# What are Dirac Magnetic Monopoles, Where Are They, and What They Mean

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**Abstract** Dirac claimed that if a magnetic monopole is a moving particle with a wave function, the size of its magnetic charge would be about (137)/2 times greater than the electric charge of the electron.

We show that confused by the CGS units system, Dirac multiplied the Vector Potential for a moving electron in an electromagnetic field by the speed of light in the vacuum $c \approx 3 \cdot 10^8 \mathrm{m/sec}$, instead of dividing by $c$, and obtained no magnetic charge whatsoever.

Dirac could not notice his error because in the CGS units, the magnetic and the electric charges have the same unit. That unit, named stat-Coulomb is fictional. There is no measurement instrument that measures it.

Using the correct Vector Potential, we show that the MKS **Elementary Magnetic Charge** is given by

$$\boxed{\mathfrak{M} = \frac{h}{\mu_0 e} = \frac{1}{2\alpha} ce \approx \frac{137}{2} 3 \cdot 10^8 e \qquad \text{Ampere} \times \text{meter}},$$

where $e = 1.60217733 \times 10^{-19}$ $\text{Ampere} \times \sec$ is the electron charge,

$h = \text{Planck Constant}$,

$\alpha = \frac{\mu_0 c e^2}{2h} \approx \frac{1}{137}$ is the Fine Structure Constant.

Could this huge magnetic charge enable us to detect a Dirac Magnetic Monopole in an experiment?

Perhaps not. We show that the Magnetic field of the elementary magnetic charge $\mathfrak{M}$ is negligible compared with the electric field of the elementary electric charge of the electron $e$. In fact,

$$\boxed{\frac{E_{\mathfrak{M}}}{E_e} \approx \frac{2.3}{10^7}}$$

Dirac concluded that the higher magnetic charges of the monopoles is the reason why North and South monopoles are locked in dipoles.

In fact, the crucial factor is the distance between the poles in a dipole. That distance must be way smaller than the distance between poles from different dipoles. Then, when a magnet breaks, it will always break between its dipoles.

Recently, we established that the Neutron is a mini-Hydrogen Atom composed of an electron and a proton orbiting each other. The electron's orbit, and the proton's orbit are magnetic dipoles.

Thus, Magnetic Monopoles must show up in a radio-active decay of a Neutron into its electron and proton. Then, the magnetic dipoles of the electron and the proton disintegrate, and either dipole breaks down into two monopoles. By energy conservation, the electromagnetic energy locked in either dipole splits into a photon that carries a North magnetic charge, and a photon that carries a South magnetic charge.

We show that the dipole magnetic energy contained the proton's orbit equals the dipole magnetic energy contained in the electron's orbit. That energy is about

$$36\text{eV}.$$

Therefore, the magnetic energy of each pole may be about

$$18\text{eV}.$$

In comparison, the energy of the electron Neutrino is about $7\text{eV}$.

By mass-energy equivalence, the mass of each photon is about

$$(3.209692764)10^{-33}\text{kg} = \frac{3.523499235}{1000} m_e,$$

where $m_e = (0.91093897)10^{-30}\text{kg}$ is the electron mass.

Magnetic monopoles are almost massless, and difficult to detect.

The frequency of the photonic magnetic monopole is about

$$\nu_{\mathfrak{M}} = (8.71925892)10^{15}\,\text{cycles/sec}\,,$$

where $h = (6.6260755)10^{-34}\,\text{J} \times \text{sec}$ is Planck constant

And its wave length

$$\lambda_{\mathfrak{M}} \sim (3.44065938)10^{-8}\,\text{meter}$$

is in the Ultra-Violet range.

The distance between the North monopole, and the South monopole in the dipole generated by the electron orbit in the Neutron is

$$(18.76328445)10^{-8}\,\text{meter}$$

That is, of the order of wave-length in the visible spectrum.

This is a million times larger than the Neutronic electron orbit around the proton. Hence, contrary to Dirac, the magnetic force between the monopoles is not greater. We show that it is about one billionth of the electric force between the Neutronic electron and proton

For the Hydrogen Atom, the magnetic dipoles of the electron, and the proton contain magnetic energy of about

$$\frac{1}{10,000}\,\text{eV}$$

Therefore, the magnetic energy of each pole may be about

$$\frac{1}{20,000}\mathrm{eV}.$$

Thus, it is unlikely that Hydrogen monopoles may be detected.

Dirac attempted to find Schrödinger's wave function of an electron moving in the radial field of quantum magnetic monopole, using his erroneous vector potential, and CGS. But even with correct MKS vector potential, we don't know its relevance to observing magnetic monopoles in experiments.

Finally, in light of our explanation of the Neutron as a mini-Hydrogen Atom composed of an electron and a proton orbiting each other, Magnetic Monopoles have crucial implication to the structure of the Neutron.

The observation of Dirac Magnetic Monopoles in Neutron Decay will confirm that the electron and the proton that appear in Neutron decay are orbiting each other the same way they do in the Hydrogen Atom. That will prove experimentally that the Neutron is composed as a condensed Hydrogen Atom.

**Keywords:** Dirac, Wave Function, Quantum Mechanics, Magnetic Dipoles, Magnetic Monopoles, Electrodynamics, Maxwell's Equations, Antimatter,

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# Contents

# 1.

# Dirac's Wave Function of the Magnetic Monopole

Dirac assumes that a moving particle carrying a single magnetic pole has a wave function

$$\psi(t,x,y,z,X) = \underbrace{A(t,x,y,z)e^{i\gamma(t,x,y,z)}}_{\psi_1(t,x,y,z)} e^{i\beta(t,x,y,z,X)},$$

$$= \psi_1(t,x,y,z)e^{i\beta(t,x,y,z,X)}$$

that depends on an unspecified variable $X$. But the partial derivatives of $\beta$ are independent of $X$. That is,

$$\partial_t\beta = \partial_0\beta = \kappa_0(t,x,y,z),$$

$$\partial_x\beta = \partial_1\beta = \kappa_1(t,x,y,z),$$

$$\partial_y\beta = \partial_2\beta = \kappa_2(t,x,y,z),$$

$$\partial_z\beta = \partial_3\beta = \kappa_3(t,x,y,z),$$

And

$$\partial_{ij}\beta \text{ need not equal } \partial_{ji}\beta.$$

The phase change around a closed space-time curve $\alpha$ is

$$\oint_{\text{space-time cycle } \alpha} d\beta = \oint_{\alpha} \kappa_0 dt + \kappa_1 dx + \kappa_2 dy + \kappa_3 dz$$

By Stokes Theorem, the line integral equals the surface integral

$$= \iint_{\text{surface bounded by } \alpha} (\partial_1\kappa_2 - \partial_2\kappa_1)dx \wedge dy + (\partial_1\kappa_3 - \partial_3\kappa_1)dx \wedge dz$$

$$+ \iint_{\text{surface bounded by } \alpha} (\partial_1\kappa_0 - \partial_0\kappa_1)dx \wedge dt + (\partial_2\kappa_3 - \partial_3\kappa_2)dy \wedge dz$$

$$+ \iint_{\text{surface bounded by } \alpha} (\partial_2\kappa_0 - \partial_0\kappa_2)dy \wedge dt + (\partial_3\kappa_0 - \partial_0\kappa_3)dz \wedge dt$$

Differentiating $\psi$, for $i = 0,1,2,3$ [1]

$$-i\hbar\partial_i\psi = e^{i\beta}(-i\hbar\partial_i\psi_1 + \hbar\kappa_i\psi_1).$$

That is, with respect to time,

$$-i\hbar\partial_0\psi = e^{i\beta}(-i\hbar\partial_0\psi_1 + \hbar\kappa_0\psi_1),$$

and with respect to space

$$-i\hbar\nabla\psi = e^{i\beta}(-i\hbar\nabla\psi_1 + \hbar\vec{\kappa}\psi_1)$$

Therefore, if Schrödinger's wave equation for $\psi$ has the Momentum operator $P$, and the Energy operator[2] $W$, then Schrödinger's wave equation for $\psi_1$ has the Momentum operator $P + \hbar\vec{\kappa}$, and the Energy operator $W - \hbar\kappa_0$.

Thus, if $\psi$ satisfies Schrödinger's wave equation for a neutral particle moving in a field free space, Then $\psi_1$ satisfies the same

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[1] Dirac denotes Planck's Constant divided by $2\pi$ by $h$. We use the common notation $\hbar$

[2] The energy operator is usually denoted by $E$

equation for an electron moving in an electromagnetic field with a CGS scalar potential[3]

$$\phi = -\frac{\hbar}{e}\kappa_0 ,$$

and a CGS vector potential,

$$\vec{A} = \frac{\hbar}{ec}\vec{\kappa} .$$

Dirac used an erroneous vector potential $\vec{A} = \frac{\hbar c}{e}\vec{\kappa}$, which yields no magnetic charge whatsoever.

We first use the correct vector potential to obtain the magnetic charge. Then we show why Dirac's vector potential led to nothing.

[3] Dirac denotes the scalar potential by $A_0$ . We use the common notation $\phi$ .

# 2.

# The Huge Magnetic Charge of a Monopole

We consider an electron moving in an electromagnetic field with a CGS scalar potential

$$\phi = -\frac{\hbar}{e}\kappa_0,$$

and a CGS vector potential,

$$\vec{A} = \frac{\hbar}{ec}\vec{\kappa}.$$

By [Woan, p.135], in the conversion from CGS to MKS, $\phi$ remains unchanged. But the MKS vector potential is multiplied by $c$.

That is, the MKS vector potential is

$$\vec{A} = \frac{\hbar}{e}\vec{\kappa}.$$

Then,

$$\nabla \times \vec{\kappa} = \frac{e}{\hbar}\underbrace{\nabla \times \vec{A}}_{\vec{B}}$$

$$\underbrace{\nabla \kappa_0}_{-\frac{e}{\hbar}\nabla\phi} - \underbrace{\partial_t \vec{\kappa}}_{\frac{e}{\hbar}\partial_t \vec{A}} = \frac{e}{\hbar}(\underbrace{-\nabla\phi - \partial_t \vec{A}}_{\vec{E}})$$

The phase change around an infinitesimal closed <u>space-curve</u> $\alpha$ is

$$\oint\limits_{\text{infinitesimal space cycle }\alpha} d\beta .$$

If $\psi$ vanishes along a line that passes through that cycle, then the phase-change around the cycle increases by a multiple of $2\pi$.

Now,

$$\oint\limits_{\alpha} d\beta = \oint\limits_{\alpha} \kappa_1 dx + \kappa_2 dy + \kappa_3 dz$$

By Stokes Theorem, the line integral equals the surface integral

$$= \iint\limits_{\text{surface bounded by }\alpha} (\partial_1\kappa_2 - \partial_2\kappa_1)dx \wedge dy + (\partial_1\kappa_3 - \partial_3\kappa_1)dx \wedge dz$$

$$+ \iint\limits_{\text{surface bounded by }\alpha} (\partial_2\kappa_3 - \partial_3\kappa_2)dy \wedge dz$$

$$= \iint\limits_{\text{surface bounded by }\alpha} \underbrace{(\nabla \times \vec{\kappa})}_{\frac{e}{\hbar}\vec{B}} \cdot d\vec{S}$$

$$= \frac{e}{\hbar} \iint\limits_{\text{surface bounded by }\alpha} \vec{B} \cdot d\vec{S}$$

$$= \frac{e}{\hbar} \times (\text{Magnetic Flux through the Surface})$$

If the surface is closed, the MKS Magnetic Flux through it is

$$\oiint\limits_{\text{closed surface }S} \vec{B} \cdot d\vec{S} = \mu_0 \times (\text{magnetic charge inside } S),$$

where $\mu_0 = \frac{4\pi}{10^7}\frac{Newton}{(Ampere)^2}$ is the permeability of the vacuum.

If the $\psi$-vanishing line is inside an infinitesimal closed surface, its ends points are an elementary magnetic monopole with magnetic charge $\mathfrak{M}$, and $\frac{e}{\hbar}\mu_0\mathfrak{M}$ equals the minimal phase change $2\pi$,

$$\frac{e}{\hbar}\mu_0\mathfrak{M} = 2\pi .$$

Therefore, the MKS elementary magnetic charge of a Dirac magnetic monopole is

$$\mathfrak{M} = \frac{2\pi}{\mu_0}\frac{\hbar}{e},$$

$$= c\frac{1}{2}\underbrace{\frac{2}{\mu_0 c}\frac{h}{e^2}}_{\approx 137}e,$$

where $\frac{2}{\mu_0 c}\frac{h}{e^2} \approx 137$ is the inverse of the Fine Structure Constant [Cohen, p.54]. Thus,

$$\boxed{\mathfrak{M} \approx 3 \cdot 10^8 \frac{137}{2} e}$$

Consequently, the MKS units of the elementary magnetic charge of a Dirac magnetic monopole are

$$\frac{\text{meter}}{\text{sec}}\text{Coulomb} = (\text{meter})(\text{Ampere})$$

We proceed to confirm these units from the Magnetic Force Formula

# 3.

# The Units of the Magnetic Charge

The electric MKS force between two electric charges $q_1$, and $q_2$ at distance $r$ is

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2},$$

where $\varepsilon_0$ is the vacuum Permittivity.

Hence,

$$\varepsilon_0 = \frac{1}{4\pi}\frac{1}{F_e}\frac{q_1 q_2}{r^2}$$

Therefore, the unit of $\varepsilon_0$ is

$$\frac{1}{\text{Newton}}\frac{(\text{Coulomb})^2}{(\text{meter})^2}.$$

The Vacuum Permeability is

$$\mu_0 = \frac{1}{\varepsilon_0 c^2}.$$

Therefore, the unit of $\mu_0$ is

$$\text{Newton}\frac{(\text{meter})^2}{(\text{Coulomb})^2}\left(\frac{\text{sec}}{\text{meter}}\right)^2 = \frac{\text{Newton}}{(\text{Ampere})^2}$$

The magnetic MKS force between two magnetic charges $\mathfrak{m}_1$, and $\mathfrak{m}_2$ at distance $r$ is

$$F_m = \frac{\mu_0}{4\pi}\frac{\mathfrak{m}_1\mathfrak{m}_2}{r^2},$$

Hence,

$$\mathfrak{m}_1\mathfrak{m}_2 = \frac{4\pi}{\mu_0}F_m r^2$$

Consequently, the unit of the magnetic charge is

$$\sqrt{\frac{(\text{Ampere})^2}{\text{Newton}}\text{Newton}(\text{meter})^2} = (\text{Ampere})(\text{meter})$$

This is precisely the unit of

$$\mathfrak{M} \approx c\frac{137}{2}e.$$

Indeed, the unit of $ce$ is

$$\frac{\text{meter}}{\text{sec}}\text{Coulomb}=(\text{meter})(\text{Ampere})$$

# 4.

# The Tiny Magnetic Field of a Dirac Magnetic Monopole

Could the huge magnetic charge enable us to detect a Dirac Magnetic Monopole in an experiment?

The answer to that question may be negative.

We show that the Magnetic Field of a Dirac Magnetic Monopole is tiny in comparison with the Electric Field of the electron's Electric charge.

The electric field of an electron with charge $e$ is

$$E_e = \frac{1}{4\pi\varepsilon_0}\frac{e}{r^2}.$$

The magnetic field of a Dirac Magnetic Monopole with the elementary magnetic charge $\mathfrak{M}$ is

$$E_{\mathfrak{M}} = \frac{\mu_0}{4\pi}\frac{\mathfrak{M}}{r^2}$$

Substituting $\mathfrak{M} \approx c\frac{137}{2}e$, and $\mu_0 = \frac{1}{\varepsilon_0 c^2}$,

$$E_{\mathfrak{M}} \approx \frac{\frac{1}{\varepsilon_0 c^2}}{4\pi}\frac{c\frac{137}{2}e}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{e}{r^2}\frac{1}{c}\frac{137}{2}$$

$$= E_e\frac{1}{c}\frac{137}{2}$$

Therefore,

$$\frac{E_{\mathfrak{M}}}{E_e} \approx \frac{1}{c}\frac{137}{2}$$

$$\approx \frac{2.3}{10^7},$$

where the quotient of the two fields, $\frac{E_{\mathfrak{M}}}{E_e}$, has the units $\frac{\text{sec}}{\text{meter}}$.

$$\boxed{\frac{E_{\mathfrak{M}}}{E_e} \approx \frac{2.3}{10^7}}$$

Thus, the huge magnetic charge of the Dirac Magnetic monopole produces a magnetic field tiny in comparison with the electron electric field.

We proceed to show that Dirac's formula for the magnetic charge yields no magnetic charge whatsoever.

# 5.

# Dirac's Incorrect Vector Potential Yields No Magnetic Charge

Dirac obtained for the elementary magnetic charge, [Dirac, p.68, formula 9],

$$\frac{1}{2}\frac{\hbar c}{e} = \frac{1}{2}\frac{\hbar c}{e^2}e = \frac{137}{2}e$$

where $\frac{\hbar c}{e^2} \approx 137$ is the CGS inverse of the Fine Structure Constant [Cohen, p.54].

Then, the magnetic charge is in units of electric charge, which is non-physical. The confusing CGS unit system leads to a result that cannot be verified even in terms of its units.

Therefore, we need to derive Dirac's result in the MKS system.

Dirac's erroneous CGS vector potential

$$\vec{A} = \frac{\hbar}{ec}\vec{\kappa},$$

Converts to the MKS potential

$$\vec{A} = \frac{\hbar}{ec^2}\vec{\kappa}$$

Then, erroneously,

$$\nabla \times \vec{\kappa} = \frac{ec^2}{\hbar} \underbrace{\nabla \times \vec{A}}_{\vec{B}}$$

and

$$\frac{ec^2}{\hbar} \mu_0 \mathfrak{M}_{\text{error}} = 2\pi .$$

Therefore, the MKS elementary magnetic charge of a Dirac magnetic monopole is

$$\mathfrak{M}_{\text{error}} = \frac{2\pi}{\mu_0} \frac{\hbar}{ec^2},$$

$$= \frac{1}{c} \frac{1}{2} \underbrace{\frac{2}{\mu_0 c} \frac{h}{e^2}}_{\approx 137} e,$$

where $\frac{2}{\mu_0 c} \frac{h}{e^2} \approx 137$ is the inverse of the Fine Structure Constant [Cohen, p.54]

$$\mathfrak{M}_{\text{error}} \approx \frac{1}{c} \frac{137}{2} e .$$

But the units of $e / c$ are not units of a magnetic charge:

$$\text{Coulomb} \frac{\text{sec}}{\text{meter}} \neq (\text{meter})(\text{Ampere}),$$

because

$$(\text{Ampere})(\text{sec}) \frac{\text{sec}}{\text{meter}} \neq (\text{meter})(\text{Ampere}),$$

because

$$(\text{sec})^2 \neq (\text{meter})^2,$$

because

$$\text{sec} \neq \text{meter} .$$

# 6.

# Dirac's Misconception of the Force between Monopoles

Dirac concludes [Dirac, p.71]

> *" …the question of why isolated magnetic poles are not observed. The experimental result*[4]
>
> $$\frac{\hbar c}{e^2} = 137$$
>
> *shows that there must be some dissimilarity between electricity and magnetism (possible connected between the cause of dissimilarity between electrons and protons) as the result of which we have,*
>
> $$\textit{not} \quad \text{elemetary magnetic charge} = \text{e}$$
>
> *but*
>
> $$\text{elemetary magnetic charge} = \frac{137}{2} \times \text{e}$$
>
> *This means that the attractive force between one-quantum poles of opposite sign is*
>
> $$\left(137 / 2\right)^2 \approx 4692$$

[4] in CGS.

*times that between electron and proton*

*This very large force may perhaps account for why poles of opposite signs have never yet been separated"*

In fact, what counts is that poles of the same dipole are way closer than poles of different dipoles.

The force between a pole of charge $\mathfrak{M}_{\text{North}}$ and a pole of charge $\mathfrak{M}_{\text{South}}$ of a dipole of length $\delta$ is

$$\frac{\mu_0}{4\pi}\frac{\mathfrak{M}_{\text{North}}\mathfrak{M}_{\text{South}}}{\delta^2}$$

The force between two monopoles from different dipoles, at a distance $\mathsf{d}$ is

$$\frac{\mu_0}{4\pi}\frac{\mathfrak{M}_{\text{North}}\mathfrak{M}_{\text{South}}}{\mathsf{d}^2}$$

Thus, the force between monopoles of the same dipole is

$$\left(\frac{\mathsf{d}}{\delta}\right)^2$$

greater than the force between monopoles from different dipoles, and a magnet always breaks between its different dipoles.

# 7.

# Dirac Magnetic Monopoles are Photons

Dirac treats magnetic monopoles on condition that they exist. We submit that due to conservation of energy, Magnetic Monopoles do exist. and may be detected whenever a magnetic dipole breaks up.

A Magnetic dipole is generated by an electric current loop, such as

1) the Hydrogen Electron orbit, or,

2) the Hydrogen Proton orbit,

or, as argued in [Dan1], when the Neutron is viewed as a mini-Hydrogen Atom. Then, the electric current is

3) the Neutronic Electron Orbit, or

4) the Neutronic Proton orbit

The magnetic energy generated by an electron loop or a proton loop is split between the two poles.

The poles stay together so long as the orbit exist. But in Neutron decay, or in Hydrogen Atom decay, the orbits of the electron, and the proton break down, the electron and proton fly away, and the magnetic energies of their orbits, is carried away by two photons.

In the breakup, Dirac magnetic monopoles appear as photons flying away along with the other products of the decay: A photon $\gamma_{\text{North}}$ carrying a North pole magnetic charge, and a photon $\gamma_{\text{South}}$ carrying a South pole magnetic charge

Hydrogen Atom decay should be.

$$\mathrm{H} \to \mathsf{p} + \mathsf{e} + 2(\gamma_{\text{North}} + \gamma_{\text{South}})$$

And Neutron decay should be

$$\mathsf{n} \to \mathsf{p} + \mathsf{e} + \nu_e^- + 2(\gamma_{\text{North}} + \gamma_{\text{South}})$$

The photonic energy contained in an electric current loop may be small, and both photons may have very small energies, and be difficult to detect.

We will show that the Neutronic photons are almost massless, although more massive than neutrinos, while the Hydrogen photons are undetectable.

# 8.

# In the Neutron, The Magnetic Monopoles of the Electron's Dipole are almost Massless

In [Dan1], we established that the Neutron is a Mini-Hydrogen Atom composed of an electron and a proton:

We showed that the Neutronic Electron has

$$1^{st} \text{ Orbit Radius } r_N \sim 9.398741807 \times 10^{-14} \text{m},$$

$$\text{Speed } v_e \sim 51,558,134 \text{ m/sec},$$

$$\sim 23.5 \times (\text{Speed in Hydrogen}).$$

$$\text{Frequency } \nu_e \sim 8.73067052 \times 10^{19} \text{cycles/second},$$

$$\sim 13,271 \times (\text{Hydrogen Electron Frequency}),$$

*in the range of Hard X Rays:*

$$\text{Quantum of Angular Momentum} \sim (0.043132065)\hbar,$$

$$\text{Zero Point Energy} \sim -7671 \text{ eV},$$

$$\sim \textit{564} \times \textit{(Hydrogen's Zero Point Energy)}$$

with photon frequency $\sim 3.708097094 \times 10^{18}$ cycles/sec

In particular, we computed the Magnetic Energy of the dipole generated by the electron's orbit. We showed [Dan1,p.23] that the Electron's Magnetic Energy in its orbit is

$$U_{\text{magnetic}} = \tfrac{1}{4}\mu_0 r_{\text{N}} e^2 \nu_e^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 v_e^2 e^2 \frac{1}{r_{\text{N}}}$$

Indeed, the current due to the electron's charge $e$ that turns $\nu_{\text{e}}$ cycles/second is

$$I = e\nu_{\text{e}}$$

The Magnetic Energy of this current is [Benson, p.486]

$$\tfrac{1}{2}LI^2.$$

By [Fischer, p.97]

$$L = \mu_0 \frac{\pi {r_{\text{N}}}^2}{2\pi r_{\text{N}}} = \tfrac{1}{2}\mu_0 r_{\text{N}}.$$

Thus, the magnetic energy due to the electron charge is

$$\tfrac{1}{2}\tfrac{1}{2}\mu_0 r_{\text{N}}(e\nu_e)^2 = \tfrac{1}{4}\underbrace{\mu_0}_{\frac{4\pi}{10^7}} r_{\text{N}} e^2 \underbrace{\nu_{\text{e}}^2}_{\frac{1}{4\pi^2}\omega_{\text{e}}^2}$$

$$= \frac{1}{(4\pi)^2}\mu_0 v_{\text{e}}^2 e^2 \frac{1}{r_{\text{N}}},$$

$$= \frac{\pi}{10^7} r_{\text{N}} e^2 \nu_{\text{e}}^2$$

Substituting

$$\mu_0 = \frac{4\pi}{10^7}\frac{\text{Newton}}{(\text{Ampere})^2}$$

$$r_N \sim 9.398741807\times 10^{-14}\text{meter}$$

$$e \sim (1.60217733)10^{-19}\text{C}$$

$$\nu_e \sim 8.73067052\times 10^{19}\text{cycles/second}$$

The units check out. Indeed,

$$\frac{\text{Newton}}{(\text{Ampere})^2}(\text{meter})(\text{Ampere}\times\text{sec})^2\left(\frac{\text{cycles}}{\text{sec}}\right)^2 = (\text{Newton})\times(\text{meter})$$

The dipole magnetic energy contained in the electron current loop is

$$\pi 10^{-7}(9.398741807)10^{-14}(1.60217733)^2 10^{-38}(8.73067052)^2 10^{38} =$$

$$= (5.777446796)10^{-18}\text{Joule}$$

$$= (5.777446796)10^{-18}\frac{1}{(1.60217733)10^{-19}}\text{electronVolt}$$

$$= (36.05997794)\text{eV}$$

$$\sim 36\text{eV}$$

In Neutron decay into its electron and proton, the electron's magnetic dipole disintegrates, the electromagnetic energy locked in the dipole splits into a photon with North magnetic charge, and a photon with South magnetic charge. The energy carried by each photon is about $18\text{eV}$.

In comparison, the electron-neutrino energy is about $7\text{eV}$

By mass-energy equivalence, the mass of each photon is about

$$\frac{\frac{1}{2}(5.777446796)10^{-18}}{c^2} = (3.209692764)10^{-33}\text{kg}$$

in terms of electron's mass, $m_e = (0.91093897)10^{-30}\text{kg}$

$$= \frac{(3.209692764)10^{-33}}{(0.91093897)10^{-30}} m_e$$

$$= \frac{3.523499235}{1000} m_e$$

With Planck constant $h = (6.6260755)10^{-34}\text{Jsec}$, the photon frequency is about

$$\nu_{\mathfrak{M}} = \frac{\frac{1}{2}(5.777446796)10^{-18}}{(6.6260755)10^{-34}}$$

$$= (4.35962946)10^{15}\text{cycles/sec}$$

Its wave length is about

$$\lambda_{\mathfrak{M}} = \frac{c}{\nu_{\mathfrak{M}}} \sim \frac{3 \cdot 10^8 \text{meter/sec}}{(4.35962946)10^{15}\text{cycles/sec}}$$

$$= (6.881318756)10^{-8}\text{meter}$$

in the Ultra-Violet range.

This level of energy is difficult to detect. Monopoles generated from the current loops of the Neutronic electron are almost massless.

# 9.

# In the Neutron, The Magnetic Monopoles of the Proton's Dipole are almost Massless

In [Dan1], we established that the Neutron is a Mini-Hydrogen Atom composed of an electron and a proton:

We showed that the Neutronic Proton has

$$1^{\text{st}} \text{ Orbit Radius } \rho_{\text{p}} \sim 2.209505336 \times 10^{-15}\text{m},$$

$$\text{Speed } V_{\text{p}} \sim 7,905,145 \text{ m/sec },$$

$$\sim 24 \times (\text{Hydrogen Proton Speed}).$$

$$\text{Frequency } \nu_{\text{p}} \sim 5.69422884 \times 10^{20} \text{ cycles/sec},$$

$$\sim 13{,}223 \times (\text{Hydrogen Proton Frequency}),$$

in the range of Gamma Rays:

$$\text{Quantum of Angular Momentum} \sim (0.277007069)\hbar,$$

$$\text{Nuclear Binding Energy} \sim -326,308 \text{ eV},$$

$$\sim 553 \times (\text{Hydrogen's Nuclear Energy Binding})$$

$$\frac{\text{Electron Orbit Radius}}{\text{Proton Orbit Radius}} = \frac{r_N}{\rho_{\text{p}}} \sim 42.5,$$

In particular, we computed the Magnetic Energy of the dipole generated by the proton's orbit. We showed [Dan1,p.27] that the Proton's Magnetic Energy in its orbit is

$$U_{\text{magnetic}} = \tfrac{1}{4}\mu_0 \rho_{\text{p}} e^2 \nu_p^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 {V_{\text{p}}}^2 e^2 \frac{1}{\rho_{\text{p}}}$$

Indeed, the current due to the proton's charge $e$ that turns $\nu_{\text{p}}$ cycles/second is

$$I = e\nu_{\text{p}}$$

The Magnetic Energy of this current is [Benson, p.486]

$$\tfrac{1}{2} L I^2 .$$

By [Fischer, p.97]

$$L = \mu_0 \frac{\pi {\rho_{\text{p}}}^2}{2\pi\rho_{\text{p}}} = \tfrac{1}{2}\mu_0\rho_{\text{p}} .$$

Thus, the magnetic energy due to the Proton's charge is

$$\tfrac{1}{2}\tfrac{1}{2}\mu_0\rho_{\text{p}}(e\nu_{\text{p}})^2 = \tfrac{1}{4}\mu_0\rho_{\text{p}} e^2 \underbrace{\nu_{\text{p}}^2}_{\frac{1}{4\pi^2}\Omega_{\text{p}}^2}$$

$$= \frac{1}{(4\pi)^2}\mu_0 V_{\text{p}}^2 e^2 \frac{1}{\rho_{\text{p}}} .$$

$$= \frac{\pi}{10^7}\rho_{\text{p}} e^2 \nu_{\text{p}}^2$$

Substituting

$$\mu_0 = \frac{4\pi}{10^7}\frac{\text{Newton}}{(\text{Ampere})^2}$$

$$\rho_{\text{p}} \sim 2.209505336 \times 10^{-15}\text{meter}$$

$$e \sim (1.60217733)10^{-19}\text{C}$$

$$\nu_{\text{p}} \sim 5.69422884 \times 10^{20} \text{ cycles/sec}$$

The units check out. Indeed,

$$\frac{\text{Newton}}{(\text{Ampere})^2}(\text{meter})(\text{Ampere} \times \text{sec})^2\left(\frac{\text{cycles}}{\text{sec}}\right)^2 = (\text{Newton})\times(\text{meter})$$

The dipole magnetic energy contained in the proton current loop is

$$\pi 10^{-7}(2.209505336)10^{-15}(1.60217733)^2 10^{-38}(5.69422884)^2 10^{40} =$$

$$= (5.777446406)10^{-18}\text{Joule}$$

$$= (5.777446406)10^{-18}\frac{1}{(1.60217733)10^{-19}}\text{electronVolt}$$

$$= (36.05905918)\text{eV}$$

$$\sim 36\text{eV}$$

Indeed, the proton current loop, and the electron current loop contain the same magnetic energy

$$\frac{\pi}{10^7}r_{\text{N}}e^2\nu_{\text{e}}^2 = \frac{\pi}{10^7}\rho_{\text{p}}e^2\nu_{\text{p}}^2$$

because

$$r_{\text{N}}\nu_{\text{e}}^2 = \rho_{\text{p}}\nu_{\text{p}}^2$$

because

$$r_{\mathrm{N}}(\underbrace{2\pi\nu_{\mathrm{e}}}_{\omega_e})^2 = \rho_{\mathrm{p}}(\underbrace{2\pi\nu_{\mathrm{p}}}_{\omega_{\mathrm{p}}})^2$$

$$\omega_e{}^2 r_{\mathrm{N}} = \omega_{\mathrm{p}}{}^2 \rho_{\mathrm{p}}$$

That is, the acceleration of the electron towards the proton equals the acceleration of the proton towards the electron.

In Neutron decay into its electron and proton, when the proton's magnetic dipole disintegrates, the electromagnetic energy locked in the dipole splits into two photons. One photon with North magnetic charge, and another photon with South magnetic charge. The energy carried by each photon is about

$$18\mathrm{eV}\,.$$

In comparison, the electron-neutrino energy is about $7\mathrm{eV}$

By mass-energy equivalence, the mass of each photon is about

$$\frac{\frac{1}{2}(5.777446406)10^{-18}}{c^2} = (3.20969244)10^{-33}\mathrm{kg}$$

in terms of electron's mass, $m_e = (0.91093897)10^{-30}\mathrm{kg}$

$$= \frac{(3.20969244)10^{-33}}{(0.91093897)10^{-30}} m_e$$

$$= \frac{3.523498888}{1000} m_e$$

Taking Planck constant $h = (6.6260755)10^{-34}\mathrm{Jsec}$, the photon's frequency is about

$$\nu_{\mathfrak{M}} = \frac{\frac{1}{2}(5.777446406)10^{-18}}{(6.6260755)10^{-34}}$$

$$= (4.35962917)10^{15}\text{cycles/sec}$$

Its wave length is about

$$\lambda_{\mathfrak{M}} = \frac{c}{\nu_{\mathfrak{M}}} \sim \frac{3 \cdot 10^{8}\text{meter/sec}}{(4.35962917)10^{15}\text{cycles/sec}}$$

$$= (6.881319221)10^{-8}\text{meter}$$

in the Ultra-Violet range.

This level of energy is difficult to detect. Monopoles generated from the current loops of the Neutronic proton are almost massless.

# 10.

# The Distance between the North and the South Poles in the Electron's Dipole in the Neutron

The magnetic dipole generated by the Neutronic electron orbiting the Neutronic proton has magnetic energy

$$\frac{\pi}{10^7} r_{\mathrm{N}} e^2 \nu_{\mathrm{e}}^2$$

The dipole is composed of two monopoles at distance

$$\delta$$

each with elementary magnetic charges

$$\mathfrak{M} \approx c\frac{137}{2}e.$$

The magnetic energy contained in the dipole is

$$\frac{\mu_0}{4\pi}\frac{\mathfrak{M}^2}{\delta}.$$

Since both expressions describe the same energy,

$$\frac{\pi}{10^7} r_{\mathrm{N}} e^2 \nu_{\mathrm{e}}^2 = \frac{\mu_0}{4\pi}\frac{\mathfrak{M}^2}{\delta}$$

$$\approx \frac{1}{10^7}\frac{1}{\delta}\left(c\frac{137}{2}e\right)^2$$

$$\pi r_{\mathrm{N}}\nu_{\mathrm{e}}^2 \approx \frac{1}{\delta}c^2\left(\frac{137}{2}\right)^2$$

$$\delta \approx \frac{1}{\pi r_{\mathrm{N}}}\left(\frac{c}{\nu_e}\frac{137}{2}\right)^2$$

We showed [Dan1] that the Neutronic Electron has

1st Orbit Radius $r_N \sim 9.398741807 \times 10^{-14}\mathrm{m}$,

Frequency $\nu_e \sim 8.73067052 \times 10^{19}\,\text{cycles/second}$,

Thus, the distance between the Neutronic electron monopoles is

$$\frac{10^{14}}{\pi 9.398741807}\left(\frac{3\cdot 10^8}{8.73067052\times 10^{19}}\frac{137}{2}\right)^2 \text{meter} =$$

$$= \frac{1}{\pi 9.398741807}\left(\frac{3}{8.73067052}\frac{137}{2}\right)^2 10^{-8}\text{meter}$$

$$= (0.1876328445)\underbrace{10^{-6}\text{meter}}_{\mu m}$$

$$= (1876.328445)\underbrace{10^{-10}\text{meter}}_{\text{Angstrom}}$$

That is, $\delta$ is of the order of wave-length in the visible spectrum

# 11.

# The Force Between the North and the South Poles in the Electron's Dipole in the Neutron

The magnetic force between two magnetic monopoles in the dipole generated by the Neutronic electron is

$$\frac{\mu_0}{4\pi}\frac{\mathfrak{M}^2}{\delta^2} \approx \frac{\mu_0}{4\pi}\frac{1}{\delta^2}\left(c\frac{137}{2}e\right)^2$$

$$= \frac{1}{10^7}\frac{1}{\delta^2}\left(3\cdot 10^8\,\frac{137}{2}e\right)^2$$

Substituting $e \sim (1.60217733)10^{-19}\mathrm{C}$

$\delta \approx (18.76328445)10^{-8}\,\mathrm{meter}$

The force is

$$\frac{1}{10^7}\frac{1}{((18.76328445)10^{-8})^2}\left(3\cdot 10^8\,\frac{137}{2}(1.60217733)10^{-19}\right)^2 =$$

$$= \left(\frac{1}{18.76328445}3\frac{137}{2}(1.60217733)\right)^2 10^{-13} =$$

$$\boxed{= (0.3079123386)10^{-6}\,\mathrm{Newton}}$$

In comparison, the electric force between the Neutronic electron and the Neutronic proton is

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_N^2} = \frac{\mu_0 c^2}{4\pi}\frac{e^2}{r_N^2}.$$

The magnetic attraction between the monopoles divided by the electric attraction between the electron and proton is

$$\frac{\frac{\mu_0}{4\pi}\frac{\mathfrak{M}^2}{\delta^2}}{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_N^2}} \approx \frac{\frac{\mu_0}{4\pi}\frac{1}{\delta^2}\left(c\frac{137}{2}e\right)^2}{\frac{\mu_0 c^2}{4\pi}\frac{e^2}{r_N^2}}$$

$$= \left(\frac{r_N}{\delta}\frac{137}{2}\right)^2$$

$$= \left(\frac{9.398741807\times 10^{-14}}{(18.76328445)10^{-8}}\frac{137}{2}\right)^2$$

$$= \frac{1.177342595}{10^9}.$$

Contrary to what Dirac believed, the huge magnetic charge does not guarantee a great magnetic force. In fact, $r_N \ll \delta$, guarantees a miniscule magnetic force.

# 12.

# In the Hydrogen Atom, the Magnetic Poles in the Electron's Dipole are Undetectable

The Hydrogen Electron has

$$1^{st} \text{ Orbit Radius } r_H \sim 5.29277249 \times 10^{-11}\,\mathrm{m}\,,$$

In [Dan1, p. 44], we showed that the Hydrogen Electron has

$$\text{Speed } v_{\mathrm{H}} \sim 2{,}189{,}781 \text{ m/sec}\,,$$

In [Dan1, p. 47], we showed that the Hydrogen Electron has

$$\text{Frequency } \nu_{\mathrm{H}} \sim= 6.58472424 \times 10^{15} \text{ cycles/second}\,,$$

The Magnetic Energy of the dipole generated by the electron's orbit is

$$U_{\text{magnetic}} = \tfrac{1}{4}\mu_0 r_{\mathrm{H}} e^2 \nu_{\mathrm{H}}^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 v_{\mathrm{H}}^2 e^2 \frac{1}{r_{\mathrm{H}}}$$

$$= \frac{\pi}{10^7} r_{\mathrm{H}} e^2 \nu_{\mathrm{H}}^2$$

Substituting

$$\mu_0 = \frac{4\pi}{10^7}\frac{\text{Newton}}{(\text{Ampere})^2}$$

$$r_{\text{H}} \sim 5.29277249 \times 10^{-11}\text{meter}$$

$$e \sim (1.60217733)10^{-19}\text{C}$$

$$\nu_{\text{H}} \sim 6.58472424 \times 10^{15} \text{ cycles/second}$$

The units check out. Indeed,

$$\frac{\text{Newton}}{(\text{Ampere})^2}(\text{meter})(\text{Ampere} \times \text{sec})^2 \left(\frac{\text{cycles}}{\text{sec}}\right)^2 = (\text{Newton})\times(\text{meter})$$

The dipole magnetic energy contained in the electron current loop is

$$\pi 10^{-7}(5.29277249)10^{-11}(1.60217733)^2 10^{-38}(6.58472424)^2 10^{30} =$$

$$= (1.850671972)10^{-23}\text{Joule}$$

$$= (1.850671972)10^{-23}\frac{1}{(1.60217733)10^{-19}}\text{electronVolt}$$

$$= (1.155068961)10^{-4}\text{eV}$$

$$\sim \frac{1}{10,000}\text{eV}$$

When a Hydrogen Atom disintegrates into its electron and proton, the electron's magnetic dipole splits into two monopoles. The electromagnetic energy locked in the dipole splits into two photons. One with North magnetic charge, and one with South magnetic charge. The energy carried by each photon is about

$$\frac{1}{20,000}\text{eV}.$$

In comparison, the electron-neutrino energy is about $7\text{eV}$

By mass-energy equivalence, the mass of each photon is about

$$\frac{\frac{1}{2}(1.850671972)10^{-23}}{c^2} = (1.028151096)10^{-40}\text{kg}$$

in terms of electron's mass, $m_e = (0.91093897)10^{-30}\text{kg}$

$$= \frac{(1.028151096)10^{-40}}{(0.91093897)10^{-30}} m_e$$

$$= \frac{1.128671766}{10,000,000,000} m_e$$

With Planck constant $h = (6.6260755)10^{-34}\text{Jsec}$, the photon frequency is about

$$\nu_{\mathfrak{M}} = \frac{\frac{1}{2}(1.850671972)10^{-23}}{(6.6260755)10^{-34}}$$

$$= (1.396506855)10^{10}\text{cycles/sec}$$

Its wave length is about

$$\lambda_{\mathfrak{M}} = \frac{c}{\nu_{\mathfrak{M}}} \sim \frac{3 \cdot 10^8 \text{meter/sec}}{(1.396506855)10^{10}\text{cycles/sec}}$$

$$= (2.148217167)10^{-2}\text{meter}$$

in the Microwave range.

This level of energy is undetectable. Monopoles generated from the current loops of the Hydrogen measure as massless

# 13.

# In the Hydrogen Atom, the Magnetic Poles in the Proton's Dipole are Undetectable

In [Dan2], we showed that the Hydrogen Proton has

1st Orbit Radius $\rho_{\text{p in H}} \sim 1.221173735 \times 10^{-12}\text{m}$,

In [Dan1, p. 36], we showed that the Hydrogen Proton

Speed is $V_{\text{p in H}} \sim 330,420 \ \text{m/sec}$ ,

In [Dan1, p. 40], we showed that the Hydrogen Proton

Frequency is $\nu_{\text{p in H}} \sim 4.30634292 \times 10^{16} \ \text{cycles/sec}$,

The Magnetic Energy of the dipole generated by the proton's orbit is

$$U_{\text{magnetic}} = \tfrac{1}{4}\mu_0 \rho_{\text{p in H}} e^2 \nu_{\text{p in H}}^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 V_{\text{p in H}}{}^2 e^2 \frac{1}{\rho_{\text{p in H}}}$$

$$= \frac{\pi}{10^7}\rho_{\text{p}} e^2 \nu_{\text{p}}^2$$

Substituting

$$\mu_0 = \frac{4\pi}{10^7}\frac{\text{Newton}}{(\text{Ampere})^2}$$

$$\rho_{\text{p in H}} \sim 1.221173735 \times 10^{-12}\text{m}$$

$$e \sim (1.60217733)10^{-19}\text{C}$$

$$\nu_{\text{p in H}} \sim 4.30634292 \times 10^{16} \text{ cycles/sec}$$

The units check out. Indeed,

$$\frac{\text{Newton}}{(\text{Ampere})^2}(\text{meter})(\text{Ampere} \times \text{sec})^2\left(\frac{\text{cycles}}{\text{sec}}\right)^2 = (\text{Newton})\times(\text{meter})$$

The dipole magnetic energy contained in the proton current loop is

$$\pi 10^{-7}(1.221173735) \times 10^{-12}(1.60217733)^2 10^{-38}(4.30634292)^2 10^{32} =$$

$$= (1.826273061)10^{-23}\text{Joule}$$

$$= (1.826273061)10^{-23}\frac{1}{(1.60217733)10^{-19}}\text{electronVolt}$$

$$= (1.139869493)10^{-4}\text{eV}$$

$$\sim \frac{1}{10,000}\text{eV}$$

Indeed, the proton current lop, and the electron current loop contain the same magnetic energy

$$\frac{\pi}{10^7}r_{\text{H}}e^2\nu_{\text{H}}^2 = \frac{\pi}{10^7}\rho_{\text{p in H}}e^2\nu_{\text{p in H}}^2$$

because

$$r_{\text{H}}\nu_{\text{e}}^2 = \rho_{\text{p in H}}\nu_{\text{p in H}}^2$$

because

$$r_{\mathrm{H}}(\underbrace{2\pi\nu_{\mathrm{H}}}_{\omega_e})^2 = \rho_{\mathrm{p}}(\underbrace{2\pi\nu_{\mathrm{p}}}_{\omega_{\mathrm{p}}})^2$$

$$\omega_{\mathrm{H}}{}^2 r_{\mathrm{H}} = \omega_{\mathrm{p}}{}^2 \rho_{\mathrm{p}}$$

That is, the acceleration of the electron towards the proton equals the acceleration of the proton towards the electron.

In the Hydrogen Atom decay into its electron and proton, the proton's magnetic dipole disintegrates, the electromagnetic energy locked in the dipole splits into two photons. A photon with North magnetic charge, and a photon with South magnetic charge. The energy carried by each photon is about

$$\frac{1}{20,000}\,\mathrm{eV}\,.$$

In comparison, the electron-neutrino energy is about $7\mathrm{eV}$

By mass-energy equivalence, the mass of each photon is about

$$\frac{\frac{1}{2}(1.826273061)10^{-23}}{c^2} = (1.014596145)10^{-40}\mathrm{kg}$$

in terms of electron's mass, $m_e = (0.91093897)10^{-30}\mathrm{kg}$

$$= \frac{(1.014596145)10^{-40}}{(0.91093897)10^{-30}}m_e$$

$$= \frac{1.113791569}{10,000,000,000}m_e$$

With Planck constant $h = (6.6260755)10^{-34}\text{Jsec}$, the photon's frequency is about

$$\nu_{\mathfrak{M}} = \frac{\frac{1}{2}(1.826273061)10^{-23}}{(6.6260755)10^{-34}}$$

$$= (1.378095572)10^{10}\text{cycles/sec}$$

Its wave length is about

$$\lambda_{\mathfrak{M}} = \frac{c}{\nu_{\mathfrak{M}}} \sim \frac{3 \cdot 10^{8}\text{meter/sec}}{(1.378095572)10^{10}\text{cycles/sec}}$$

$$= (2.176917234)10^{-2}\text{meter}$$

in the Microwave range.

This level of energy is undetectable. Monopoles generated from the current loops of the Hydrogen proton measure as massless.

# 14.

# Observation of Dirac Magnetic Monopoles in Neutron Decay Will Confirm that the Neutron is a Condensed Hydrogen Atom

In light of our explanation of the Neutron as a mini-Hydrogen Atom composed of an electron and a proton orbiting each other, Magnetic Monopoles have crucial implication to the structure of the Neutron.

The observation of Dirac Magnetic Monopoles in Neutron Decay will confirm that the electron and the proton that appear in Neutron decay are orbiting each other the same way they do in the Hydrogen Atom. That will prove experimentally that the Neutron is composed as a condensed Hydrogen Atom. Then,

$$\mathsf{n} \to \mathsf{p} + \mathsf{e} + \nu_e^- + 2(\gamma_{\text{North}} + \gamma_{\text{South}}).$$

And $\gamma_{\text{North}}$ or $\gamma_{\text{South}}$ may have very short lifetimes before they may merge into one photon.

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