Elementary Particle Mixing for Maximum Channel Capacity in Measured Decays

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Abstract

We adopt the channel capacity formalism from quantum information theory and suggest that it is fundamental to particle decays at the smallest space-time scale. A key to this application is to represent mixing, decay, and measurement as a quantum channel in which auxiliary (non-system) decay products form the 'environment'. A new type of classical-to-quantum complementarity is suggested to apply to particle identity. The stochastic nature of this complimentarity is described by an 'implied ensemble' that involves particle identity, particle quantum states, and transformations as revealed through repeated measurements. In the theory of quantum channels there is also the concept of exchange entropy, which leads to a measure of the reversibility of the channel operation. We compute the measure for various decay mechanism. It is suggested that channel capacity-based mixing is part of the underlying physics of the Cabibbo-Kobayashi-Maskawa mixing between fermionic generations. Keywords: quantum channel capacity, elementary particle decays, lepton/quark generations. PACS Number: 03.65.BZ. Please direct correspondence to R.Y. Levine, bob@spectral.com.

1 Introduction

In this paper we consider, and push to its logical limit, the idea that quantum measurement reveals classical information about a system. In a previous article, we proposed an analogous form of classical-to-quantum complementarity [1]. It was suggested that the chiral left-right mixing of massive fermions is complementary to position and momentum; and that the generational (u,c,t), (d,s,b), (e,μ,τ), (ν_e,ν_μ,ν_τ) elementary fermionic patterns complement spin. In this picture fermionic generations arise as a Poincare group representation that generalizes the natural left-right mixing in the Dirac equation. This paper further considers particle mixing as arising from complementarity. However, in this case, the formalism is applied to particle identity.

As information, particle identity is non-quantum, but the particle itself resides in a quantum state and undergoes the transformations of scattering and decay that are quantum mechanical. The stochastic nature of classical-quantum complementarity is best described by an 'implied ensemble' that involves particle identity, particle quantum states, and transformations as revealed through repeated measurements. In an extended quantum mechanics, the implied ensemble is part of the physics describing the particle state which is determined by criteria that generalize statistical mechanics. The quantum statistics of the implied ensemble maximizes the number of 'accessibly distinguishable' classical states achievable by repeated quantum measurements. The ensemble itself is the set of possible assignments of particle identity resulting from these measurements. In this case, the expression 'accessible distinguishability' refers to classically distinguishable particle identities that are made indistinguishable by quantum mixing and transformations [2]. For a given quantum transformation, which in this paper is particle decay, there are unique particle mixings that maximize the number of accessibly distinguishable inputs to the decay channel.

In recent years the physics of the implied ensemble has been worked out by quantum information theorists. Channel capacity has been defined as a measure of the number of accessibly distinguishable classical states, and shown to be computable using the Holevo bound [3, 4]. In this paper, we adopt the quantum channel capacity formalism and suggest that it is fundamental to particle decays at the smallest space-time scale. As far as we know, there is no direct low energy evidence for this extension of quantum mechanics. However, quantum space-time dislocation of the sort that creates virtual particles and decay channels could effectively allow the existence of an implied ensemble. If a particle samples a vast number of virtual decay products before an actual decay, then the number of such measurably accessible states would be proportional to the quantum channel capacity. Maximizing the number of accessibly distinguishable states would be roughly analogous to maximizing the number of microscopic configurations associated with a macroscopic state in classical statistical mechanics. A particle consists fundamentally of its quantum state and the implied ensemble of product states.

A key to this application of accessible distinguishability is to represent the mixing and decay process as a quantum channel in which auxiliary (non-system) decay products form the 'environment', the context of which will be defined in the next section. For the sake of discussion, we take the auxiliary decay products to be gamma rays, but they could in general represent any particle not included in the original implied ensemble. For weakly interacting fermions, the environment consists of W and Z bosons. In the theory of quantum channels there is also the concept of exchange entropy, which leads to a measure of the reversibility of the channel operation. The interpretation of reversibility for decay processes is not clear at this time. Nevertheless, we compute the quantity for various decay mechanisms, and note its connection to the mixing of the system with a copy of the system known as the 'purifying reservoir' - a measure called fidelity in quantum information theory [5, 6]. In Section II, the channel capacity formalism is set up for a quantum channel defined by simple particle decay. Examples of branching and cascade decay modes are worked out in Section III. In Sections II and III, optimum particle mixings associated with particular decays are computed. A conclusion follows in Section IV.

2 Simple Spontaneous Decay

In this section we formulate spontaneous emission as a quantum channel and define the key operators and functions necessary to enumerate the accessibly distinguishable states in the implied ensemble. That number is equivalent to the Holevo bound on mutual information in a particular type of construction - which we denote the QRE construction [2, 7, 8]. In addition to the universality of the construction in determining channel capacity, the possibility that QRE mixing is found among quark multiplets motivates its consideration.

2.1 Operator Elements

Consider the transition $\Sigma \to \Lambda + \gamma$, which we assume to occur spontaneously due to an arbitrary interaction among elementary particles Σ, Λ, and γ. The γ-ray represents the emitted photon $\gamma(\vec{k}, \hat{\epsilon})$ with wavevector $\vec{k}$ and polarization $\hat{\epsilon}$. The emitted photon Hilbert space, $\mathcal{H}_{\mathcal{E}}$, represents the environment, 'E' in the QRE construction. The ('Q') system Hilbert space $\mathcal{H}_{\mathcal{Q}}$ has a basis $\{|\Sigma>, |\Lambda>\}$, which is assumed to be orthonormal and complete. Note that in order to isolate the problem to particle identity only, we are ignoring the kinematics of the system particles Σ and Λ [9]. The environmental vacuum with zero photon number is denoted $|0>$. The evolution operator $U(t)$ for the decay couples the system to the environment, and is mathematically an operator on the basis set $\{|\Sigma, 0>, |\Lambda, 0>, \{|\Sigma, \gamma(\vec{k}, \hat{\epsilon})>; \forall(\vec{k}, \hat{\epsilon})\}\}$ for the Hilbert space $\mathcal{H}_{\mathcal{Q}} \times \mathcal{H}_{\mathcal{E}}$ of the system and environment.

The quantum channel describing the possible Σ-to-Λ transition is defined by a set of operator elements *in* $\mathcal{H}_{\mathcal{Q}}$ given by

$$E_0 =< 0|U|0>, \tag{1}$$

and

$$E_\gamma =< \gamma(\vec{k}, \hat{\epsilon})|U|0>, \tag{2}$$

where an operator $E_{\gamma(\vec{k},\hat{\epsilon})}$ exists for every $(\vec{k}, \hat{\epsilon})$-state of the emitted photon [10]. Because $\{|\Sigma>, |\Lambda>\}$ forms a complete basis set of $\mathcal{H}_{\mathcal{Q}}$, we can evaluate the operator element E_0 as

$$E_0 =< \Sigma, 0|U|\Sigma, 0> |\Sigma><\Sigma| + <\Lambda, 0|U|\Lambda, 0> |\Lambda><\Lambda|, \tag{3}$$

using the fact that the states $|\Sigma, 0>$ and $|\Lambda, 0>$ are only connected to themselves through system evolution if no photon appears. The infinite set of operator elements $\{E_\gamma\}$ can be evaluated as

$$\begin{aligned} E_\gamma &= <\gamma(\vec{k}, \hat{\epsilon})|U|0> \\ &= <\gamma(\vec{k}, \hat{\epsilon})|U|\Sigma, 0><\Sigma| + <\gamma(\vec{k}, \hat{\epsilon})|U|\Lambda, 0><\Lambda| \\ &= <\Lambda, \gamma(\vec{k}, \hat{\epsilon})|U|\Sigma, 0> |\Lambda><\Sigma|, \end{aligned} \tag{4}$$

where $< \Lambda, \gamma(\vec{k}, \hat{\epsilon})|U|\Sigma, 0 >$ is the amplitude for spontaneous $\gamma(\vec{k}, \hat{\epsilon})$ emission.

That the quantum channel defined by operators $\{E_0, \{E_{\gamma(\vec{k},\hat{\epsilon})}\}\}$ is trace preserving can be seen by considering (for notational purposes let γ represent $\gamma(\vec{k}, \hat{\epsilon})$ and $\sum_\gamma$ the sum over $\vec{k}$ and $\hat{\epsilon}$ photon states)

$$E_0^\dagger E_0 + \sum_\gamma E_\gamma^\dagger E_\gamma = \Lambda_0|\Lambda >< \Lambda| + (\Sigma_0 + \Lambda_\gamma)|\Sigma >< \Sigma|, \qquad (5)$$

where

$$\Lambda_0 = | < \Lambda, 0|U|\Lambda, 0 > |^2, \qquad (6)$$

$$\Sigma_0 = | < \Sigma, 0|U|\Sigma, 0 > |^2, \qquad (7)$$

and

$$\Lambda_\gamma = \sum_\gamma | < \Lambda, \gamma(\vec{k}, \hat{\epsilon})|U|\Sigma, 0 > |^2. \qquad (8)$$

The expression in Eq. (5) is the identity operator, $I = |\Sigma >< \Sigma| + |\Lambda >< \Lambda|$, if

$$\Lambda_0 = 1, \qquad (9)$$

and

$$\Sigma_0 + \Lambda_\gamma = 1, \qquad (10)$$

conditions that are equivalent to the unitarity of U.

2.2 QRE Construction for Simple Decays

The implied ensemble for simple Σ decay contains vectors of particle identity, Σ or Λ in each component, resulting from measurements of decay (or non-decay) products as the number of such measurements approaches infinity. The implied ensemble is encoded using the density matrix,

$$\rho = p|\Sigma >< \Sigma| + (1 - p)|\Lambda >< \Lambda| \qquad (11)$$

for input to the decay channel, and the parameter p is optimized to obtain the channel capacity reflecting a maximum number of accessibly distinguishable states. For a given (Σ, Λ)- system mixing, the enumeration of accessibly distinguishable states in the implied ensemble can be accomplished using the QRE-construction [2].

In addition to the system with basis $\{|\Sigma >, |\Lambda >\}$ and the environmental photon Hilbert spaces, we introduce the reservoir space $\{\Sigma' >, |\Lambda' >\}$ that 'purifies' the density matrix in Eq. (11). The initial QRE-state is then given by

$$|QRE >= [\sqrt{p}|\Sigma, \Sigma' > +\sqrt{1-p}|\Lambda, \Lambda' >]|0 >, \qquad (12)$$

which evolves to the state

$$\begin{aligned} |Q'R'E' > &= E_0[\sqrt{p}|\Sigma, \Sigma' > +\sqrt{1-p}|\Lambda, \Lambda' >]|0 > + \\ & \sum_\gamma E_\gamma[\sqrt{p}|\Sigma, \Sigma' > +\sqrt{1-p}|\Lambda, \Lambda' >]|\gamma > \end{aligned} \qquad (13)$$

as output from the decay channel [2]. The state $|Q'R'E' >$ can be computed easily from Eqs. (3) and (4) with the result

$$\begin{aligned} |Q'R'E' > &= \sqrt{p} < \Sigma, 0|U|\Sigma, 0 > |\Sigma, \Sigma', 0 > + \\ & \sqrt{1-p}e^{i\alpha t}|\Lambda, \Lambda', 0 > + \sum_\gamma \sqrt{p} < \Lambda, \gamma|U|\Sigma, 0 > \times \\ & |\Lambda, \Sigma', \gamma >, \end{aligned} \qquad (14)$$

where from Eq. (9) we take $< \Lambda, 0|U|\Lambda, 0 >= e^{i\alpha t}$ for pure phase α.

Relevant quantities derivable from Eq. (14) include the number of accessibly distinguishable states in the implied ensemble (channel capacity) and the exchange entropy. The latter is the rate that entropy is released to the environment as decays and measurements proceed [11]. Exchange entropy is a measure of the reversibility of the decay; that is, the extent to which the input density matrix ρ in Eq. (11) can be recovered from the decay products. In order to compute these quantities we need the entropy of the output *system* density matrix

$$\mathcal{T}(\rho) = Trace_{RE}(|Q'R'E' >< Q'R'E'|), \qquad (15)$$

which from Eq. (14) is given by

$$\mathcal{T}(\rho) = p\Sigma_0|\Sigma >< \Sigma| + ((1-p) + p\Lambda_\gamma)|\Lambda >< \Lambda|. \qquad (16)$$

In Eq. (16) $\mathcal{T}(\rho)$ denotes the channel operation on system input ρ. The *environmental* density matrix

$$\rho_{E'} = Trace_{QR}(|Q'R'E' >< Q'R'E'|) \qquad (17)$$

is given by

$$\begin{aligned} \rho_{E'} &= (p\Sigma_0 + (1-p))|0 >< 0| + \\ & \quad p \sum_{\gamma} \sum_{\tilde{\gamma}} < \Lambda, \gamma|U|\Sigma, 0 >< \Lambda, \tilde{\gamma}|U|\Sigma, 0 >^* |\gamma >< \tilde{\gamma}| \\ &= (p\Sigma_0 + (1-p))|0 >< 0| + (p < \phi|\phi >)|\hat{\phi} >< \hat{\phi}|, \end{aligned} \tag{18}$$

where $|\hat{\phi} >$ is the normalized state corresponding to

$$|\phi >= \sum_{\gamma} < |\Lambda, \gamma|U|\Sigma, 0 > |\gamma > . \tag{19}$$

The entropies of the mixed states in Eqs. (16) and (18) are easily computed as

$$S(\mathcal{T}(\rho)) = H(p\Sigma_0), \tag{20}$$

and

$$S(\rho_{E'}) = H((1-p) + p\Sigma_0), \tag{21}$$

where for $0 \leq x \leq 1$,

$$H(x) = -x\ln(x) - (1-x)\ln(1-x). \tag{22}$$

The expression in Eq. (21) is the exchange entropy [11].

The channel capacity of the decay is given by

$$C(\mathcal{T}) = \max_p [H(p\Sigma_0) - pH(\Sigma_0)]. \tag{23}$$

The channel capacity in Eq. (23) is obtained from the Holevo bound

$$S(\mathcal{T}(\rho)) - pS(\mathcal{T}(|\Sigma >< \Sigma|)) - (1-p)S(\mathcal{T}(|\Lambda >< \Lambda|)) \tag{24}$$

on mutual *classical* information across the quantum channel, where the classical information is the particle identity, either Σ or Λ. The value of p implied by Eq. (23) is obtained from Eq. (22) as

$$p_{\max} = \frac{(1-\Sigma_0)^{(1/\Sigma_0 - 1)}}{1 + \Sigma_0(1-\Sigma_0)^{(1/\Sigma_0 - 1)}}, \tag{25}$$

with

$$\Sigma_0(t) = | < \Sigma, 0|U|\Sigma, 0 > |^2 = e^{-t/\tau}, \tag{26}$$

Figure 1: Optimum particle mixing versus time for simple decay with $\tau = 1$(A.U.).

Figure 2: Channel capacity versus time for simple decay with $\tau = 1$(A.U.).

where τ is the lifetime of the Σ particle. The mixing in Eq. (25) is optimal in that it maximizes the number of accessibly distinguishable states in the implied ensemble.

In the limit $t \to 0$, $\Sigma_0(t) \to 1$, and from Eq. (23) we have $p_{\max} = 0.5$. The QRE mixing in Eq. (12) then corresponds to an angle θ of $45°$. Taking the limit $t \to \infty$, we obtain the optimum mixing for time scales much greater than the particle lifetime,

$$p_{\max} = e^{-1} = 0.368, \tag{27}$$

which corresponds to a QRE mixing angle of $52.7°$. A plot of $p_{\max}$ in Eq. (25) versus time (for $\tau = 1$(A.U.)) between these two extremes is shown in Figure 1. The corresponding channel capacity is shown in Figure 2 to decrease as the likelihood of particle decay increases.

Reversibility is a measure of the existence of a channel that reverses the particle decay. While the quantum process described by unitary operator U is clearly reversible, the establishment of input and output particle identities from the channel involves coding and measurement. The entire process has a reversing channel if and only if the irreversibility

$$Irr(\rho, \mathcal{T}) = S(\rho) - S(\mathcal{T}(\rho)) + S(\rho_{E'}) \tag{28}$$

vanishes [5]. Substitution of Eqs. (20) and (21) yields

$$Irr(\rho, \mathcal{T}) = H(p) - H(p\Sigma_0) + H((1-p) + p\Sigma_0), \tag{29}$$

where Σ_0 is given in Eq. (26). As expected, $Irr(\rho, \mathcal{T})$ vanishes for p of zero or one, situations in which the particle identity exactly matches the quantum mixing. Also the channel is reversible at $\Sigma_0 = 1$ because no decays have occurred. The physical interpretation of nonzero $Irr(\rho, \mathcal{T})$ for decay processes is not completely understood and is the subject of continuing research. It has been shown that the channel is reversible if and only if the fidelity

Figure 3: Irreversibility versus time at the optimum particle mixing for simple decay with $\tau = 1$ (A.U.).

$< QR|\rho^{Q'R'}|QR >$ is unity [5, 6]. Effectively, this condition corresponds to no mixing between the system and the reservoir in the process of decays and measurements. Figure 3 demonstrates irreversibility at the optimum mixing values in Figure 1. As decays progress and channel capacity decreases, the decay channel becomes less reversible.

The purification in Eq. (12) uses a 'reservoir' $\{\Sigma', \Lambda'\}$ that replicates the number of basis elements in the original system. The reservoir elements interact with neither the system nor the environment. It is interesting to consider the Cabibbo rotation between weakly interacting quarks as a purification that connects quarks to the vacuum [12, 13]. The Cabibbo rotation $(\cos\theta_C|d > + \sin\theta_C|s >)$ is written as a purification $(\cos\theta_C|d, 0 > + \sin\theta_C|0, s >)$. In this case, the strange quark Hilbert space forms a complementary reservoir to the down quark space. This purification may be part of the QRE construction for both down and strange quark weak decays as quantum channels. Of course the patterns of decay in the standard model are much more complicated that described in this paper. However, in Section III decay modes of higher complexity are considered that more closely resemble decay modes of weakly interacting quarks and leptons.

3 Branching and Cascade Decay Processes

The discussion of the previous section postulates that particle mixing is determined by maximizing distinguishability in an implied ensemble generated by simple spontaneous decay. In general, different decay processes are reflected in an ensemble through different particle identities and blurring mechanisms between ensemble elements. In this section, optimized accessible distinguishability is applied to the more complicated processes of branching and cascade decays.

3.1 Branching Decay

In this section we define the implied ensemble, channel capacity, and irreversibility for a particle Σ that can decay to either κ or Λ. The system space $\mathcal{H}_Q$ is defined by basis set $\{|\Sigma>, |\kappa>, |\Lambda>\}$ and the environment E is the same as for simple decay; that is, the Hilbert space $\mathcal{H}_E$ of photons $\gamma(\vec{k}, \hat{\epsilon})$. The channel evolves with unitary operator U coupling the system and the environmental particles. The operator elements are given by

$$\begin{aligned} E_0 &= <0|U|0> \\ &= <\Sigma, 0|U|\Sigma, 0> |\Sigma><\Sigma| + <\kappa, 0|U|\kappa, 0> |\kappa><\kappa| + \\ & \quad <\Lambda, 0|U|\Lambda, 0> |\Lambda><\Lambda| \end{aligned} \tag{30}$$

and

$$\begin{aligned} E_\gamma &= <\gamma(\vec{k}, \hat{\epsilon})|U|0> \\ &= <\kappa, \gamma(\vec{k}, \hat{\epsilon})|U|\Sigma, 0> |\kappa><\Sigma| + <\Lambda, \gamma(\vec{k}, \hat{\epsilon})|U|\Sigma, 0> \times \\ & \quad |\Lambda><\Sigma|, \end{aligned} \tag{31}$$

for each photon state $|\gamma(\vec{k}, \hat{\epsilon})>$. As in the previous section, a trace preserving decay channel is equivalent to unitarity of U, which in terms of the specific amplitudes is equivalent to

$$\kappa_0 = |<\kappa, 0|U|\kappa, 0>|^2 = 1, \tag{32}$$

$$\Lambda_0 = |<\Lambda, 0|U|\Lambda, 0>|^2 = 1, \tag{33}$$

and

$$\Sigma_0 + \kappa_\gamma + \Lambda_\gamma = 1, \tag{34}$$

where

$$\Sigma_0 = |<\Sigma, 0|U|\Sigma, 0>|^2, \tag{35}$$

$$\kappa_\gamma = \sum_\gamma |<\kappa, \gamma(\vec{k}, \epsilon)|U|\Sigma, 0>|^2, \tag{36}$$

and

$$\Lambda_\gamma = \sum_\gamma |<\Lambda, \gamma(\vec{k}, \hat{\epsilon})|U|\Sigma, 0>|^2. \tag{37}$$

The encoding density matrix is given by

$$\rho = a|\Sigma >< \Sigma| + b|\kappa >< \kappa| + c|\Lambda >< \Lambda| \tag{38}$$

with $a + b + c = 1$. The purification and initial QRE-constructed state is given by

$$|QRE >= (\sqrt{a}|\Sigma, \Sigma' > +\sqrt{b}|\kappa, \kappa' > +\sqrt{c}|\Lambda, \Lambda' >)|0 > \tag{39}$$

involving the reservoir Hilbert space $\mathcal{H}_R$ with basis $\{|\Sigma' >, |\kappa' >, |\Lambda' >\}$. The evolved purified state is given by

$$|Q'R'E' >= E_0|QR > |0 > + \sum_\gamma E_\gamma|QR > |\gamma >, \tag{40}$$

which upon substitution of Eqs. (30) and (31) yields,

$$\begin{aligned} |Q'R'E' > &= \sqrt{a} < \Sigma, 0|U|\Sigma, 0 > |\Sigma, \Sigma', 0 > +\sqrt{b}e^{i\alpha_\kappa t}|\kappa, \kappa', 0 > + \\ &\quad \sqrt{c}e^{i\alpha\Lambda t}|\Lambda, \Lambda', 0 > +\sqrt{a}\sum_\gamma < \kappa, \gamma|U|\Sigma, 0 > \times \\ &\quad |\kappa, \Sigma', \gamma > +\sqrt{a}\sum_\gamma < \Lambda, \gamma|U|\Sigma, 0 > |\Lambda, \Sigma', \gamma >, \end{aligned} \tag{41}$$

where we have used Eqs. (32) and (33) to represent $< \Lambda, 0|U|\Lambda, 0 >$ and $< \kappa, 0|U|\kappa, 0 >$ with pure phases α_Λ and α_κ. As in the previous section, in order to obtain the channel capacity and exchange entropy, we consider $\mathcal{T}(\rho) = Trace_{RE}(|Q'R'E' >< Q'R'E'|)$ and $\rho_{E'} = Trace_{QR}(|Q'R'E' >< Q'R'E'|)$, respectively. By substitution of Eq. (41), we obtain

$$\begin{aligned} \mathcal{T}(\rho) &= a\Sigma_0|\Sigma >< \Sigma| + a\kappa_0|\kappa >< \kappa| + a\Lambda_\gamma|\Lambda >< \Lambda| + \\ &\quad a\sum_\gamma < \kappa, \gamma|U|\Sigma, 0 >< \Lambda, \gamma|U|\Sigma, 0 >^* |\kappa >< \Lambda| + \\ &\quad a\sum_\gamma < \Lambda, \gamma|U|\Sigma, 0 >< \kappa, \gamma|U|\Sigma, 0 >^* |\Lambda >< \kappa| + \\ &\quad b|\kappa >< \kappa| + c|\Lambda >< \Lambda|, \end{aligned} \tag{42}$$

and

$$\begin{aligned} \rho_{E'} &= (a\Sigma_0 + b + c)|0 >< 0| + a\sum_\gamma \sum_{\tilde{\gamma}} < \Lambda, \gamma|U|\Sigma, 0 > \times \\ &\quad < \Lambda, \gamma|U|\Sigma, 0 >^* |\gamma >< \tilde{\gamma}|. \end{aligned} \tag{43}$$

Evaluation and interpretation of these quantities is made easier if it is assumed that the photon $\vec{k}$ - value for decay to κ is different than that for decay to Λ. This is the case if the κ and Λ particles have different rest masses. Operationally, we assume that for any given γ either $<\kappa,\gamma|U|\Sigma,0>=0$ or $<\Lambda,\gamma|U|\Sigma,0>=0$ to remove the cross terms in Eq. (42) and diagonalize Eq. (43). From Eq. (42) we then obtain the channel capacity from the Holevo bound

$$S(\mathcal{T}(\rho))-aS(\mathcal{T}(|\Sigma><\Sigma|))-bS(\mathcal{T}(|\kappa><\kappa|))-cS(\mathcal{T}(\Lambda><\Lambda|)) \tag{44}$$

as

$$\begin{aligned} C(\mathcal{T}) \; = \; & \max_{a,b,c}[-a\Sigma_0\ln(a\Sigma_0)-(a\kappa_\gamma+b)\ln(a\kappa_\gamma+b)- \\ & (a\Lambda_\gamma+c)\ln(a\Lambda_\gamma+c)+a(\Sigma_0\ln(\Sigma_0)+\kappa_\gamma\ln(\kappa_\gamma)+ \\ & \Lambda_\gamma\ln(\Lambda_\gamma)]. \end{aligned} \tag{45}$$

Note from Eq. (34) we have that the expression

$$(a\Lambda_\gamma+c)=1-(a\Sigma_0)-(a\kappa_\gamma+b) \tag{46}$$

forms a constraint on logarithm arguments in Eq. (45).

Assuming non-overlapping photon decay products, the environmental density matrix in Eq. (43) can be written,

$$\begin{aligned} \rho_{E'} \; = \; & (a\Sigma_0+b+c)|0><0|+a|\psi(\kappa,\gamma)><\psi(\kappa,\gamma)|+ \\ & a|\psi(\Lambda,\gamma)><\psi(\Lambda,\gamma)|, \end{aligned} \tag{47}$$

where the vectors $|\psi(\kappa,\gamma)>=\sum_\gamma|\gamma><\kappa,\gamma|U|\Sigma,0>$ and $|\psi(\Lambda,\gamma)>=\sum_\gamma|\gamma><\Lambda,\gamma|U|\Sigma,0>$ are orthogonal, with normalizations $<\psi(\kappa,\gamma)|\psi(\kappa,\gamma)>=\kappa_\gamma$ and $<\psi(\Lambda,\gamma)|\psi(\Lambda,\gamma)>=\Lambda_\gamma$, respectively. The exchange entropy is given by

$$S(\rho_{E'})=-(a\Sigma_0+b+c)\ln(a\Sigma_0+b+c)-(a\kappa_\gamma)\ln(a\kappa_\gamma)-(a\Lambda_\gamma)\ln(a\Lambda_\gamma). \tag{48}$$

Using Eqs. (42) and (48), we have an expression for irreversibility in Eq. (28) given by

$$\begin{aligned} Irr(\rho,\mathcal{T}) \; = \; & -a\ln a-b\ln b-c\ln c+a\Sigma_0\ln(a\Sigma_0)+ \\ & (a\kappa_\gamma+b)\ln(a\kappa_\gamma+b)+(a\Lambda_\gamma+c)\ln(a\Lambda_\gamma+c)- \\ & (a\Sigma_0+b+c)\ln(a\Sigma_0+b+c)-(a\kappa_\gamma)\ln(a\kappa_\gamma)- \\ & (a\Lambda_\gamma)\ln(a\Lambda_\gamma). \end{aligned} \tag{49}$$

Figure 4: Optimum particle mixing versus time for branching decay with $\tau_\kappa = \tau_\Lambda = 1.0$(A.U.).

The evaluation of channel capacity and irreversibility requires the computation of Σ_0, κ_γ, and Λ_γ for Eqs. (45) and (49). Assuming lifetimes τ_κ and τ_Λ of Σ to the κ and Λ particles, respectively, we obtain

$$\Sigma_0 = | < \Sigma, 0|U|\Sigma, 0 > |^2 = e^{-t/\tau_\Sigma}, \tag{50}$$

$$\kappa_\gamma = \sum_\gamma | < \kappa, \gamma|U|\Sigma, 0 > |^2 = \frac{\tau_\Sigma}{\tau_\kappa}(1 - e^{-t/\tau_\Sigma}), \tag{51}$$

and

$$\Lambda_\gamma = \sum_\gamma | < \Lambda, \gamma|U|\Sigma, 0 > |^2 = \frac{\tau_\Sigma}{\tau_\Lambda}(1 - e^{-t/\tau_\Sigma}), \tag{52}$$

where

$$\frac{1}{\tau_\Sigma} = \frac{1}{\tau_\kappa} + \frac{1}{\tau_\Lambda}. \tag{53}$$

The optimum mixing can be determined from Eq. (45) as a function of time. In the limit $t \to 0$, we have $\Sigma_0 \to 1$, and $\kappa_\gamma, \Lambda_\gamma \to 0$ with $C(\mathcal{T}) = \max_{a,b,c}(-a \ln a - b \ln b - c \ln c)$ maximum at $a = b = c = 1/3$. In the limit $t \to \infty$, we have $\Sigma_0 \to 0$, $\kappa_\gamma \to \tau_\Sigma/\tau_\kappa$, and $\Lambda_\gamma \to \tau_\Sigma/\tau_\Lambda$, reflecting the branching ratios to the decay products. In this limit, the channel capacity is given by

$$\begin{aligned} C(\mathcal{T}) \;&=\; \max_{a,b}[-(a\kappa_\gamma + b)\ln(a\kappa_\gamma + b) - \\ &\quad (1 - a\kappa_\gamma - b)\ln(1 - a\kappa_\gamma - b) + a(\kappa_\gamma \ln(\kappa_\gamma) + \\ &\quad \Lambda_\gamma \ln(\Lambda_\gamma))], \end{aligned} \tag{54}$$

which is maximum for $(a\kappa_\gamma + b) = 0.5$ and $a = 0$. Consequently, we obtain 50% mixing of the κ and Λ decay products in the initial density matrix and a channel capacity of one. While it is possible to obtain $a_{\max}(t)$ and $b_{\max}(t)$ by setting the gradient of Eq. (45) to zero, we instead performed a grid search on the domain (a, b) with $c(t) = 1 - a(t) - b(t)$. Plots of optimum mixings $a(t)$ and $b(t)$ are shown in Figure 4 for the case $\tau_\kappa = \tau_\Lambda = 1.0$ (A.U.). The corresponding channel capacity from Eq. (45) is shown in Figure 5.

Figure 5: Channel capacity versus time for branching decay with $\tau_\kappa = \tau_\Lambda = 1.0$(A.U.).

Figure 6: Irreversibility versus time at the optimum particle mixing for branching decay with $\tau_\kappa = \tau_\Lambda = 1.0$(A.U.).

Figure 6 contains the irreversibility versus time from Eq. (49) with optimum particle mixing in Figure 4. Figures 5 and 6 demonstrate the decrease in channel capacity and increase in irreversibility as the decay progresses. Note, however, that as $t \to \infty$ the quantum states evolve to an reversible equal mixture of κ and Λ. The $t \to \infty$ asymptotic values for irreversibility are achieved at $t < 7$(A.U.).

3.2 Cascading Decay

In this section, channel capacity and reversibility are computed for the $\{\Sigma, \kappa, \Lambda\}$ system in which the implied ensemble is observed through a $\Sigma \to \kappa \to \Lambda$ cascading decay. The environment consists of the non-system decay products, γ-rays $\gamma(\vec{k}, \hat{\epsilon})$ from either the first, second, *or combined* decays. The operator elements are given by

$$\begin{aligned} E_0 &= <0|U|0> \\ &= <\Sigma, 0|U|\Sigma, 0> |\Sigma><\Sigma| + <\kappa, 0|U|\kappa, 0> |\kappa><\kappa| + \\ &\quad <\Lambda, 0|U|\Lambda, 0> |\Lambda><\Lambda|, \end{aligned} \tag{55}$$

$$\begin{aligned} E_\gamma &= <\gamma(\vec{k}, \hat{\epsilon})|U|0> \\ &= <\kappa, \gamma(\vec{k}, \hat{\epsilon})|U|\Sigma, 0> |\kappa><\Sigma| + <\Lambda, \gamma(\vec{k}, \hat{\epsilon})|U|\kappa, 0> \times \\ &\quad |\Lambda><\kappa|, \end{aligned} \tag{56}$$

and

$$\begin{aligned} E_{\gamma\gamma'} &= <\gamma(\vec{k}, \hat{\epsilon}), \gamma'(\vec{k}', \hat{\epsilon}')|U|0> \\ &= <\Lambda, \gamma(\vec{k}, \hat{\epsilon}), \gamma'(\vec{k}', \hat{\epsilon}')|U|\Sigma, 0> |\Lambda><\Sigma|, \end{aligned} \tag{57}$$

for each photon state $|\gamma(\vec{k}, \hat{\epsilon})$ or $|\gamma(\vec{k}, \hat{\epsilon}), \gamma'(\vec{k}', \hat{\epsilon}') >$. By direct substitution, the trace preserving condition

$$E_0^\dagger E_0 + \sum_\gamma E_\gamma^\dagger E_\gamma + \sum_{\gamma\gamma'} E_{\gamma\gamma'}^\dagger E_{\gamma\gamma'} = I \tag{58}$$

is equivalent to

$$\Sigma_0 + \kappa_\gamma + \Lambda_{\gamma\gamma'} = 1, \tag{59}$$

$$\kappa_0 + \Lambda_\gamma = 1, \tag{60}$$

and

$$\Lambda_0 = 1, \tag{61}$$

all following as well from the unitarity of U, and where

$$\Sigma_0 = | < \Sigma, 0|U|\Sigma, 0 > |^2, \tag{62}$$

$$\kappa_0 = | < \kappa, 0|U|\kappa, 0 > |^2, \tag{63}$$

$$\Lambda_0 = | < \Lambda, 0|U|\Lambda, 0 > |^2, \tag{64}$$

$$\kappa_\gamma = \sum_\gamma | < \kappa, \gamma|U|\Sigma, 0 > |^2, \tag{65}$$

$$\Lambda_\gamma = \sum_\gamma | < \Lambda, \gamma|U|\kappa, 0 > |^2, \tag{66}$$

and

$$\Lambda_{\gamma\gamma'} = \sum_{\gamma\gamma'} | < \Lambda, \gamma, \gamma'|U|\Lambda, 0 > |^2, \tag{67}$$

are the cascade decay parameters.

As discussed in Section 3.1, the encoding density matrix and QRE-purification are given by Eqs. (38) and (39), respectively, involving a reservoir Hilbert space $\mathcal{H}_R$ with basis $\{|\Sigma' >, |\kappa' >, |\Lambda' >\}$. The evolution of this purification is given by

$$\begin{aligned} |Q'R'E' > &= E_0|QR > |0 > + \sum_\gamma E_\gamma|QR > |\gamma > + \\ & \sum_{\gamma\gamma'} E_{\gamma\gamma'}|QR > |\gamma\gamma' > \end{aligned} \tag{68}$$

with the result

$$
\begin{aligned}
|Q'R'E'> \;\; = \;\; & \sqrt{a} < \Sigma, 0|U|\Sigma, 0 > |\Sigma, \Sigma', 0 > + \\
& \sqrt{b} < \kappa, 0|U|\kappa, 0 > |\kappa, \kappa', 0 > + \sqrt{c} e^{i\alpha t}|\Lambda, \Lambda', 0 > + \\
& \sqrt{a} \sum_{\gamma} < \kappa, \gamma|U|\Sigma, 0 > |\kappa, \Sigma', \gamma > + \\
& \sqrt{b} \sum_{\gamma} < \Lambda, \gamma|U|\kappa, 0 > |\Lambda, \kappa', \gamma > + \\
& \sqrt{a} \sum_{\gamma\gamma'} < \Lambda, \gamma, \gamma'|U|\Sigma, 0 > |\Lambda, \Sigma', \gamma, \gamma' >, \qquad (69)
\end{aligned}
$$

where a pure phase α is substituted for $< \Lambda, 0|U|\Lambda, 0 >$ from Eq. (61). Tracing out the environment and reservoir Hilbert spaces in the state $|Q'R'E' >< Q'R'E'|$, we obtain the evolved system density matrix

$$
\begin{aligned}
\mathcal{T}(\rho) \;\; = \;\; & a\Sigma_0|\Sigma >< \Sigma| + (b\kappa_0 + a\kappa_\gamma)|\kappa >< \kappa| + \\
& (c\Lambda_0 + b\Lambda_\gamma + c\Lambda_{\gamma\gamma'})|\Lambda >< \Lambda|. \qquad (70)
\end{aligned}
$$

Tracing out the system and reservoir Hilbert spaces in $|Q'R'E' >< Q'R'E'|$, we obtain the environmental density matrix

$$
\begin{aligned}
\rho_{E'} \;\; = \;\; & (a\Sigma_0 + b\kappa_0 + c\Lambda_0)|0 >< 0| + (a\kappa_\gamma)|\hat{\psi}_\kappa >< \hat{\psi}_\kappa| + \\
& (b\Lambda_\gamma)|\hat{\psi}_\Lambda >< \hat{\psi}_\Lambda| + (a\Lambda_{\gamma\gamma'})|\hat{\phi}_\Lambda >< \hat{\phi}_\Lambda|, \qquad (71)
\end{aligned}
$$

with the vectors

$$|\psi_\kappa >= \sum_{\gamma} < \kappa, \gamma|U|\Sigma, 0 > |\gamma >, \qquad (72)$$

$$|\psi_\Lambda >= \sum_{\gamma} < \Lambda, \gamma|U|\kappa, 0 > |\gamma >, \qquad (73)$$

and

$$|\phi_\Lambda >= \sum_{\gamma\gamma'} < \Lambda, \gamma, \gamma'|U|\Sigma, 0 > |\gamma, \gamma' >, \qquad (74)$$

having normalizations $< \psi_\kappa|\psi_\kappa >= \kappa_\gamma$, $< \psi_\Lambda|\psi_\Lambda >= \Lambda_\gamma$, and $< \phi_\Lambda|\phi_\Lambda >= \Lambda_{\gamma\gamma'}$. Note that we assume non-overlapping γ-rays for the decays $\Sigma \to \kappa$ and $\kappa \to \Lambda$ in order to have $< \psi_\kappa|\psi_\Lambda >= 0$.

Computing the entropies of the density matrices in Eq. (70) for arbitrary a, b, and $c = (1 - a - b)$, and in turn for $a = 1$, $b = 1$, and $c = 1$, we obtain the channel capacity from Eq. (44) given by

$$\begin{aligned} C(\mathcal{T}) &= \max_{a,b,c}[-a\Sigma_0 \ln(a\Sigma_0) - (b\kappa_0 + a\kappa_\gamma)\ln(b\kappa_0 + a\kappa_\gamma) - \\ &\quad (1 - a\Sigma_0 - b\kappa_0 - a\kappa_\gamma)\ln(1 - a\Sigma_0 - b\kappa_0 - a\kappa_\gamma) + \\ &\quad a(\Sigma_0 \ln(\Sigma_0) + \kappa_\gamma \ln(\kappa_\gamma) + \Lambda_{\gamma\gamma'} \ln(\Lambda_{\gamma\gamma'})) + b(\kappa_0 \ln(\kappa_0) + \\ &\quad \Lambda_\gamma \ln(\Lambda_\gamma))]. \end{aligned} \tag{75}$$

Assuming non-overlapping γ-rays, we obtain the exchange entropy from Eq. (71) given by,

$$\begin{aligned} S(\rho_{E'}) &= -(a\Sigma_0 + b\kappa_0 + c\Lambda_0)\ln(a\Sigma_0 + b\kappa_0 + c\Lambda_0) - \\ &\quad (a\kappa_\gamma)\ln(a\kappa_\gamma) - (b\Lambda_\gamma)\ln(b\Lambda_\gamma) - (a\Lambda_{\gamma\gamma'}) \times \\ &\quad \ln(a\Lambda_{\gamma\gamma'}), \end{aligned} \tag{76}$$

for an irreversibility measure from Eqs. (28), (70) and (76) given by

$$\begin{aligned} Irr(\rho, \mathcal{T}) &= -a\ln a - b\ln b - c\ln c - \\ &\quad (a\Sigma_0 + b\kappa_0 + c\Lambda_0)\ln(a\Sigma_0 + b\kappa_0 + c\Lambda_0) - \\ &\quad (a\kappa_\gamma)\ln(a\kappa_\gamma) - (b\Lambda_\gamma)\ln(b\Lambda_\gamma) - (a\Lambda_{\gamma\gamma'})\ln(a\Lambda_{\gamma\gamma'}) + \\ &\quad (a\Sigma_0)\ln(a\Sigma_0) + (b\kappa_0 + a\kappa_\gamma)\ln(b\kappa_0 + a\kappa_\gamma) + \\ &\quad (1 - a\Sigma_0 - b\kappa_0 - a\kappa_\gamma)\ln(1 - a\Sigma_0 - b\kappa_0 - a\kappa_\gamma). \end{aligned} \tag{77}$$

In order to compute the channel capacity and irreversibility, we need to input the cascade parameters in Eqs. (75) and (77),

$$\Sigma_0 = |<\Sigma, 0|U|\Sigma, 0>|^2 = e^{-t/\tau_\Sigma}, \tag{78}$$

$$\kappa_0 = |<\kappa, 0|U|\kappa, 0>|^2 = e^{-t/\tau_\kappa}, \tag{79}$$

$$\Lambda_\gamma = (1 - \kappa_0) = (1 - e^{-t/\tau_\kappa}), \tag{80}$$

$$\kappa_\gamma = \frac{\tau_\kappa}{\tau_\Sigma - \tau_\kappa}(e^{-t/\tau_\Sigma} - e^{-t/\tau_\kappa}), \tag{81}$$

and

$$\Lambda_{\gamma\gamma'} = (1 - \Sigma_0 - \kappa_\gamma), \tag{82}$$

in terms of the Σ and κ lifetimes τ_Σ and τ_κ, respectively. The expression in Eq. (81) was derived by solving the coupled differential

Figure 7: Optimum particle mixing versus time for cascade decay with $\tau_\Sigma = 2\tau_\kappa = 2.0$ (A.U.).

Figure 8: Channel capacity versus time for cascade decay with $\tau_\Sigma = 2\tau_\kappa = 2.0$ (A.U.).

equations for the $\Sigma \to \kappa \to \Lambda$ cascaded decay, and interpreting the κ particle growth in terms of κ_γ in Eq. (65).

In the limit $t \to 0$, we have $\Sigma_0 \to 1$, $\kappa_0 \to 1$, $\Lambda_\gamma \to 0$, $\kappa_\gamma \to 0$, and $\Lambda_{\gamma\gamma'} \to 0$ to obtain $C(\mathcal{T}) = \max_{a,b}[-a\ln a - b\ln b - c\ln c]$, given by $a = b = c = 1/3$. In the limit $t \to \infty$, we have $\Sigma_0 \to 0$, $\kappa_0 \to 0$, $\Lambda_\gamma \to 1$, $\kappa_\gamma \to 0$ and $\Lambda_{\gamma\gamma'} \to 1$ to obtain $C(\mathcal{T}) \to 0$ so that the mixing is indeterminate. A grid search on the domain (a, b) to obtain the solution to Eq. (75) resulted in time dependent $a_{\max}(t)$ and $b_{\max}(t)$ shown in Figure 7 (for $\tau_\Sigma = 2\tau_\kappa = 2.0$ (A.U.)). The channel capacity versus time is shown in Figure 8 to decrease to zero, and the irreversibility versus time as the decay progresses is shown in Figure 9. In this case the asymptotic values were obtained for $t > 10$.

4 Conclusion

In this paper we developed a framework from quantum information theory for interpreting particle decays and measurement as a quantum channel. It was shown that channel capacity, a measure of the most efficient transfer of particle identity across the channel, implies optimum mixings between particles.

We are searching for an extended quantum mechanics that explains fermionic generations and mixings. This phenomenon is utterly mysterious, arises at the scale of the weak interaction, and may point to an incomplete understanding of high energy quantum mechanics. Note that the postulated physics of the implied

Figure 9: Irreversibility versus time for cascade decay at optimum mixing for $\tau_\Sigma = 2\tau_\kappa = 2.0$ (A.U.).

ensemble depends on the particle interaction only through decay modes and lifetimes. In a previous paper, the 50% left-right mixing in the Dirac equation, arising at length scales $\hbar/mc$ with the appearance of negative energy states, was suggested to extend to the Cabibbo-Kobayashi-Maskawa (CKM) generational mixing matrix. The principles involved in the extension, classical-quantum complementarity, suggested a generational structure but resulted in no quantitative estimates of the mixing. In this paper, mixing is explained naturally by maximal accessible distinguishability in the implied ensemble. Accessible distinguishability as a principle for CKM phenomenology will be explored in a forthcoming paper.

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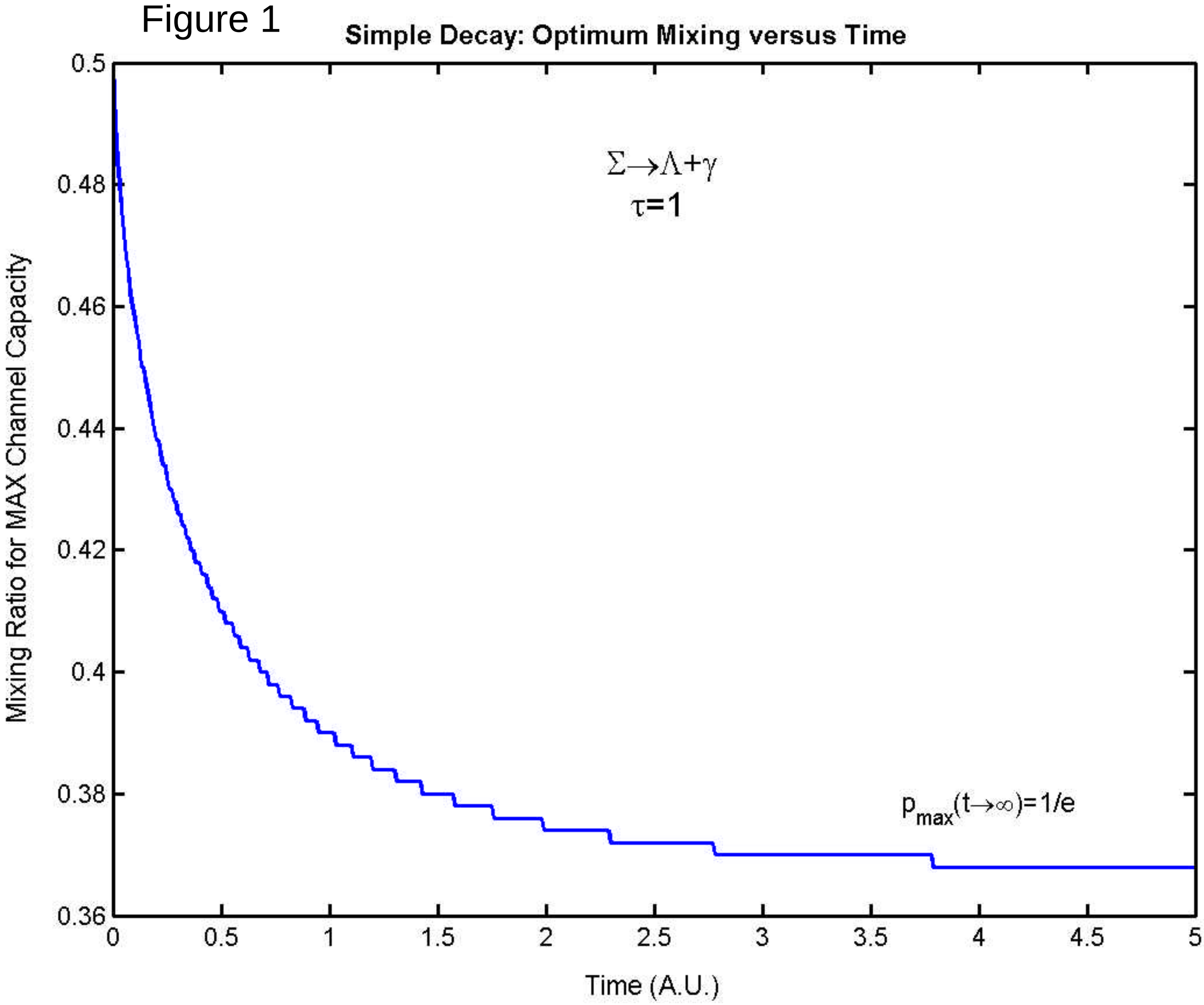
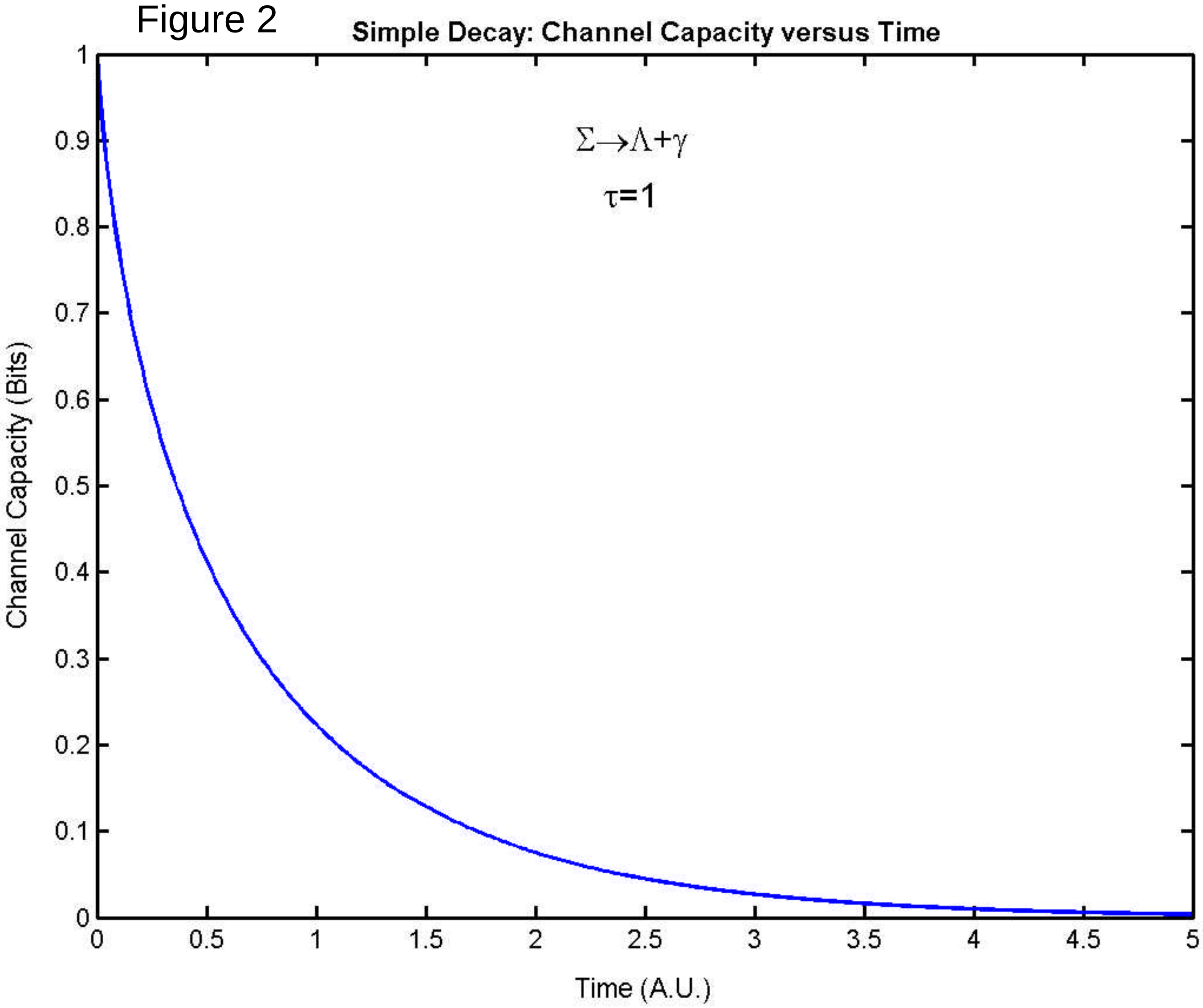
Figure 1



Simple Decay: Optimum Mixing versus Time
Σ→Λ+γ
τ=1
p max(t→∞)=1/e
Mixing Ratio for MAX Channel Capacity
Time (A.U.)
0.5
0.48
0.46
0.44
0.42
0.4
0.38
0.36
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Figure 2



Simple Decay: Channel Capacity versus Time
Σ→Λ+γ
τ=1
Channel Capacity (Bits)
Time (A.U.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Figure 3

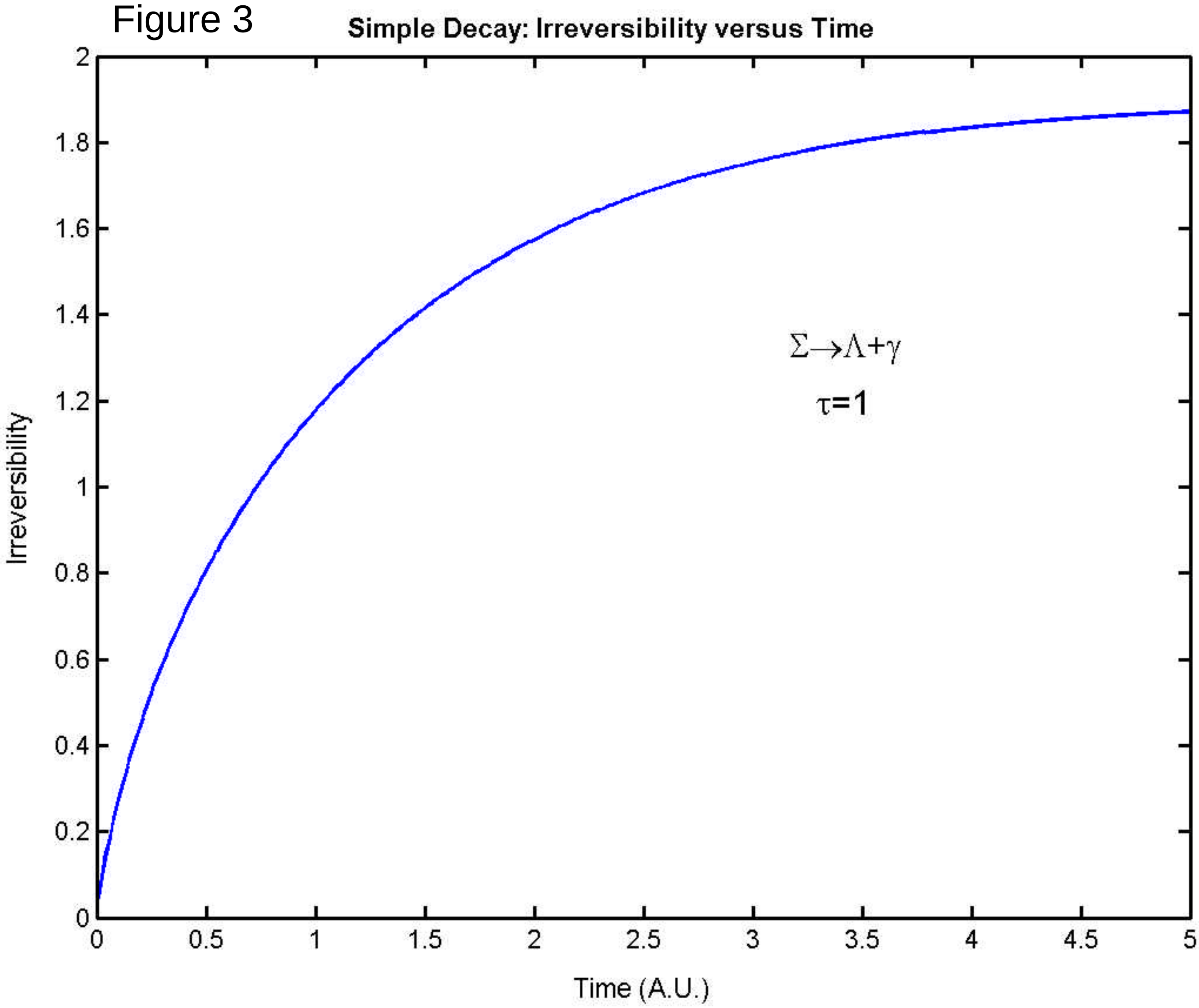
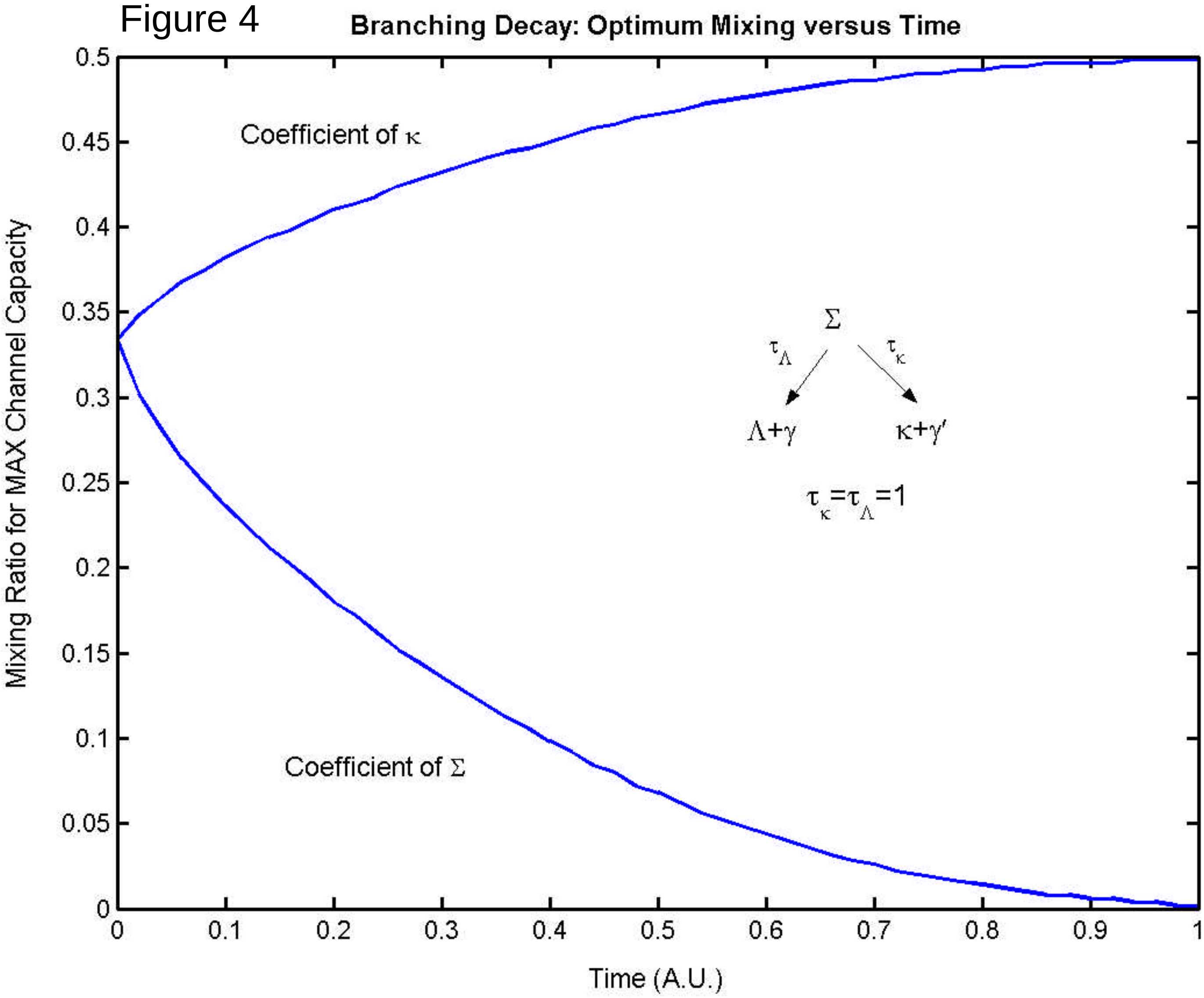
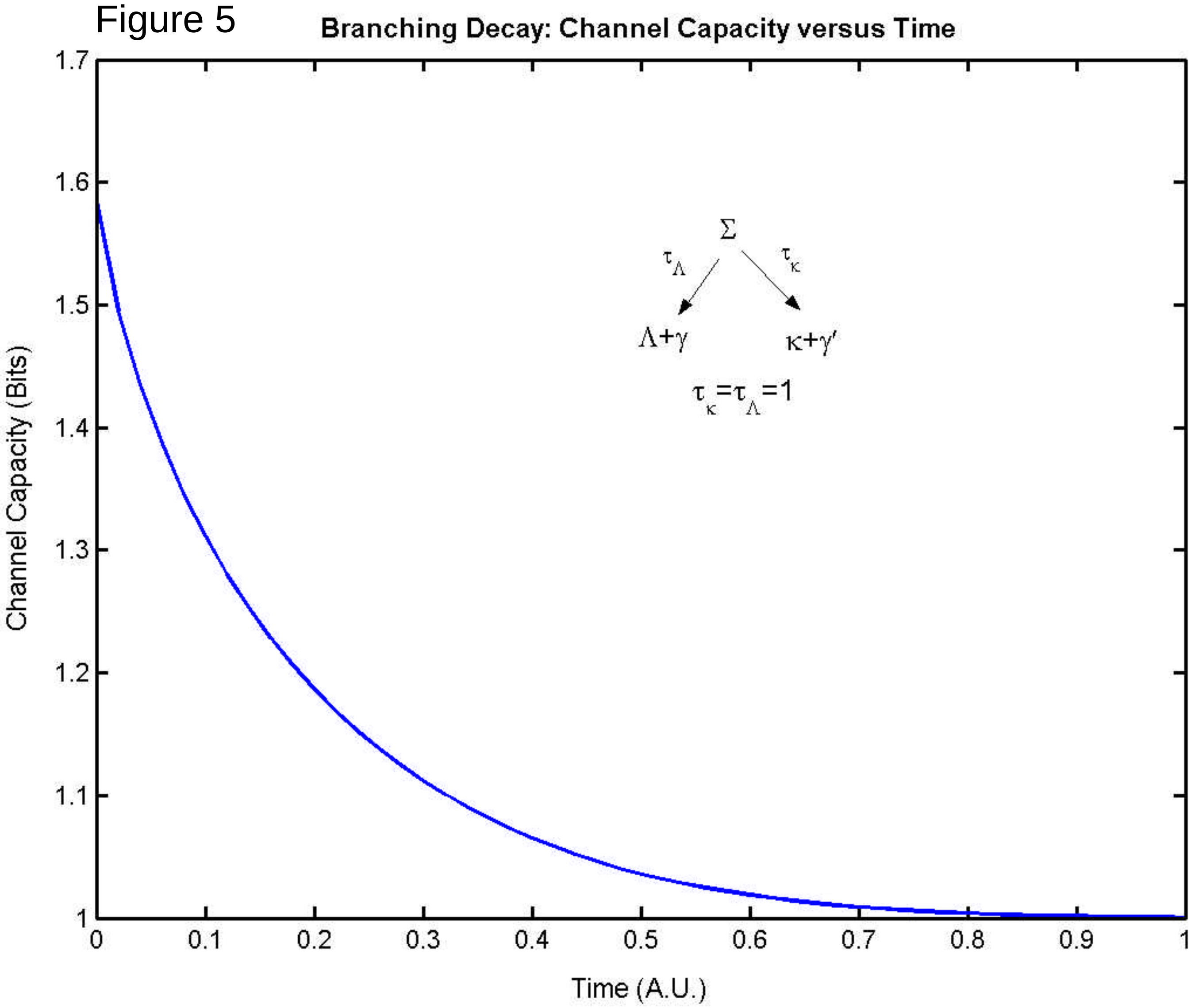

Simple Decay: Irreversibility versus Time
Irreversibility
Time (A.U.)
Σ→Λ+γ
τ=1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Figure 4



Branching Decay: Optimum Mixing versus Time
Mixing Ratio for MAX Channel Capacity
Time (A.U.)
Coefficient of κ
Coefficient of Σ
Σ
τ_Λ
τ_κ
Λ+γ
κ+γ′
τ_κ=τ_Λ=1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 5



Branching Decay: Channel Capacity versus Time
Channel Capacity (Bits)
Time (A.U.)
Σ
τ_Λ
τ_κ
Λ+γ
κ+γ′
τ_κ=τ_Λ=1

Figure 6

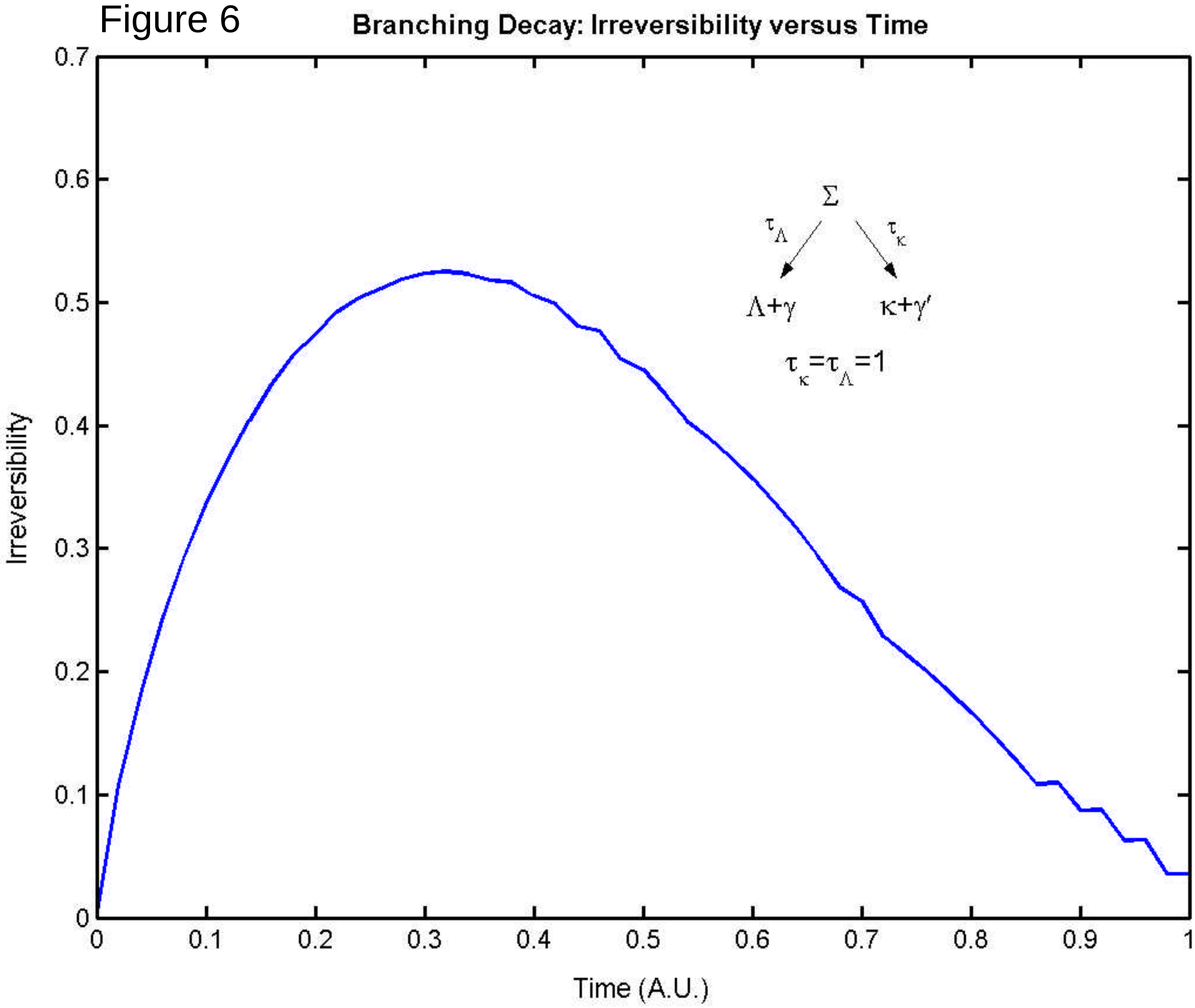

Branching Decay: Irreversibility versus Time
Irreversibility
Time (A.U.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Σ
τ_Λ
τ_κ
$\Lambda+\gamma$
$\kappa+\gamma'$
$\tau_\kappa=\tau_\Lambda=1$

Figure 7

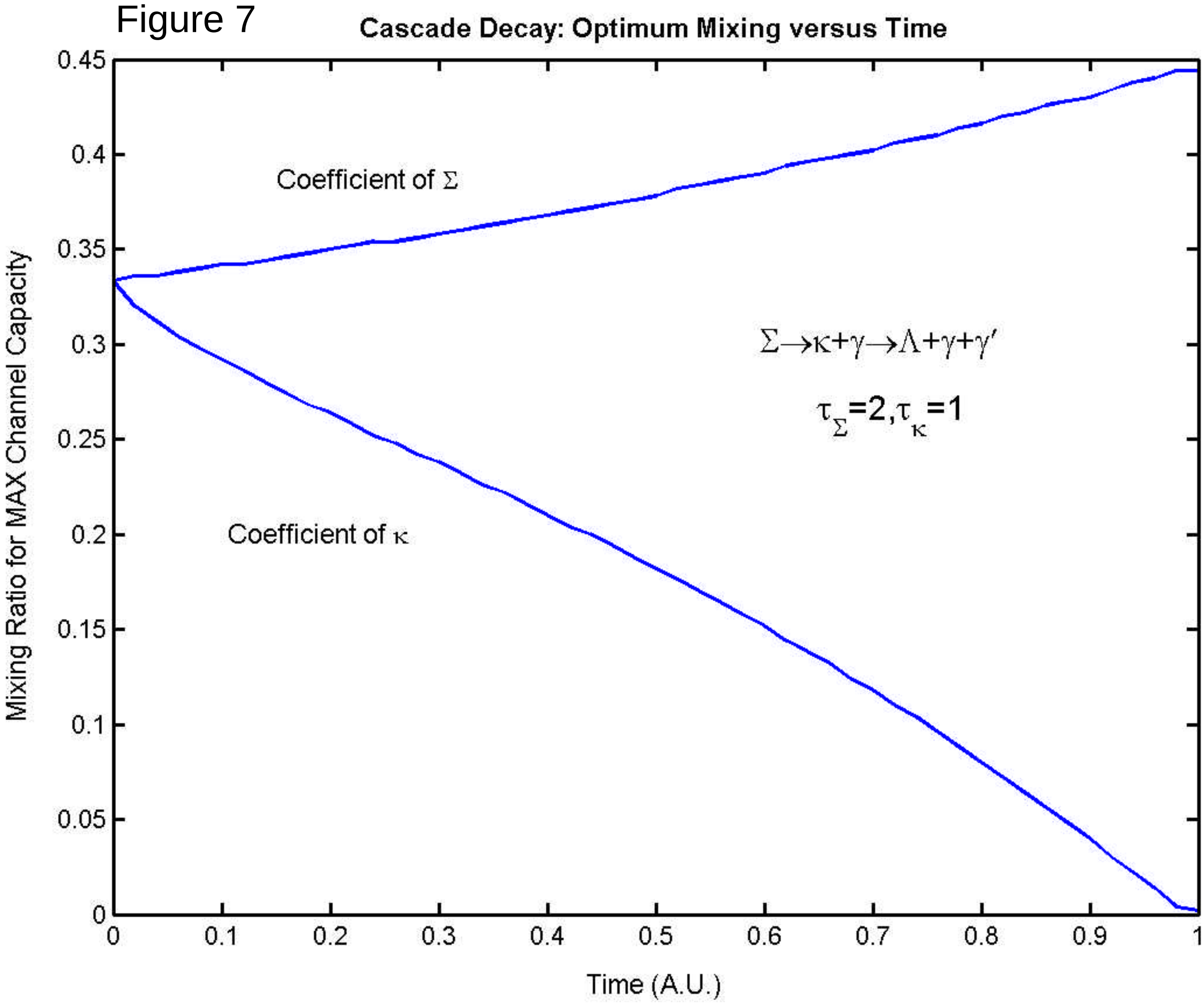
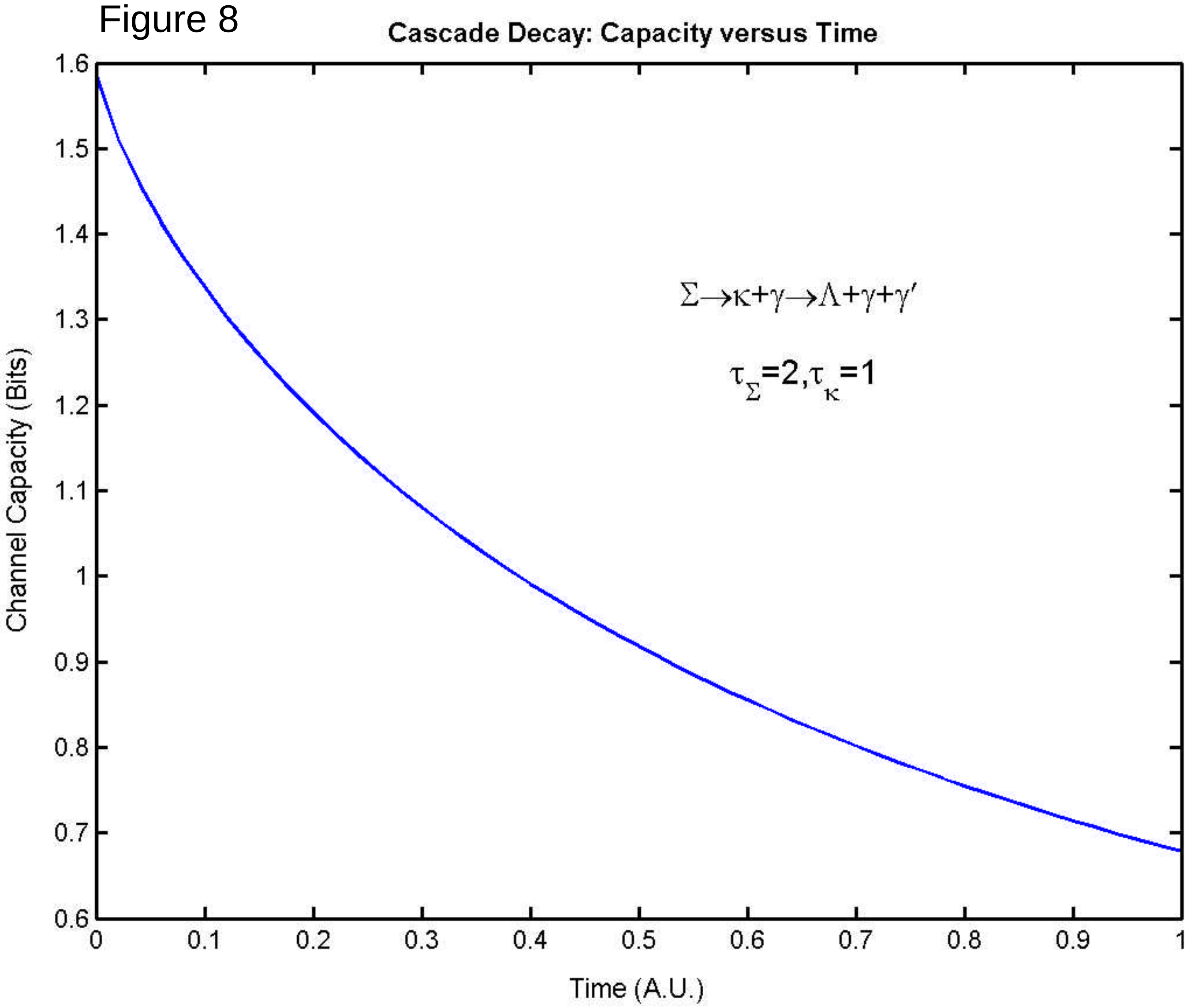
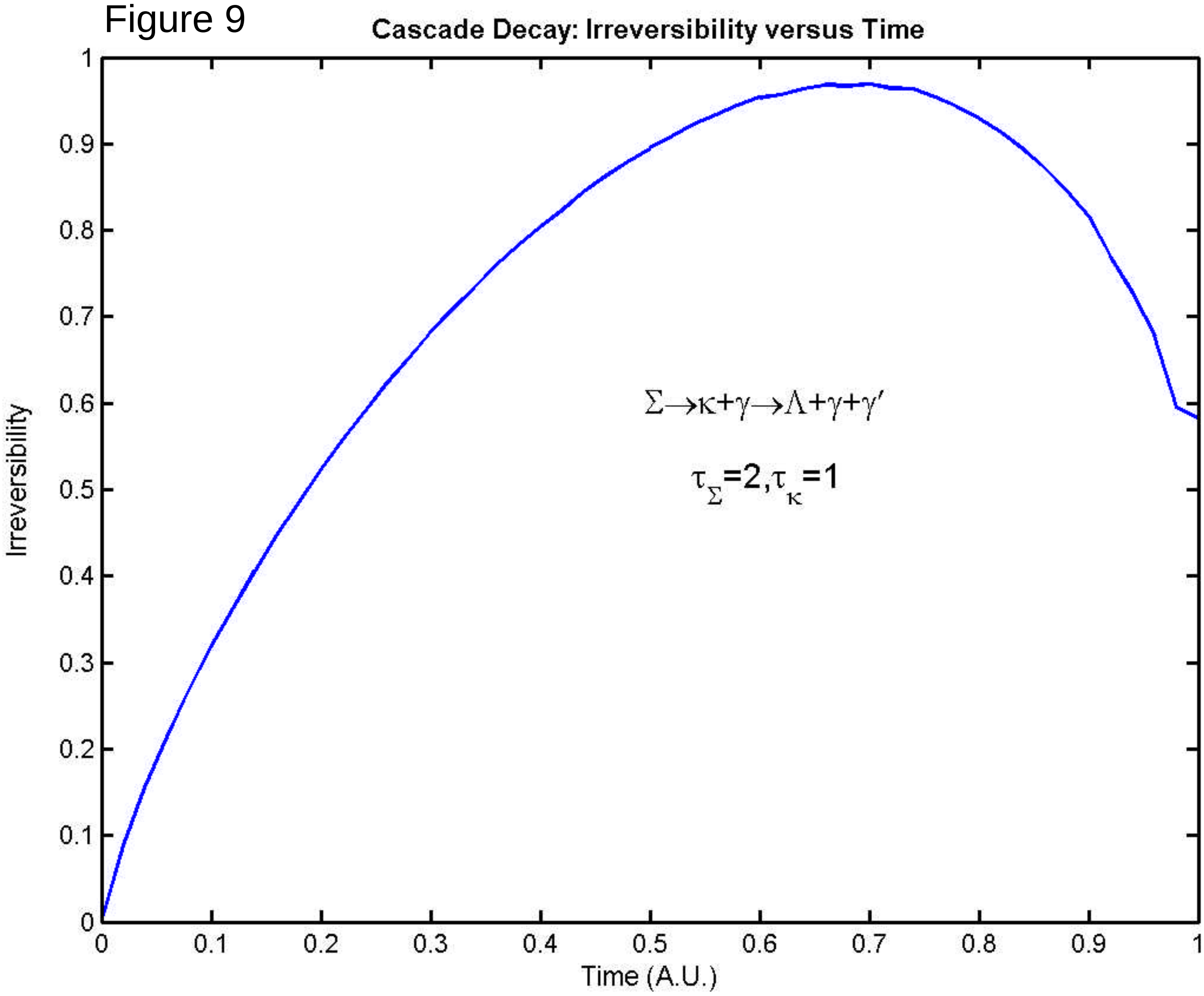

Cascade Decay: Optimum Mixing versus Time
Coefficient of Σ
Coefficient of κ
Σ→κ+γ→Λ+γ+γ′
τ_Σ=2,τ_κ=1
Mixing Ratio for MAX Channel Capacity
Time (A.U.)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 8



Cascade Decay: Capacity versus Time
Channel Capacity (Bits)
Time (A.U.)
$\Sigma \rightarrow \kappa + \gamma \rightarrow \Lambda + \gamma + \gamma'$
$\tau_\Sigma = 2, \tau_\kappa = 1$
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 9



Cascade Decay: Irreversibility versus Time
Irreversibility
Time (A.U.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Σ→κ+γ→Λ+γ+γ′
τ_Σ=2,τ_κ=1