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Power Means Calculus

Power Means Calculus: Product Calculus, Harmonic Mean Calculus, and Quadratic Mean Calculus

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Abstract We describe a generalized calculus that was suggested by Michael Spivey's [Spiv] observation of the relation between the Geometric Mean of a function over an interval, and its product integral.

We will see that each Power Mean of order $r \neq 0$,

$$\left(\frac{a_1^r + a_2^r + \ldots a_n^r}{n}\right)^{\frac{1}{r}}$$

is associated with a *Power Mean Derivative of order* r,

$$D^{(r)}.$$

The Fermat/Newton/Leibnitz Derivative

$$D^{(1)} = D \equiv \frac{d}{dx}$$

is associated with the Arithmetic Mean

$$\frac{a_1 + a_2 + \ldots a_n}{n},$$

which is Power Mean of order $r = 1$.

The Geometric Mean Derivative

$$D^{(0)}$$

is associated with the Geometric Mean

$$\left(a_1 a_2 \ldots a_n\right)^{1/n}$$

which is Power Mean of order $r \to 0$ [Kaza].

Product Integration is an operation inverse to the Geometric Mean Derivative. Both are multiplicative operations, that apply naturally to products, and in particular to $\Gamma(z)$, the analytic extension of the factorial function

The Harmonic Mean Derivative

$$D^{(-1)}$$

is associated with the Harmonic Mean

$$\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\ldots\frac{1}{a_n}}$$

which is Power Mean of order $r = -1$.

The Quadratic Mean Derivative

$$D^{(2)}$$

is associated with the Power Mean of order $r = 2$,

$$\left(\frac{a_1^2+a_2^2+\ldots a_n^2}{n}\right)^{\frac{1}{2}}.$$

The inverse operation, the Quadratic Mean Integration transforms a function to its L^2 norm squared.