < \operatorname{Im} z < 1468$$

lie exactly on the line $x = 1/2$. Following that, Touring (1953) extended the upper limit to $\operatorname{Im} z = 1540$, and Lehmer (1956) confirmed that the first 25,000 zeros of Zeta in $0 < x < 1$ with

$$0 < \operatorname{Im} z < 21{,}944$$

lie on the line $x = 1/2$.

By 2002, the first 50 billion Zeta zeros have been located on the line $x = 1/2$, and in a 2008 meeting a far greater number was mentioned.

The fact that the first 50 billion zeros are on $x = 1/2$, does not constitute a proof for the infinitely many zeros in the infinite area of the strip $0 < x < 1$.

But expecting massive endless computations to disprove the Riemann Hypothesis by finding a zero off the line $x = 1/2$, is statistically implausible.

In fact, statistical tests indicate that the Riemann Hypothesis holds with extremely high statistical certainty. In 2008, we applied a Chi-Squared Goodness-of-Fit-Test to the Riemann formula for the count of the primes, and confirmed that the Riemann Hypothesis holds with certainty that is limited only by the software [Dan2].

In sections 1 to 4, we present Riemann's function $\xi(z)$, and known facts about its zeros.

In section 5, we recall that $\left|\xi(x - iy)\right| = \left|\xi(x + iy)\right|$. This fact serves as a key result for the proof.

In 8.2, we present a key result that follows from the Weierstrass factorization of $\xi(z)$, and in 9.1 a key result due to Hadamard.

The last key result due to Titchmarsh, is presented in section 10.

There, we prove that in $0 < x < 1$, any factor of $\xi(z)$ must have its zero on the line $x = 1/2$.

Since $\xi(z)$ has in $0 < x < 1$ the same zeros as $\zeta(z)$, this proves that all the zeros of $\zeta(z)$ in $0 < x < 1$, are on $x = 1/2$.

Riemann Zeta Function: the Riemann Hypothesis Origin, the Factorization Error, and the Count of the Primes **H. Vic Dannon.**

Abstract: Riemann's 1859 Zeta paper defines the Zeta function and uses its properties to approximate the count of prime numbers up to a number t, and the density of the primes at the number t

The few pages paper outlines a book that was never written by Riemann. The paper sums up Riemann's results on the Zeta function, and on the count of the prime numbers, with a few connecting words, and no proofs/explanations.

Attempts to write the book were made by Titchmarsh, and by Edwards, but none followed through Riemann's writing.

Only by staying faithful to Riemann's development of the Zeta function we can

Get to the origin and meaning of the Riemann Hypothesis,

Correct the factorization error,

Apply Riemann's Formula for the Count of the Primes,

Acquire the tools needed for the Hypothesis Proof

This is a detailed account of the Riemann Zeta paper.

Chi-Squared Goodness-of-fit test of the Riemann Hypothesis **H. Vic Dannon**

Abstract The computing of the number of the primes up to a number x, started with Gauss who approximated it by the logarithmic integral.

In his 1859 Zeta paper, Riemann obtained an intricate formula that uses all the zeros of the Zeta function on the line $x = \frac{1}{2}$, to solve the problem completely, provided that all the zeros of the Zeta function in $0 < x < 1$, are on the line $x = \frac{1}{2}$.

The Riemann formula has four terms. But only the first and the third of these terms have non-negligible values. The first is a dominant term that can be computed precisely. The third term is smaller and depends on the provision regarding the zeros of the Zeta function.

This provision became known as the Riemann Hypothesis, but it was never hypothesized by Riemann. Not seeing an easy proof for it, Riemann used only the first term of his formula and obtained an approximation far superior to Gauss for the count of the primes. This term is known as the Riemann Approximation term.

We shall refer to the third term that depends on the Hypothesis, and was neglected since Riemann, as the Riemann-Hypothesis-term.

Here, we will confirm that the Hypothesis holds with high statistical certainty, and the Riemann formula can be used with that high certainty.

The uncertainty we found was way under 10^{-16}. Indeed, the software produced certainty of 1.0. Consequently, we did not try to use many more Zeta zeros to obtain a lower uncertainty.

Our computations indicate that if not for the limitations of the software, the Hypothesis can be confirmed to any degree of certainty.

Riemann's Formula for Count of the Primes: The effect of the Hypothesis Series **H. Vic Dannon**

Abstract How many prime numbers up to 10?

2, 3, 5, and 7 are the four prime numbers up to 10.

Lehmer have listed the prime numbers up to 10,006,721, and we can count with his list.

A supercomputer was used to count how many primes are up to a Billion times a Trillion.

But isn't there some formula to replace counting?

The computing of the number of the primes up to a number x, started with Gauss who approximated it by the logarithmic integral.

In his 1859 Zeta paper, Riemann obtained a formula for the count of the primes, that uses all the zeros of the Zeta function on the line $x = \frac{1}{2}$, to solve the problem completely, provided that all the zeros of the Zeta function in $0 < x < 1$, are on the line $x = \frac{1}{2}$.

The Riemann formula has four terms. But only the first and the third of these terms have non-negligible values. The first is a dominant term that can be computed precisely. The third term is smaller and depends on the provision regarding the zeros of the Zeta function.

This provision became known as the Riemann Hypothesis, but it was never hypothesized by Riemann. Not seeing an easy proof for it, Riemann used only the first term of his formula, and obtained an approximation far superior to Gauss for the count of the primes. Thus, the first term in Riemann's Formula is known as the Riemann Approximation term.

We shall refer to the third term that depends on the Hypothesis, and was neglected since Riemann, as the Riemann-Hypothesis-Series.

It is obtained provided that all the zeros of the Zeta function in the strip $0 < x < 1$, lie on the line $x = \frac{1}{2}$.

Each term of the Hypothesis Series is evaluated at a zero of the Zeta function on the line $x = \frac{1}{2}$. Since there are infinitely many such zeros, the Series has infinitely many terms

Riemann wondered about the effect of the Hypothesis series, but left it out of his approximation formula [Riem].

We have proved in [Dan, Theorem 14.1] that

> *Riemann's Formula for the Count of the Primes is valid with Riemann Hypothesis Series, with uncertainty under* 10^{-16}.

This allows us to use Riemann's formula for the count of the primes with great certainty.

Actually, our computations indicated that if not for the limitations of the software, Riemann's Formula can be confirmed to any degree of certainty.

Here, we approximate $F(10\,\hat{}\,7)$ by Riemann's Formula, and compare it to Lehmer's count. We confirm Riemann's suspicion that *the Hypothesis Series convergence is unpredictable.*

Even with many Hypothesis Series terms, the convergence is slow. The Cesaro-Arithmetic-Means converge more smoothly but not faster.

The slow convergence of the Hypothesis series, forces us to use statistics to estimate $F(10\,\hat{}\,21)$.