

The Entropy Constant k , The Radiation Constant h , The Quantized 2nd Law, The Quantized Temperature, &The Temperature Quantum

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Abstract Planck's Electromagnetic Radiation laws involve the entropy constant, k , and the radiation constant, h . Both Planck Constants¹.

But no account of the Radiation laws after Planck tells where either one of these two constants comes from.

To date, no one was able to follow Planck's confused writings about his two constants, and about finding the discrete probability distribution of the quantized energy, and the quantized entropy.

¹ While Planck articulates that the Entropy Constant k is his, for apparent lack of confidence, he attributes his own entropy equation $S = k \log W$ to Boltzmann, Thus, k became known as Boltzmann Constant. But it was defined and determined by Planck.

Instead of seeking maximal entropy depending on the probabilities of quantized radiation energy, Planck sought maximal entropy depending on kinetic energy. This leads to the continuous Maxwell's distribution, which does not represent quantized energy, and quantized entropy.

Planck missed the quantum nature of the Radiation Entropy. Its distribution is discrete, and any of Planck's derivations that assume continuous entropy distribution are erroneous.

Furthermore, Planck missed the quantum nature of the Temperature. Its distribution is discrete, and there is a Temperature Quantum.

Assuming that quantum energies are distributed with discrete probability, we obtain here the

Quantized 2nd Law of Thermodynamics:

The Probability Distribution of Quantized Energy, that maximizes the Quantized Entropy is

$$p(nh\nu) = (1 - e^{-n\frac{h\nu}{kT}})e^{-n\frac{h\nu}{kT}}.$$

This is what Planck failed to state, and derive in all his Black Body Radiation publications.

We conclude the clarifying of Planck's claims with the computation of h , and k .

$$h = 6.626\ 083\ 463 \cdot 10^{-34} \text{ J sec}$$

$$k = 1.38\ 065\ 933 \cdot 10^{-23} \text{ J/degree}$$

Then, we establish the crucial Physics that went unnoticed for over hundred years: that the Temperature is Quantized, and has a Temperature Quantum.

Under probabilistic entropy, $s = k \log W$,

$$T = \frac{W}{k} \frac{\partial u}{\partial W}$$

Under the Quantized 2nd Law, $p(nh\nu) = (1 - e^{-n\frac{h\nu}{kT}})e^{-n\frac{h\nu}{kT}}$

$$T = \frac{h\nu}{k} \frac{1}{\log(1 + \frac{h\nu}{u_\nu})}$$

Consequently, the Temperature is Quantized, and the Temperature Quantum is

$$\delta T = \frac{h\nu}{k \log 2} \text{ degrees Kelvin}$$

The Quantized Temperature is

$$T \approx \frac{u_\nu}{k}$$

If $u_\nu = Mh\nu$, then

$$T \approx M \frac{h\nu}{k}$$

And the Entropy Quantum is

$$\delta s = \frac{k}{M}$$

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1.

The Electromagnetic Radiation is Quantized

In 1893, Wien reasoned that the electromagnetic radiation density per unit volume at frequencies between ν , and $\nu + \delta\nu$ of an ideal radiator (black body) is

$$u(\nu, T)d\nu = \frac{\nu^3}{c^4} \Phi\left(\frac{\nu}{T}\right)d\nu.$$

Radiation measurements indicated that Φ should be of the form

$$\Phi = C_1 e^{-\frac{C_2\nu}{T}},$$

with some constants C_1 , and C_2 .

That is,

$$u(\nu, T)d\nu = \frac{\nu^3}{c^4} C_1 e^{-\frac{C_2\nu}{T}} d\nu$$

Assuming that at frequencies between

$$\nu, \text{ and } \nu + \delta\nu,$$

a unit volume has

$$N_\nu$$

radiating oscillators, each with average radiation energy

$$\overline{\epsilon_\nu},$$

we have

$$u(\nu, T)d\nu = N_\nu \overline{\epsilon_\nu} d\nu.$$

Rayleigh computed N_ν as the number of standing waves (per unit volume) in the modes between ν , and $\nu + \delta\nu$,

$$N_\nu d\nu = \frac{8\pi}{c^3} \nu^2 d\nu.$$

It is assumed that there is constant

$$k$$

named after Boltzmann so that at frequency ν , the radiation energy ε_ν of an oscillator, is distributed with Probability Density Function

$$f(\varepsilon_\nu) = \alpha e^{-\frac{\varepsilon_\nu}{kT}},$$

again, named after Boltzmann...²

Rayleigh assumed that the distribution of $f(\varepsilon_\nu)$ was continuous.

Then, the possible probabilities sum up to 1.

$$\begin{aligned} 1 &= \int_{\varepsilon_\nu=0}^{\varepsilon_\nu=\infty} f(\varepsilon_\nu) d\varepsilon_\nu \\ &= \int_{\varepsilon_\nu=0}^{\varepsilon_\nu=\infty} \alpha e^{-\frac{\varepsilon_\nu}{kT}} d\varepsilon_\nu \\ &= \alpha kT \int_{\varepsilon_\nu=0}^{\varepsilon_\nu=\infty} e^{-\frac{\varepsilon_\nu}{kT}} d\left(\frac{\varepsilon_\nu}{kT}\right) \end{aligned}$$

Since $\int_{\xi=0}^{\xi=\infty} e^{-\xi} d\xi = 1,$

² Boltzmann never had it. Planck guessed it but failed to drive it. No textbook has the derivation. We supply the derivation here in chapters 3,5, and 6.

$$= \alpha kT$$

$$\alpha = \frac{1}{kT}$$

$$f(\varepsilon_\nu) = \frac{1}{kT} e^{-\frac{\varepsilon_\nu}{kT}}$$

Thus,

$$\begin{aligned} \overline{\varepsilon_\nu} &= \int_{\varepsilon_\nu=0}^{\varepsilon_\nu=\infty} \varepsilon_\nu f(\varepsilon_\nu) d\varepsilon_\nu \\ &= \int_{\varepsilon_\nu=0}^{\varepsilon_\nu=\infty} \varepsilon_\nu \frac{1}{kT} e^{-\frac{\varepsilon_\nu}{kT}} d\varepsilon_\nu \\ &= kT \int_{\varepsilon_\nu=0}^{\varepsilon_\nu=\infty} \frac{\varepsilon_\nu}{kT} e^{-\frac{\varepsilon_\nu}{kT}} d\left(\frac{\varepsilon_\nu}{kT}\right) \end{aligned}$$

Since $\int_{\xi=0}^{\xi=\infty} \xi e^{-\xi} d\xi = 1,$

$$= kT$$

Since $N_\nu = \frac{8\pi}{c^3} \nu^2$, Rayleigh obtained

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} kT.$$

This disagrees with Wien's law, and with the measurements that

$$u(\nu, T) \rightarrow 0, \text{ for large } \nu.$$

Wien's approximated law requires that $\overline{\varepsilon_\nu}$ will depend on ν .

To avoid the Rayleigh-Jeans failure, we must assume that ε_ν may take only discrete values,

multiples of $h\nu$, where h = the Radiation Energy Constant

$$\begin{aligned} \varepsilon_{\nu,0} &= 0 \\ \varepsilon_{\nu,1} &= h\nu, \\ \varepsilon_{\nu,2} &= 2h\nu, \\ \varepsilon_{\nu,3} &= 3h\nu, \\ &\dots\dots\dots \\ \varepsilon_{\nu,n} &= nh\nu, \\ &\dots\dots\dots \end{aligned}$$

Then, the probability distribution is discrete,

$$\begin{aligned} f(\varepsilon_{\nu,n}) &= \alpha e^{-\frac{nh\nu}{kT}} \\ 1 &= \alpha + \alpha e^{-\frac{h\nu}{kT}} + \alpha e^{-\frac{2h\nu}{kT}} + \alpha e^{-\frac{3h\nu}{kT}} + \alpha e^{-\frac{4h\nu}{kT}} + \dots \\ &= \alpha(1 + e^{-\frac{h\nu}{kT}} + e^{-\frac{2h\nu}{kT}} + e^{-\frac{3h\nu}{kT}} + e^{-\frac{4h\nu}{kT}} + \dots) \\ &= \alpha \frac{1}{1 - e^{-\frac{h\nu}{kT}}} \end{aligned}$$

$$1 - e^{-\frac{h\nu}{kT}} = \alpha$$

$$f(\varepsilon_{\nu,n}) = (1 - e^{-\frac{h\nu}{kT}}) e^{-\frac{nh\nu}{kT}}$$

$$\begin{aligned} \overline{\varepsilon_\nu} &= \varepsilon_{\nu,0}f(\varepsilon_{\nu,0}) + \varepsilon_{\nu,1}f(\varepsilon_{\nu,1}) + \varepsilon_{\nu,2}f(\varepsilon_{\nu,2}) + \varepsilon_{\nu,3}f(\varepsilon_{\nu,3}) + \varepsilon_{\nu,4}f(\varepsilon_{\nu,4}) + \dots \\ &= h\nu(1 - e^{-\frac{h\nu}{kT}})e^{-\frac{h\nu}{kT}} + 2h\nu(1 - e^{-\frac{h\nu}{kT}})e^{-2\frac{h\nu}{kT}} + 3h\nu(1 - e^{-\frac{h\nu}{kT}})e^{-3\frac{h\nu}{kT}} + \dots \end{aligned}$$

Put $\xi = \frac{1}{kT}$

$$= (1 - e^{-h\nu\xi}) \{ h\nu e^{-h\nu\xi} + 2h\nu e^{-2h\nu\xi} + 3h\nu e^{-3h\nu\xi} + 4h\nu e^{-4h\nu\xi} + \dots \}$$

$$\begin{aligned}
&= (1 - e^{-h\nu\xi}) \left\{ -\frac{d}{d\xi}(e^{-h\nu\xi}) - \frac{d}{d\xi}e^{-2h\nu\xi} - \frac{d}{d\xi}e^{-3h\nu\xi} + \dots \right\} \\
&= (1 - e^{-h\nu\xi}) \frac{d}{d\xi} \left\{ -e^{-h\nu\xi} - e^{-2h\nu\xi} - e^{-3h\nu\xi} + \dots \right\} \\
&= (1 - e^{-h\nu\xi}) \frac{d}{d\xi} (-e^{-h\nu\xi} [1 + e^{-h\nu\xi} + e^{-2h\nu\xi} + \dots]) \\
&= (1 - e^{-h\nu\xi}) \frac{d}{d\xi} \frac{-e^{-h\nu\xi}}{1 - e^{-h\nu\xi}} \\
&= (1 - e^{-h\nu\xi}) \frac{d}{d\xi} \frac{-1}{e^{h\nu\xi} - 1} \\
&= (1 - e^{-h\nu\xi}) \frac{e^{h\nu\xi} h\nu}{(e^{h\nu\xi} - 1)^2} \\
&= \frac{h\nu}{e^{h\nu\xi} - 1} \\
&= \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}.
\end{aligned}$$

By Planck's 1901 radiation law, the radiation density per unit volume at frequency ν and temperature T is

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}.$$

But where does the k come from? how does the k get into the probability distribution

$$f(\varepsilon_{\nu,n}) = (1 - e^{-\frac{h\nu}{kT}}) e^{-\frac{nh\nu}{kT}}.$$

Does k play for entropy the role that h plays for radiation energy?

2.

The Electromagnetic Entropy is Quantized

The average radiation energy per mode between ν , and $\nu + d\nu$ is

$$u_\nu = \frac{h\nu}{\frac{h\nu}{e^{kT}} - 1}$$

Solving for T ,

$$\frac{u_\nu}{h\nu} = \frac{1}{\frac{h\nu}{e^{kT}} - 1},$$

$$\frac{u_\nu}{h\nu} (e^{\frac{h\nu}{kT}} - 1) = 1,$$

$$\frac{u_\nu}{h\nu} e^{\frac{h\nu}{kT}} = \frac{u_\nu}{h\nu} + 1,$$

$$\frac{h\nu}{e^{kT}} = \frac{\frac{u_\nu}{h\nu} + 1}{\frac{u_\nu}{h\nu}},$$

$$\frac{h\nu}{kT} = \log \frac{\frac{u_\nu}{h\nu} + 1}{\frac{u_\nu}{h\nu}},$$

$$\frac{1}{T} = \frac{k}{h\nu} \log \frac{\frac{u_\nu}{h\nu} + 1}{\frac{u_\nu}{h\nu}},$$

$$\frac{1}{T} = \frac{k}{h\nu} \left\{ \log\left(\frac{u_\nu}{h\nu} + 1\right) - \log\left(\frac{u_\nu}{h\nu}\right) \right\}$$

Since $\frac{\partial s}{\partial u} = \frac{1}{T}$,

$$\frac{\partial s_\nu}{\partial u_\nu} = \frac{k}{\varepsilon} \left\{ \log\left(\frac{u_\nu}{h\nu} + 1\right) - \log\left(\frac{u_\nu}{h\nu}\right) \right\}$$

Integrating with respect to u_ν , the average entropy per mode is

$$\begin{aligned} s_\nu &= k \left\{ \left(\frac{u_\nu}{h\nu} + 1\right) \log\left(\frac{u_\nu}{h\nu} + 1\right) - \left(\frac{u_\nu}{h\nu}\right) \log\left(\frac{u_\nu}{h\nu}\right) \right\} \\ &= k \left\{ \left(\frac{u_\nu}{h\nu}\right) \log\left(\frac{u_\nu}{h\nu} + 1\right) - \left(\frac{u_\nu}{h\nu}\right) \log\left(\frac{u_\nu}{h\nu}\right) + \log\left(\frac{u_\nu}{h\nu} + 1\right) \right\} \\ &= k \left\{ \left(\frac{u_\nu}{h\nu}\right) \log \frac{\frac{u_\nu}{h\nu} + 1}{\frac{u_\nu}{h\nu}} + \log\left(\frac{u_\nu}{h\nu} + 1\right) \right\} \end{aligned}$$

Substituting from above

$$\log \frac{\frac{u_\nu}{h\nu} + 1}{\frac{u_\nu}{h\nu}} = \frac{h\nu}{kT}.$$

Also, from $u_\nu = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$

$$\frac{u_\nu}{h\nu} = \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

$$\frac{u_\nu}{h\nu} + 1 = \frac{e^{\frac{h\nu}{kT}}}{e^{\frac{h\nu}{kT}} - 1},$$

Therefore,

$$\begin{aligned}
 s_\nu &= k \left\{ \frac{1}{\frac{h\nu}{e^{kT}} - 1} \frac{h\nu}{kT} + \log \frac{\frac{h\nu}{e^{kT}}}{\frac{h\nu}{e^{kT}} - 1} \right\} \\
 &= k \left\{ \frac{1}{\frac{h\nu}{e^{kT}} - 1} \frac{h\nu}{kT} + \frac{h\nu}{kT} + \log \frac{1}{\frac{h\nu}{e^{kT}} - 1} \right\} \\
 &= \frac{h\nu}{T} \left(\frac{1}{\frac{h\nu}{e^{kT}} - 1} + 1 \right) + k \log \frac{1}{\frac{h\nu}{e^{kT}} - 1}.
 \end{aligned}$$

This is the average entropy per mode in $[\nu, \nu + d\nu]$.

There are $\frac{8\pi\nu^2}{c^3}$ modes in $[\nu, \nu + d\nu]$, and the average entropy per unit volume in $[\nu, \nu + d\nu]$ is

$$\frac{8\pi\nu^2}{c^3} s_\nu(T) = \frac{8\pi\nu^2}{c^3} \left\{ \frac{h\nu}{T} \left(\frac{1}{\frac{h\nu}{e^{kT}} - 1} + 1 \right) + k \log \frac{1}{\frac{h\nu}{e^{kT}} - 1} \right\}.$$

The electromagnetic entropy is discrete, and its quantum is

$$\sigma_\nu = \frac{\varepsilon_\nu}{T} = \frac{h\nu}{T}$$

We shall see later³ that the temperature T depends uniquely on the radiation frequency ν .

$$T = T(\nu)$$

Hence the entropy quantum is uniquely determined by ν .

The role of the constant k remains unclear.

³ chapter 9

We shall call

k the entropy constant⁴

although k does not play for entropy a role similar to the one h plays for energy.

In fact, h plays that role for both the energy, and the entropy:

Radiation Energy is made of photons, which are units of $h\nu$.

Electromagnetic Entropy is made of photons/degree which are

units of $\frac{h\nu}{T(\nu)} = h \frac{\nu}{T(\nu)}$.

⁴ because it appears in Planck's Entropy Equation $S = k \log W$

3.

The Probabilistic Entropy

Planck attempted to derive the probability distribution for the quantized energy, and quantized entropy by applying Boltzmann Probabilistic approach to Entropy.

We assume

N Molecules

distributed in a

Volume V .

$$N_1 \text{ in volume } \delta V_1, \text{ with probability } p_1 = \frac{N_1}{N}$$

$$N_2 \text{ in volume } \delta V_2, \text{ with probability } p_2 = \frac{N_2}{N}$$

$$N_3 \text{ in volume } \delta V_3, \text{ with probability } p_3 = \frac{N_3}{N}$$

.....

$$N = N_1 + N_2 + N_3 + \dots,$$

$$1 = p_1 + p_2 + p_3 + \dots$$

The state of the molecule system

$$A(N_1, N_2, N_3, \dots)$$

is determined by the numbers

$$N_1, N_2, N_3, \dots$$

or, alternatively, by the probabilities

$$p_1, p_2, p_3, \dots$$

The state A has

$$W(A) = W(N_1, N_2, N_3, \dots) = \frac{N!}{N_1! N_2! N_3! \dots}$$

such combinations.

The probability of each of these combinations is

$$p(A) = p(N_1, N_2, N_3, \dots) = \frac{1}{W(N_1, N_2, N_3, \dots)} = \frac{N_1! N_2! N_3! \dots}{N!}.$$

We define the entropy of the state,

$$S(A) = S(W) = S(N_1, N_2, N_3, \dots),$$

to depend only on the number of the combinations so that

For independent states A_1 , and A_2

$$(I) \quad p(A_1 \wedge A_2) = p(A_1)p(A_2)$$

$$(II) \quad S(W_1) + S(W_2) = S(\underbrace{W_1 W_2}_W).$$

$$\partial_{W_1}(II) \Rightarrow$$

$$\partial_{W_1} S(W_1) + \underbrace{\partial_{W_1} S(W_2)}_0 = \partial_{W_1} S(W_1 W_2)$$

$$\partial_{W_1} S(W_1) = \underbrace{\partial_W S(W)}_{S'(W)} \underbrace{\partial_{W_1} \{W_1 W_2\}}_{W_2}$$

$$\partial_{W_1} S(W_1) = S'(W) W_2$$

$$\underbrace{\partial_{W_2} \{\partial_{W_1} S(W_1)\}}_0 = \partial_{W_2} \{S'(W) W_2\}$$

$$0 = \underbrace{\partial_W S'(W)}_{S''(W)} \underbrace{\partial_{W_2}(W)}_{W_1} W_2 + S'(W)$$

$$0 = S''(W) \underbrace{W_1 W_2}_W + S'(W)$$

$$0 = WS''(W) + S'(W).$$

This equation has the integrating factor

$$e^{\int \frac{1}{W} dW} = e^{\log W} = W$$

Thus,

$$(WS'(W))' = 0$$

$$WS'(W) = k = \text{the entropy constant}$$

$$S'(W) = \frac{k}{W}$$

$$S(W) = k \int \frac{1}{W} dW + \text{const}$$

$$S(W) = k \log W + \text{const}$$

By Sterling's approximation, for large n , $n! \sim n^n e^{-n} \sqrt{2\pi n}$

Therefore,

$$N_1! \sim N_1^{N_1} e^{-N_1} \sqrt{2\pi N_1},$$

$$N_2! \sim N_2^{N_2} e^{-N_2} \sqrt{2\pi N_2},$$

.....

$$W = \frac{N!}{N_1! N_2! N_3! \dots}$$

$$= \frac{N^N e^{-N} \sqrt{2\pi N}}{N_1^{N_1} e^{-N_1} \sqrt{2\pi N_1} N_2^{N_2} e^{-N_2} \sqrt{2\pi N_2} N_3^{N_3} e^{-N_3} \sqrt{2\pi N_3} \dots}$$

$$= \frac{N^N \sqrt{2\pi N}}{N_1^{N_1} \sqrt{2\pi N_1} N_2^{N_2} \sqrt{2\pi N_2} N_3^{N_3} \sqrt{2\pi N_3} \dots}$$

$$\log W = (N \log N + \frac{1}{2} \log 2\pi N) - (N_1 \log N_1 + \frac{1}{2} \log 2\pi N_1) \\ - (N_2 \log N_2 + \frac{1}{2} \log 2\pi N_2) - \dots$$

$$\approx N \log N - \underbrace{N_1}_{p_1 N} \log N_1 - \underbrace{N_2}_{p_2 N} \log N_2 - \dots$$

$$= N \log N - N p_1 (\log p_1 + \log N) - N p_2 (\log p_2 + \log N) - \dots$$

$$= N \log N \underbrace{(1 - p_1 - p_2 - \dots)}_0 - N(p_1 \log p_1 + p_2 \log p_2 + \dots)$$

$$= -N(p_1 \log p_1 + p_2 \log p_2 + \dots)$$

4.

Planck's Erroneous Probability Density Function

Each gas molecule has mass

$$m,$$

coordinates

$$\vec{r} = (x, y, z)$$

and momentums

$$m\vec{v}$$

Choosing a continuous probability density function, means that instead of

$$p(\vec{r}_1, \vec{v}_1) + p(\vec{r}_2, \vec{v}_2) + p(\vec{r}_3, \vec{v}_3) + \dots = 1$$

we will have

$$\iint_{\vec{r}, \vec{v}} f(\vec{r}, \vec{v}) d\vec{r} d\vec{v} = 1,$$

leading to a possibly erroneous result.

Recall Rayleigh's choice of continuous density function

$$1 = \int_{\varepsilon_\nu=0}^{\varepsilon_\nu=\infty} f(\varepsilon_\nu) d\varepsilon_\nu,$$

that led to the ultra-violet singularity

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} kT.$$

That violates Wien's law, and the measurements that

$$u(\nu, T) \rightarrow 0, \text{ for large } \nu.$$

Applying the probabilistic version of the second law of Thermodynamics, Planck obtains Maxwell's probability density

$$f(x, y, z, \dot{x}, \dot{y}, \dot{z}) = \alpha e^{-\beta(x^2 + \dot{y}^2 + \dot{z}^2)}$$

which does not represent the discrete radiation.

Then,

$$N = V\alpha \iiint_{(-\infty, \infty)} e^{-\beta(x^2 + \dot{y}^2 + \dot{z}^2)} d\dot{x}d\dot{y}d\dot{z} = V\alpha \left(\frac{\pi}{\beta}\right)^{\frac{3}{2}},$$

which cannot be correct.

And

$$\begin{aligned} U &= \frac{m}{2} \int (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) f(x, y, z, \dot{x}, \dot{y}, \dot{z}) d\dot{x}d\dot{y}d\dot{z} \\ &= \frac{m}{2} V\alpha \int (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) e^{-\beta(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)} d\dot{x}d\dot{y}d\dot{z} \\ &= \frac{3}{4} Vm\alpha \frac{1}{\beta} \left(\frac{\pi}{\beta}\right)^{\frac{3}{2}}, \end{aligned}$$

where Planck had no use for the Jacobian necessary to transform the integration from Cartesian to Spherical coordinates.⁵

Then, by algebra, he obtained

$$\alpha = \frac{N}{V} \left(\frac{3mN}{4\pi U}\right)^{\frac{3}{2}}, \quad \text{and} \quad \beta = \frac{3mN}{4U},$$

which are based on the above wrong statements.

Consequently, Planck had

⁵ In case one wonders, it would be $U = \frac{1}{2} m\alpha V \left[\frac{\pi}{\beta}\right]^{\frac{5}{2}} = \frac{1}{2} mN \frac{\pi}{\beta}$.

$$f(x, y, z, \dot{x}, \dot{y}, \dot{z}) = \frac{N}{V} \left(\frac{3mN}{4\pi U} \right)^{\frac{3}{2}} e^{-\frac{3mN}{4U}(x^2 + y^2 + z^2)}$$

$$\log f = \log \frac{N}{V} \left(\frac{3mN}{4\pi U} \right)^{\frac{3}{2}} - \frac{3mN}{4U}(x^2 + y^2 + z^2)$$

And concluded with

$$\begin{aligned} S &= \text{const} - k \int (f \log f) dx dy dz d\dot{x} d\dot{y} d\dot{z} \\ &= \text{const} + kN \left(\frac{3}{2} \log U + \log V \right). \end{aligned}$$

A collection of errors. None of it should be trusted.

5.

The 2nd Law, and Thermodynamic Equilibrium

The 2nd Law of Thermodynamics says that in a thermodynamical process, the entropy S of a Thermodynamic system increases:

$$\boxed{\Delta S \geq 0}.$$

In a reversible process,

$$\Delta S = 0.$$

In an irreversible process,

$$\Delta S > 0.$$

This leads to

5.1 The Thermodynamic Equilibrium 2nd Law

In Thermodynamic Equilibrium

the Entropy S is maximal

Proof: Else, the state of the system will change to states with higher, and higher entropies.

6.

The Quantized 2nd Law

We shall assume that the probability density is discrete, and depends on the discrete energies

$$\begin{aligned}
 \varepsilon_0 &= 0, \text{ with probability } p_0 \\
 \varepsilon_1 &= h\nu, \text{ with probability } p_1 \\
 \varepsilon_2 &= 2h\nu, \text{ with probability } p_2 \\
 \varepsilon_3 &= 3h\nu, \text{ with probability } p_3 \\
 &\dots\dots\dots \\
 \varepsilon_n &= nh\nu, \text{ with probability } p_n \\
 &\dots\dots\dots
 \end{aligned}$$

Then, we derive

6.1 The Quantized 2nd law

The Probability Distribution of Quantized Energy that Maximizes the Quantized Entropy is

$$\boxed{p(nh\nu) = (1 - e^{-\frac{h\nu}{kT}})e^{-n\frac{h\nu}{kT}}}$$

This is what Planck failed to state and derive in all his Black Body Radiation publications:

Proof: By the Thermodynamic Equilibrium 2nd Law,

Given N molecules

$$p_0N + p_1N + p_2N + p_3N + \dots = N,$$

and Internal Energy

$$\varepsilon_1p_1N + \varepsilon_2p_2N + \varepsilon_3p_3N + \dots = U,$$

at Thermodynamic Equilibrium, the entropy

$$S(p_1, p_2, \dots) = -kN \{ p_1 \log p_1 + p_2 \log p_2 + p_3 \log p_3 + \dots \}$$

is Maximal.

To find that maximum, we define the Lagrange Function

$$G(p_1, p_2, \dots, \lambda, \mu) = -kN \{ p_1 \log p_1 + p_2 \log p_2 + p_3 \log p_3 + \dots \} \\ - \lambda \{ (p_0 + p_1 + p_2 + p_3 + \dots)N - N \} - \mu \{ \varepsilon_1p_1N + \varepsilon_2p_2N + \dots - U \}.$$

λ , and μ are the Lagrange Multipliers for this problem.

Setting the first derivatives of G to zero,

$$\partial_{p_1} G(p_1, p_2, \dots, \lambda, \mu) = 0 \Rightarrow -kN \{ \log p_1 + 1 \} - \lambda N - \mu \varepsilon_1 N = 0,$$

$$\partial_{p_2} G(p_1, p_2, \dots, \lambda, \mu) = 0 \Rightarrow -kN \{ \log p_2 + 1 \} - \lambda N - \mu \varepsilon_2 N = 0,$$

.....

$$\partial_{p_n} G(p_1, p_2, \dots, \lambda, \mu) = 0 \Rightarrow -kN \{ \log p_n + 1 \} - \lambda N - \mu \varepsilon_n N = 0,$$

.....

That is, for each $n = 1, 2, 3, \dots$

$$k \{ \log p_n + 1 \} = -\lambda - \mu \varepsilon_n$$

$$\begin{aligned} \log p_n + 1 &= -\frac{\lambda}{k} - \mu \frac{\varepsilon_n}{k} \\ \log p_n &= -\frac{\lambda}{k} - 1 - \mu \frac{\varepsilon_n}{k} \\ p_n &= \exp\left(-\frac{\lambda}{k} - 1 - \mu \frac{\varepsilon_n}{k}\right) \\ &= \underbrace{\exp\left(-\frac{\lambda}{k} - 1\right)}_{\alpha} \exp\left(-\mu \frac{\varepsilon_n}{k}\right) \\ p_n &= \alpha \exp\left(-\mu \frac{nh\nu}{k}\right) \end{aligned}$$

That is how k appears at the denominator of the exponent in the probability density function of the radiation energy

We need to find α , and μ .

7.

α , and μ of the Quantized 2nd Law

To determine α , we plug $p_n = \alpha \exp\left(-\mu \frac{nh\nu}{k}\right)$ in

$$p_0 + p_1 + p_2 + p_3 + \dots = 1$$

$$\alpha + \alpha \exp\left(-\mu \frac{h\nu}{k}\right) + \alpha \exp\left(-\mu \frac{2h\nu}{k}\right) + \alpha \exp\left(-\mu \frac{3h\nu}{k}\right) + \dots = 1$$

$$\alpha \frac{1}{1 - e^{-\mu \frac{h\nu}{k}}} = 1$$

$$\alpha = 1 - e^{-\mu \frac{h\nu}{k}}$$

To determine μ , we plug $p_n = \alpha \exp\left(-\mu \frac{nh\nu}{k}\right)$ in

$$\varepsilon_1 p_1 N + \varepsilon_2 p_2 N + \varepsilon_3 p_3 N + \dots = U$$

$$u = \frac{U}{N} = \varepsilon_1 p_1 + \varepsilon_2 p_2 + \varepsilon_3 p_3 + \dots$$

$$= h\nu(1 - e^{-\mu \frac{h\nu}{k}})e^{-\mu \frac{h\nu}{k}} + 2h\nu(1 - e^{-\mu \frac{h\nu}{k}})e^{-2\mu \frac{h\nu}{k}} + 3h\nu(1 - e^{-\mu \frac{h\nu}{k}})e^{-3\mu \frac{h\nu}{k}} + \dots$$

Put $\xi = \mu \frac{1}{k}$

$$= (1 - e^{-h\nu\xi}) \left\{ h\nu e^{-h\nu\xi} + 2h\nu e^{-2h\nu\xi} + 3h\nu e^{-3h\nu\xi} + 4h\nu e^{-4h\nu\xi} + \dots \right\}$$

$$= (1 - e^{-h\nu\xi}) \left\{ -\frac{d}{d\xi}(e^{-h\nu\xi}) - \frac{d}{d\xi}e^{-2h\nu\xi} - \frac{d}{d\xi}e^{-3h\nu\xi} + \dots \right\}$$

$$\begin{aligned}
&= (1 - e^{-h\nu\xi}) \frac{d}{d\xi} \{-e^{-h\nu\xi} - e^{-2h\nu\xi} - e^{-3h\nu\xi} + \dots\} \\
&= (1 - e^{-h\nu\xi}) \frac{d}{d\xi} (-e^{-h\nu\xi} [1 + e^{-h\nu\xi} + e^{-2h\nu\xi} + \dots]) \\
&= (1 - e^{-h\nu\xi}) \frac{d}{d\xi} \frac{-e^{-h\nu\xi}}{1 - e^{-h\nu\xi}} \\
&= (1 - e^{-h\nu\xi}) \frac{d}{d\xi} \frac{-1}{e^{h\nu\xi} - 1} \\
&= (1 - e^{-h\nu\xi}) \frac{e^{h\nu\xi} h\nu}{(e^{h\nu\xi} - 1)^2} \\
&= \frac{h\nu}{e^{h\nu\xi} - 1} \\
&= \frac{h\nu}{e^{\mu \frac{h\nu}{k}} - 1}.
\end{aligned}$$

That is

$$u = \frac{h\nu}{e^{\mu \frac{h\nu}{k}} - 1}.$$

So we need to show that

$$\mu = \frac{1}{T}.$$

But $\frac{\partial s}{\partial u} = \frac{1}{T}$.

So we want to show that $\frac{\partial s}{\partial u} = \mu$.

$$S(p_1, p_2, \dots) = -kN \{p_1 \log p_1 + p_2 \log p_2 + p_3 \log p_3 + \dots\}$$

$$\frac{S(p_1, p_2, \dots)}{N} = s = -k \{p_1 \log p_1 + p_2 \log p_2 + p_3 \log p_3 + \dots\}$$

$$\begin{aligned}
 &= -k(1 - e^{-\mu \frac{h\nu}{k}})e^{-\mu \frac{h\nu}{k}} \left[\log(1 - e^{-\mu \frac{h\nu}{k}}) - \mu \frac{h\nu}{k} \right] \\
 &\quad -k(1 - e^{-\mu \frac{h\nu}{k}})e^{-2\mu \frac{h\nu}{k}} \left[\log(1 - e^{-\mu \frac{h\nu}{k}}) - 2\mu \frac{h\nu}{k} \right] \\
 &\quad -k(1 - e^{-\mu \frac{h\nu}{k}})e^{-3\mu \frac{h\nu}{k}} \left[\log(1 - e^{-\mu \frac{h\nu}{k}}) - 3\mu \frac{h\nu}{k} \right] \\
 &\quad \dots\dots\dots \\
 &= -k(1 - e^{-\mu \frac{h\nu}{k}}) [\log(1 - e^{-\mu \frac{h\nu}{k}})] \underbrace{\left[e^{-\mu \frac{h\nu}{k}} + e^{-2\mu \frac{h\nu}{k}} + e^{-3\mu \frac{h\nu}{k}} + \dots \right]}_{\frac{e^{-\mu \frac{h\nu}{k}}}{1 - e^{-\mu \frac{h\nu}{k}}}} \\
 &\quad -k(1 - e^{-\mu \frac{h\nu}{k}}) \left\{ e^{-\mu \frac{h\nu}{k}} \left[-\mu \frac{h\nu}{k} \right] + e^{-2\mu \frac{h\nu}{k}} \left[-2\mu \frac{h\nu}{k} \right] + e^{-3\mu \frac{h\nu}{k}} \left[-3\mu \frac{h\nu}{k} \right] + \dots \right\} \\
 &= -ke^{-\mu \frac{h\nu}{k}} [\log(1 - e^{-\mu \frac{h\nu}{k}})] \\
 &\quad + \mu(1 - e^{-\mu \frac{h\nu}{k}}) \left\{ e^{-\mu \frac{h\nu}{k}} h\nu + 2h\nu e^{-2\mu \frac{h\nu}{k}} + 3h\nu e^{-3\mu \frac{h\nu}{k}} + \dots \right\} \\
 &\text{Put } \xi = \mu \frac{1}{k} \\
 &= -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})] \\
 &\quad + \mu(1 - e^{-h\nu\xi}) \left\{ h\nu e^{-h\nu\xi} + 2h\nu e^{-2h\nu\xi} + 3h\nu e^{-3h\nu\xi} + 4h\nu e^{-4h\nu\xi} + \dots \right\} \\
 &= -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})] \\
 &\quad + \mu(1 - e^{-h\nu\xi}) \left\{ -\frac{d}{d\xi} (e^{-h\nu\xi}) - \frac{d}{d\xi} e^{-2h\nu\xi} - \frac{d}{d\xi} e^{-3h\nu\xi} + \dots \right\} \\
 &= -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})]
 \end{aligned}$$

$$\begin{aligned}
& +\mu(1 - e^{-h\nu\xi}) \frac{d}{d\xi} \left\{ -e^{-h\nu\xi} - e^{-2h\nu\xi} - e^{-3h\nu\xi} + \dots \right\} \\
= & -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})] \\
& +\mu(1 - e^{-h\nu\xi}) \frac{d}{d\xi} (-e^{-h\nu\xi} [1 + e^{-h\nu\xi} + e^{-2h\nu\xi} + \dots]) \\
= & -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})] \\
& +\mu(1 - e^{-h\nu\xi}) \frac{d}{d\xi} \frac{-e^{-h\nu\xi}}{1 - e^{-h\nu\xi}} \\
= & -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})] \\
& +\mu(1 - e^{-h\nu\xi}) \frac{d}{d\xi} \frac{-1}{e^{h\nu\xi} - 1} \\
= & -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})] \\
& +\mu(1 - e^{-h\nu\xi}) \frac{e^{h\nu\xi} h\nu}{(e^{h\nu\xi} - 1)^2} \\
= & -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})] \\
& +\mu \frac{h\nu}{e^{h\nu\xi} - 1} \\
= & -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})] \\
& +\mu \frac{h\nu}{e^{\frac{\mu}{k} h\nu} - 1} . \\
= & -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})] + \mu u
\end{aligned}$$

That is,

$$s = -ke^{-h\nu\xi} [\log(1 - e^{-h\nu\xi})] + \mu u .$$

$$\frac{\partial s}{\underbrace{\partial u}_{\frac{1}{T}}} = \mu.$$

Thus,

$$\mu = \frac{1}{T}.$$

Consequently, the Probability Distribution of quantized energy that maximizes the quantized entropy is

$$p(nh\nu) = (1 - e^{-\frac{h\nu}{kT}})^{-1} e^{-n\frac{h\nu}{kT}}$$

This is what Planck failed to state and derive in all his Black Body Radiation publications.

But neither Planck, nor anyone after him imagined that the Temperature is Quantized, and that there is a well defined Temperature Quantum.⁶

⁶ After the next Chapter.

8.

Determining h , and k

The radiation density at temperature T due to all the frequencies is the summation

$$\begin{aligned}
 \int_{\nu=0}^{\nu=\infty} u(\nu, T) d\nu &= \frac{8\pi h}{c^3} \int_{\nu=0}^{\nu=\infty} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \nu^3 d\nu \\
 &= \frac{8\pi h}{c^3} \int_{\nu=0}^{\nu=\infty} \frac{1}{e^{\frac{h\nu}{kT}}} \frac{1}{1 - e^{-\frac{h\nu}{kT}}} \nu^3 d\nu \\
 &= \frac{8\pi h}{c^3} \int_{\nu=0}^{\nu=\infty} e^{-\frac{h\nu}{kT}} \frac{1}{1 - e^{-\frac{h\nu}{kT}}} \nu^3 d\nu \\
 &= \frac{8\pi h}{c^3} \int_{\nu=0}^{\nu=\infty} e^{-\frac{h\nu}{kT}} \{1 + e^{-\frac{h\nu}{kT}} + e^{-2\frac{h\nu}{kT}} + e^{-3\frac{h\nu}{kT}} + \dots\} \nu^3 d\nu \\
 &= \frac{8\pi h}{c^3} \int_{\nu=0}^{\nu=\infty} \{e^{-\frac{h\nu}{kT}} + e^{-2\frac{h\nu}{kT}} + e^{-3\frac{h\nu}{kT}} + \dots\} \nu^3 d\nu \\
 &= \frac{8\pi h}{c^3} \left(\frac{kT}{h}\right)^4 \int_{\nu=0}^{\nu=\infty} \{e^{-\frac{h\nu}{kT}} + e^{-2\frac{h\nu}{kT}} + e^{-3\frac{h\nu}{kT}} + \dots\} \left(\frac{h\nu}{kT}\right)^3 d\left(\frac{h\nu}{kT}\right)
 \end{aligned}$$

Denoting $\eta = \frac{h\nu}{kT}$

$$= \frac{8\pi h}{c^3} \left(\frac{kT}{h}\right)^4 \int_{\nu=0}^{\nu=\infty} \{e^{-\eta} + e^{-2\eta} + e^{-3\eta} + \dots\} \eta^3 d\eta$$

Since
$$\int_{\nu=0}^{\nu=\infty} e^{-m\eta} \eta^3 d\eta = \frac{\Gamma(3+1)}{m^{3+1}} = \frac{3!}{m^4},$$

$$= \frac{48\pi h \left(\frac{kT}{h}\right)^4}{c^3} \underbrace{\left(1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \dots\right)}_{\zeta(4)}$$

By Euler,

$$\zeta(4) = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \frac{1}{6^4} + \dots = \frac{\pi^4}{90} = 1.082\ 323\ 234.$$

$$\int_{\nu=0}^{\nu=\infty} u(\nu, T) d\nu = \frac{48\pi h \left(\frac{kT}{h}\right)^4}{c^3} \frac{\pi^4}{90}$$

$$= \frac{48\pi h \left(\frac{kT}{h}\right)^4}{c^3} \frac{\pi^4}{90}$$

By Stephen Radiation Law, the radiation density per unit volume is

$$u = aT^4.$$

Therefore,

$$a = \frac{48\pi}{c^3} \zeta(4) \frac{k^4}{h^3}.$$

The Second Radiation Constant is

$$c_2 = c \frac{h}{k}$$

From a , and c_2 , we obtain h , and k .

$$\frac{a}{h} = \frac{48\pi \zeta(4) k^4}{c^3 h^4}$$

$$\begin{aligned}\frac{h}{a} &= \frac{c^3}{48\pi\zeta(4)} \left(\frac{h}{k}\right)^4 \\ &= \frac{c^3}{48\pi\zeta(4)} \left(\frac{c_2}{c}\right)^4 \\ &= c_2^4 \frac{1}{48\pi\zeta(4)c}.\end{aligned}$$

Hence,

$$h = ac_2^4 \frac{1}{48\pi\zeta(4)c}$$

The entropy constant is

$$\begin{aligned}k &= \frac{c}{c_2} h \\ &= \frac{c}{c_2} ac_2^4 \frac{1}{48\pi\zeta(4)c} \\ &= ac_2^3 \frac{1}{48\pi\zeta(4)}\end{aligned}$$

a , and c_2 are determined by measurements

$$a = 4 \frac{\sigma}{c}$$

In 1912, Westphal, and Rubens, measured

$$\sigma = 5.54 \cdot 10^{-8} \frac{\text{Watt}}{\text{m}^2(\text{degree})^4}.$$

and obtained

$$a = 4 \frac{\sigma}{c} = 7.39 \cdot 10^{-15}.$$

And the Physically Technical Capital⁷ Laboratory measured

⁷ physikalisch technisch reichsanstalt

$$c_2 = 0.01436 \text{ meter} \cdot \text{degree}$$

Using these, Planck computed

$$h = 6.415 \cdot 10^{-34} \text{ Joule} \cdot \text{sec}$$

$$k = 1.34 \cdot 10^{-23} \text{ Joule/degree}$$

Now, from [Cohen],

$$\sigma = 5.67051 \cdot 10^{-8} \frac{\text{Watt}}{\text{m}^2(\text{degree})^4}$$

$$c = 299\,792\,458 \text{ m/sec}$$

$$a = 4 \frac{\sigma}{c} = 7.56\,591\,415 \cdot 10^{-15}$$

$$c_2 = 0.01438\,769 \text{ meter} \cdot \text{degree}$$

$$\begin{aligned} h &= (7.56591415 \cdot 10^{-15})(1.438\,769 \cdot 10^{-2})^4 \frac{1}{48\pi(1.082\,323\,234)} \times \\ &\quad \times \frac{1}{299\,792\,458} \\ &= 6.626\,083\,463 \cdot 10^{-34} \text{ J sec} \end{aligned}$$

In tables we have

$$6.626\,0755 \text{ (40)}$$

$$\begin{aligned} k &= (7.56591415 \cdot 10^{-15})(1.438\,769 \cdot 10^{-2})^3 \frac{1}{48\pi(1.082\,323\,234)} \\ &= 1.38\,065\,933 \cdot 10^{-23} \text{ J/degree} \end{aligned}$$

In tables we have

$$= 1.38\,065\,8(12)$$

9.

The Temperature Under $s = k \log W$

9.1

$$T = \frac{W}{k} \frac{\partial u}{\partial W}$$

From the 2nd Law of Thermodynamics

$$ds = \frac{du + pdv}{T}$$

$$du = Tds - pdv$$

Using

$$s = k \log W,$$

$$ds = k \frac{1}{W} dW,$$

$$du = T \frac{k}{W} dW - pdv$$

$$\frac{\partial u}{\partial W} = T \frac{k}{W}$$

$$T = \frac{W}{k} \frac{\partial u}{\partial W}.$$

W = the number of combinations of N in N_1, N_2, N_3, \dots

$\partial_W u$ = the rate of change of the radiation energy with respect to the change of the combinations.

Too abstract to be stated in physical terms.

10.

The Temperature Under the Quantized 2nd law

10.1

$$T = \frac{h\nu}{k} \frac{1}{\log\left(1 + \frac{h\nu}{u_\nu}\right)}$$

Proof: Under the quantized 2nd Law, the Probability Distribution of quantized energy that maximizes the quantized entropy is

$$p(nh\nu) = \left(1 - e^{-\frac{h\nu}{kT}}\right) e^{-n\frac{h\nu}{kT}}$$

The Entropy

$$s = -k \left\{ p_1 \log p_1 + p_2 \log p_2 + p_3 \log p_3 + \dots \right\}$$

is Maximal, and the average radiation energy per mode between ν , and $\nu + d\nu$ is

$$u_\nu = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

Solving for T ,

$$\frac{u_\nu}{h\nu} = \frac{1}{e^{\frac{h\nu}{kT}} - 1},$$

$$\frac{u_\nu}{h\nu} (e^{\frac{h\nu}{kT}} - 1) = 1,$$

$$\frac{u_\nu}{h\nu} e^{\frac{h\nu}{kT}} = \frac{u_\nu}{\varepsilon_\nu} + 1,$$

$$e^{\frac{h\nu}{kT}} = \frac{\frac{u_\nu}{h\nu} + 1}{\frac{u_\nu}{h\nu}},$$

$$\frac{h\nu}{kT} = \log \frac{\frac{u_\nu}{h\nu} + 1}{\frac{u_\nu}{h\nu}},$$

$$\frac{1}{T} = \frac{k}{h\nu} \log \frac{\frac{u_\nu}{h\nu} + 1}{\frac{u_\nu}{h\nu}},$$

$$\frac{1}{T} = \frac{k}{h\nu} \left\{ \log\left(\frac{u_\nu}{h\nu} + 1\right) - \log\left(\frac{u_\nu}{h\nu}\right) \right\}$$

$$T = \frac{h\nu}{k} \frac{1}{\log\left(\frac{u_\nu}{h\nu} + 1\right) - \log\left(\frac{u_\nu}{h\nu}\right)}$$

$$\boxed{T = \frac{h\nu}{k} \frac{1}{\log\left(1 + \frac{h\nu}{u_\nu}\right)}}$$

This is the dependence of the Temperature on the quantized radiation energy under the Quantized 2nd Law.

Since the Radiation Energy is Quantized the Temperature is Quantized, and has a Temperature Quantum.

11.

The Temperature is Quantized, and the Entropy Quantum Depends Uniquely on ν

The relation

$$T = \frac{h\nu}{k} \frac{1}{\log(1 + \frac{h\nu}{u_\nu})}$$

tells us that the Temperature corresponds uniquely to the radiation energy.

At times, we may manipulate the radiation energy, as continuous, But it is well known that it is discrete, made of photons $h\nu$ that are emitted or absorbed in the transition between orbits of electrons around the nucleus.

Since the number of possible orbits is finite, the number of the transitions between them is finite.

For instance, if the number of possible orbits is 5, the number of transitions between them is 10:

4 transitions $(5 \rightleftharpoons 1), (5 \rightleftharpoons 2), (5 \rightleftharpoons 3), (5 \rightleftharpoons 4),$

3 transitions $(4 \rightleftharpoons 1), (4 \rightleftharpoons 2), (4 \rightleftharpoons 3),$

2 transitions $(3 \rightleftharpoons 1), (3 \rightleftharpoons 2),$

1 transition ($2 \rightleftharpoons 1$).

Thus, the corresponding values of the temperature are discrete, and their number is finite. The quantization of the radiation energy quantizes the Temperature.

We may write

$$T = T(\nu),$$

and the correspondence is unique. A one-one function with an inverse function

$$\nu = \nu(T).$$

Consequently, the quantum of entropy

$$h \frac{\nu}{T(\nu)}$$

is determined uniquely by the radiation frequency ν .

For each photon $h\nu$, there is a unique Temperature $T(\nu)$.

The quantum of entropy is not a family of values depending on a varying temperature. But is determined uniquely by the radiation frequency ν .

12.

The Temperature Quantum

12.1 The Temperature Quantum is

$$\boxed{\delta T = \frac{h\nu}{k \log 2}}$$

δT is proportional to ν

Proof: From

$$T = \frac{h\nu}{k} \frac{1}{\log\left(1 + \frac{h\nu}{u_\nu}\right)}$$

The Temperature Quantum is obtained for $\delta u_\nu = h\nu$

$$\begin{aligned} \delta T &= \frac{h\nu}{k} \frac{1}{\log\left(1 + \frac{h\nu}{\delta u_\nu}\right)} \\ &= \frac{h\nu}{k} \frac{1}{\log\left(1 + \frac{h\nu}{h\nu}\right)} \\ &= \frac{h\nu}{k \log 2} \end{aligned}$$

13.

The Quantized Temperature

13.1

$$T \approx \frac{u_\nu}{k}$$

T is proportional to u_ν

Proof: Since the radiation energy is a collection of photons $h\nu$,

each u_ν is the sum of M photons

$$u_\nu = Mh\nu.$$

Substituting in

$$\begin{aligned} T &= \frac{h\nu}{k} \frac{1}{\log\left(1 + \frac{h\nu}{u_\nu}\right)} \\ &= \frac{h\nu}{k} \frac{1}{\log\left(1 + \frac{1}{M}\right)} \end{aligned}$$

By Euler,

$$\log(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$$

$$\log\left(1 + \frac{1}{M}\right) = \frac{1}{M} - \frac{1}{2M^2} + \frac{1}{3M^3} - \frac{1}{4M^4} + \dots$$

Therefore,

$$T = \frac{h\nu}{k} \frac{1}{\log\left(1 + \frac{1}{M}\right)}$$

$$\begin{aligned}
&= \frac{h\nu}{k} \frac{1}{\frac{1}{M} - \frac{1}{2M^2} + \frac{1}{3M^3} - \frac{1}{4M^4} + \dots} \\
&= M \frac{h\nu}{k} \frac{1}{1 - \frac{1}{2} \frac{1}{M} + \frac{1}{3M^2} - \frac{1}{4M^3} + \dots}
\end{aligned}$$

Practically,

$$\frac{1}{M} = \text{an infinitesimal } \delta.$$

Thus, the denominator

$$\begin{aligned}
1 - \frac{1}{2} \frac{1}{M} + \frac{1}{3M^2} - \frac{1}{4M^3} + \dots &= 1 - \frac{1}{2} \delta + \frac{1}{3} \delta^2 - \frac{1}{4} \delta^3 + \dots \\
&\approx 1 - \frac{1}{2} \delta
\end{aligned}$$

Hence, to first order in δ ,

$$T \approx M \frac{h\nu}{k} \frac{1}{1 - \frac{1}{2} \delta}$$

But to first order in δ , $\frac{1}{1 - \frac{1}{2} \delta} \approx 1 + \frac{1}{2} \delta$.

Therefore,

$$\begin{aligned}
T &\approx M \frac{h\nu}{k} \left(1 + \frac{1}{2} \delta\right) \\
&= M \frac{h\nu}{k} \left(1 + \frac{1}{2M}\right)
\end{aligned}$$

$$= \left(M + \frac{1}{2}\right) \frac{h\nu}{k}$$

$$\approx M \frac{h\nu}{k}$$

$$= \frac{u_\nu}{h\nu} \frac{h\nu}{k}$$

$$= \frac{u_\nu}{k}$$

13.2

$$u_\nu = Mh\nu \Rightarrow T \approx M \frac{h\nu}{k}$$

14.

The Entropy Quantum

14.1 The Entropy Quantum is

$$\boxed{\delta s = k \frac{h\nu}{u_\nu}}$$

δs *is proportional to* $\frac{\nu}{u_\nu}$

Proof:

The Entropy Quantum is

$$\frac{h\nu}{T} \approx \frac{h\nu}{\left(\frac{u_\nu}{k}\right)} = k \frac{h\nu}{u_\nu}$$

14.2

$$\boxed{u_\nu = Mh\nu \Rightarrow \delta s = \frac{k}{M}}$$

The Smaller u_ν , the Greater δs

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