EPR, and the Paradoxial Axiomatic Quantum Mechanics

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Abstract In 1925, in response to no experiment that needed explanation, the quantum theory was axiomatized. The new theory did not explain better any experiment, did not increase physical understanding, and introduced into physics the non-physical notions of mathematical functional analysis.

Heisenberg's Uncertainty Principle restates the trivial fact that illuminating an electron to find its location will change its velocity, and momentum.

Dirac's representation of a particle state as an element of a Hilbert space has no physical meaning.

Axiomatic Quantum Mechanics postulates that the particle state is completely represented by Schrödinger's wave function.

But the meaning of this wave function was never clarified, and it remained obscure. Its interpretation by Born's as the probability to find a particle at some location remains a speculation, and to date, chemists may be misled to believe that orbitals are orbits.

Einstein was dissatisfied with the statistical interpretation of Schrödinger's wave function. He believed that the wave function was an incomplete description of the quantum system, that some variables, unknown at the time, were missing, and that had these variables been known, the description of the wave function will be complete, and the theory will be correct.

The 1935 EPR paper presents Einstein's belief in the incomplete description of the wave function due to hidden variables.

To that end, the 1935 EPR paper proves that Schrödinger's Wave Function for a system of two particles, allows simultaneous measurement of location, and momentum of either particle.

Here, we refine that proof from the 1935 EPR paper, and establish that Schrödinger's wave function contradicts the Uncertainty Principle of Axiomatic Quantum Mechanics, and violates the elementary physics that there is really no way to simultaneously determine the location, and the speed of an electron.

Consequently, since the Schrödinger wave function is a central notion in Axiomatic Quantum Mechanics, this paradox leads to the demise of the Axiomatic Quantum Mechanics, and whatever theories which follow from it, and depend on it.

Keywords: Quantum Hypothesis, Black Body Radiation, photon, photoelectric effect, Compton effect, Axiomatic Quantum Mechanics, Schrödinger Wave Function, Electron, Hidden

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The Quantum Hypothesis, and Axiomatic Quantum Mechanics

0.1 The Quantum of Electromagnetic Radiation

In 1901, Planck showed that the electromagnetic energy density per unit volume at frequencies between ν, and $\nu + \delta\nu$ of an ideal radiator (black body) is

$$u(\nu, T) = \frac{8\pi}{c^3}\nu^2 \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}.$$

To that end, Planck had to assume that Electromagnetic radiation at frequency ν is emitted in packets of energy

$$h\nu.$$

The experimental confirmation of Planck's Radiation Law proved that electromagnetic energy was discrete, and established Planck's hypothesis as a fact of nature.

But that conflicted with Planck's belief in radiation of continuous waves. He viewed his hypothesis as an assumption, to be removed once a more believable radiation law will be found.

To reconcile his quantum hypothesis with his conception of wave radiation, Planck avoided the conclusion that radiation energy must be made of particles, and postulated that radiation is a transition between the energy levels of an oscillator. Furthermore, ignoring the symmetry between emission and absorption, he maintained that the absorption of radiation energy is continuous.

Einstein's 1905 explanation of the photo-electric effect, established that electromagnetic energy is also absorbed in discrete packets.

In Bohr's explanation of the Hydrogen Atom, an electron moves to a higher orbit when it absorbs a photon, and returns to a lower orbit when it emits a photon.

In the 1923 Compton Effect, some of the scattered radiation has a lower frequency than the incident radiation due to the conservation of momentum and energy of the photons.

Thus, the fact that electromagnetic energy is made of photons, explains successfully atomic, and subatomic interactions.

In 1925 it became inferior to the "new quantum mechanics", because in the beyond comprehension, we trust.

0.2 The 1925 Axiomatic Quantum Mechanics

In 1925, in response to no experiment that needed explanation, the quantum theory was axiomatized. The new theory did not explain better any experiment, did not increase physical understanding, and introduced into physics the non-physical notions of mathematical functional analysis.

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Dirac's representation of a particle state as an element of a

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Axiomatic Quantum Mechanics postulates that the particle state is completely represented by Schrödinger's wave function.

But the meaning of this wave function was never clarified, and it remained obscure. Its interpretation by Born's as the probability to find a particle at some location remains a speculation, and to date, chemists may be misled to believe that orbitals are orbits.

0.3 Einstein, and Schrödinger's Wave Function

Einstein was dissatisfied with the statistical interpretation of Schrödinger's wave function. Einstein believed that the wave function was an incomplete description of the quantum system, that some variables, unknown at the time, were missing, and had these variables been known, the description of the wave function will be complete, and the theory will be correct.

The 1935 EPR paper presents Einstein's belief in the incomplete description of the wave function due to hidden variables.

Einstein's argument for hidden variables is not necessary: the incompleteness of the Schrödinger wave function is inherent in the physics: There is really no way to simultaneously determine the location, and the speed of an electron.

To forward the hidden variables hypothesis, the 1935 EPR paper proves that Schrödinger's Wave Function for a system of two particles, allows simultaneous measurement of location, and momentum of either particle.

Here, we refine that proof from the 1935 EPR paper, and establish that Schrödinger's wave function contradicts the Uncertainty Principle of Axiomatic Quantum Mechanics, and violates the elementary physics that there is really no way to simultaneously determine the location, and the speed of an electron.

Consequently, since the Schrödinger wave function is a central notion in Axiomatic Quantum Mechanics, this paradox leads to the demise of the Axiomatic Quantum Mechanics, and whatever theories which follow from it, and depend on it.

We proceed with the proof, refined from the 1935 EPR paper.

For ease of reference, we kept most of the EPR notations.

One exception is the location coordinate, which the EPR denotes as x, and we denote, as is common, by q.

We note that the EPR use of x, is loose: at some point, it is momentum, and location. At another point, it is location alone.

1.

The Axiomatic Quantum Mechanics of One Particle

At time t, a particle moving along the q axis has momentum

$$p = p(t)$$

at the location

$$q = q(t).$$

Axiomatic Quantum Mechanics postulates that the particle state is completely represented by the wave function

$$\psi(p,q) = e^{\frac{2\pi i}{h}p_0 q}$$

where

$$h = \text{planck constant},$$

and

$$p_0 = \text{ constant number}.$$

$\psi(p,q)$ is an eigen-function of the momentum operator

$$P = \frac{h}{2\pi i}\partial_q,$$

with the eigen-value p_0 because

$$P\psi = \frac{h}{2\pi i}\partial_q\left\{e^{\frac{2\pi i}{h}p_0 q}\right\},$$

$$= \underbrace{\frac{h}{2\pi i}\frac{2\pi i}{h}}_{1} p_0 \underbrace{e^{\frac{2\pi i}{h}p_0 q}}_{\psi},$$

$$= p_0\psi .$$

Therefore, at the state given by $\psi(p,q)$, the particle's momentum is

$$p(t)\Big|_{\psi(p,q)} = p_0$$

But $\psi(p,q)$ is not an eigen function of the location operator

$$Q = q(t)$$

because

$$Q\psi = q(t)e^{\frac{2\pi i}{h}p_0 q},$$

and the variable $q(t)$ is not a constant number, and not an eigen-value.

Therefore, at the state given by $\psi(p,q)$, the particle's location

$$q(t)\Big|_{\psi(p,q)}$$

has no definite value.

The probability that the particle's location is between a, and b is

$$\int_{q=a}^{q=b} \psi(p,q)\overline{\psi}(p,q)dq = \int_{q=a}^{q=b} \underbrace{e^{\frac{2\pi i}{h}p_0 q} e^{-\frac{2\pi i}{h}p_0 q}}_{1} dq$$

$$= b - a .$$

Thus, all locations are equally probable, and the particle may be found at any location $q(t)$.

Direct measurement of the particle's location will alter its state, demolish its wave function $\psi(p,q)$, and eliminate the knowledge of its momentum.

Therefore, when the particle's momentum is known with certainty, its location cannot be determined.

That is, the axiomatic quantum mechanics of one particle is consistent with what we expect to happen in nature.

In general, it is shown in Axiomatic Quantum Mechanics that since the momentum, and location operators of a quantum system do not commute,

$$PQ \neq QP,$$

then the precise knowledge of the system's momentum, precludes the knowledge of its location, and the precise knowledge of the system's location precludes the knowledge of its momentum.

We shall see that this result does not hold even for a two particles' quantum system.

2.

The Axiomatic Quantum Mechanics of Two Particles

At time

$$0 \le t \le T,$$

a particle moving along the q axis with momentum

$$p_1 = p_1(t),$$

at the location

$$q_1 = q_1(t),$$

interacts with a second particle moving along the q axis with momentum

$$p_2 = p_2(t),$$

at the location

$$q_2 = q_2(t).$$

For each p, the first particle momentum operator

$$P_1 = \frac{h}{2\pi i}\partial_{q_1},$$

has the eigen functions

$$u_p(p_1, q_1) = e^{\frac{2\pi i}{h} p q_1}$$

with the eigen-value p because

$$P_1 u_p = \frac{h}{2\pi i}\partial_{q_1}\left\{ e^{\frac{2\pi i}{h} p q_1} \right\},$$

$$= \underbrace{\frac{h}{2\pi i}\frac{2\pi i}{h}}_{1} p \underbrace{e^{\frac{2\pi i}{h} p q_1}}_{u_p},$$

$$= p u_p .$$

Thus, the measurement of $p_1(t)$ at $u_p(p_1, q_1)$ gives

$$p_1(t)\Big|_{u_p} = p .$$

The wave function of the two particles, $\Psi(p_1, q_1; p_2, q_2)$ can be expanded in the eigen functions u_p, with coefficients

$$\psi_p(p_2, q_2) = e^{\frac{2\pi i}{h}(-p)q_2} e^{\frac{2\pi i}{h} p p_0},$$

by integrating over the continuous spectrum of the momentum p

$$\Psi(p_1, q_1; p_2, q_2) = \int_{p=-\infty}^{p=\infty} \underbrace{u_p(p_1, q_1)}_{e^{\frac{2\pi i}{h} p q_1}} \underbrace{\psi_p(p_2, q_2)}_{e^{\frac{2\pi i}{h}(-p)q_2} e^{\frac{2\pi i}{h} p p_0}} dp ,$$

$$= \int_{p=-\infty}^{p=\infty} e^{\frac{2\pi i}{h}(q_1 - q_2 + q_0)p} dp .$$

For each p, the coefficients $\psi_p(p_2, q_2)$ are eigen functions of the second particle momentum operator

$$P_2 = \frac{h}{2\pi i} \partial_{q_2} ,$$

with the eigen-value $-p$ because

$$P_2 \psi_p = \frac{h}{2\pi i} \partial_{q_2} \left\{ e^{\frac{2\pi i}{h}(-p)q_2} e^{\frac{2\pi i}{h} p p_0} \right\},$$

$$= \underbrace{\frac{h}{2\pi i}\frac{2\pi i}{h}}_{1}(-p)\underbrace{e^{\frac{2\pi i}{h}(-p)q_2}e^{\frac{2\pi i}{h}pp_0}}_{\psi_p},$$

$$= (-p)\psi_p.$$

Thus, the measurement of $p_2(t)$ at the state $\psi_p(p_2, q_2)$ gives

$$p_2(t)\Big|_{\psi_p} = -p.$$

Alternatively, for each q, the first particle location operator

$$Q_1 = q_1,$$

has the eigen functions

$$v_q(p_1, q_1) = \delta(q - q_1)$$

with the eigen-value q because

$$Q_1 v_q = q_1\delta(q - q_1),$$

$$= q\delta(q - q_1),$$

$$= qv_q.$$

Thus, the measurement of $q_1(t)$ at the state $v_q(p_1, q_1)$ gives

$$q_1(t)\Big|_{v_q} = q.$$

Therefore, the wave function of the two particles $\Psi(p_1, q_1; p_2, q_2)$ can be expanded also in the eigen functions v_q, with coefficients

$$\varphi_q(p_2, q_2) = h\delta(q - q_2 + q_0),$$

by integrating over the continuous spectrum of the location q

$$\Psi(p_1,q_1;p_2,q_2) = \int_{q=-\infty}^{q=\infty} \underbrace{v_q(p_1,q_1)}_{\delta(q_1-q)} \underbrace{\varphi_q(p_2,q_2)}_{h\delta(q-q_2+q_0)} dq$$

$$= h\delta(q_1 - q_2 + q_0).$$

Since $\delta(x) = \frac{1}{2\pi}\int_{\sigma=-\infty}^{\sigma=\infty} e^{i\sigma x} d\sigma$, $h\delta(q_1 - q_2 + q_0)$ is the wave packet

$$h\frac{1}{2\pi}\int_{\sigma=-\infty}^{\sigma=\infty} e^{i\sigma(q_1-q_2+q_0)} d\sigma = \int_{\sigma=-\infty}^{\sigma=\infty} e^{i\frac{2\pi}{h}(\frac{h}{2\pi}\sigma)(q_1-q_2+q_0)} d\,\underbrace{(\frac{h}{2\pi}\sigma)}_{p},$$

$$= \int_{p=-\infty}^{p=\infty} e^{\frac{2\pi i}{h}p(q_1-q_2+q_0)} dp\,.$$

For each q, the coefficients $\varphi_q(p_2,q_2)$ are eigen functions of the second particle location operator

$$Q_2 = q_2,$$

with the eigen-value $q + q_0$ because

$$Q_2\varphi_q = q_2 h\delta(q - q_2 + q_0),$$

$$= (q + q_0)h\delta(q - q_2 + q_0),$$

$$= (q + q_0)\varphi_q.$$

Thus, the measurement of $q_2(t)$ at the state $\varphi_q(p_2,q_2)$ gives

$$q_2(t)\Big|_{\varphi_q} = q + q_0\,.$$

After the interaction between the two particles ends, for the measurement of the momentum, the first particle is at the state $u_p(p_1, q_1)$, and the second particle is at the state $\psi_p(p_2, q_2)$.

For the measurement of the location, the first particle is at the state $v_q(p_1, q_1)$, and the second particle is at the state $\varphi_q(p_2, q_2)$.

Since the measurements are done after the interaction ends, the second particle is not affected by the first, and we can determine simultaneously the momentum, and the location of the second particle with

$\psi_p(p_2, q_2)$, an eigen function of P_2,

and

$\varphi_q(p_2, q_2)$, an eigen function of Q_2.

But P_2, and Q_2 do not commute. In fact,

$$\left(P_2Q_2 - Q_2P_2\right)\psi = \frac{h}{2\pi i}\partial_{q_2}\left\{q_2\psi\right\} - q_2\frac{h}{2\pi i}\partial_{q_2}\psi,$$

$$= \frac{h}{2\pi i}\left(\psi + q_2\partial_{q_2}\psi\right) - q_2\frac{h}{2\pi i}\partial_{q_2}\psi,$$

$$= \frac{h}{2\pi i}\psi.$$

Thus, the basic notion of a state represented by the Schrödinger Wave Function contradicts Heisenberg's Uncertainty Principle that when the particle's momentum is known with certainty, its location cannot be determined.

It contradicts Axiomatic Quantum Mechanics that since the momentum, and location operators of a quantum system do not commute, then the precise knowledge of the system's momentum, precludes the knowledge of its location, and the precise knowledge of the system's location precludes the knowledge of its momentum. Thus, Schrödinger's wave function is paradoxial already for a system of two particles. Axiomatic Quantum Mechanics works only for one particle, and fails when applied to a system of two particles.

3.

Conclusions

Einstein's argument for hidden variables is not necessary: the incompleteness of the Schrödinger wave function is inherent in the physics: There is really no physical way to simultaneously measure the location, and the speed of an electron.

The location and the momentum operators can be Naimark extended [Nagy] into commutative operators in a Banach Space. But these extended operators have no physical realization. The simultaneous measurement of location and speed in abstract mathematical spaces, cannot be realized physically.

Meanwhile, the EPR paper misses the meaning of what it proves: That the Axiomatic Quantum Mechanics Foundations are at best shaky, if not plain wrong. That perceiving physical states in terms of mathematical functional analysis, leads to contradiction, and paradox.

The EPR's seeking of hidden variables that no physics supports, does not save the Schrödinger's Wave Function from its failure, and the Axiomatic Quantum Mechanics from its demise.

Seeking hidden variables, the EPR failed to see the meaninglessness of the Axiomatic Quantum Mechanics, and to prevent the Axiomatic Quantum Field theory that follows from it.

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