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Gauge Institute Journal

Feynman Integral

Feynman Integral and Convolution Products **H. Vic Dannon**

Abstract: De-Broglie associated with a moving particle a wave, and Schrodinger derived for a particle of mass m moving in the direction of x, free of forces, the wave equation

$$\frac{\partial \psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial x^2}.$$

Born suggested that $\left|\psi(x,t)\right|^2$ is the probability to find the particle in the time interval $[t, t+dt]$, and in the length interval $\left[x, x+dx\right]$.

If the particle is at x_a, at time t_a, and at x_b, at time t_b, its Schrodinger's wave function is given by

$$\psi(x_b, t_b) = \int_{x_a=-\infty}^{x_a=\infty} K(x_b, t_b; x_a, t_a)\psi(x_a, t_a)dx_a,$$

where

$$K(x_b,t_b;x_a,t_a) = \left[\frac{m}{2\pi i\hbar(t_b - t_a)}\right]^{1/2} \exp\frac{i}{\hbar}\int_{t=t_a}^{t=t_b} \mathcal{L}(x_{\min},\dot{x}_{\min},t)dt\,,$$

is Schrodinger's Propagator, and $x_{\min}(t)$ is the least-energy-path of the particle.

Feynman attempted to express $K(x_b,t_b;x_a,t_a)$ by what is known as an Infinite Convolution Product.

He showed that if the path is partitioned into the segments

$$a = x_0, x_1, x_2 ... x_N = b$$

so that

$$t_a = t_0 < t_1 < t_2 < ... < t_N = t_b\,,$$

then

$$K(x_b,t_b;x_{N-1},t_{N-1}) * K(x_{N-1},t_{N-1};x_{N-2},t_{N-2}) * ... * K(x_1,t_1;x_a,t_a) = K(x_a,t_a;x_b,t_b)\,,$$

where the symbol $*$ denotes the Convolution Product.

This equality fails to yield an Infinite Convolution Product.

To obtain a genuine Infinite Convolution Product, the path partition points must be determined by zeros of the Laplace Transform of the Kernel $K(x_a,t_a;x_b,t_b)$, and the Kernel has to be generated by basic Non-Schrodinger kernels that depend on those zeros.

Feynman's taking the limit $N \to \infty$ makes no difference, because the equality holds for any natural number N, and the limit process does not obtain any new result.

Feynman's Path Integral amounts to impossible expectations of the Schrodinger Propagator.

In the following, we derive the Schrodinger Propagator for a particle in a force-free motion, and for a harmonic oscillator, in order to understand Feynman's failed attempt to express these Propagators as Infinite Convolution Products.

Then, we use Polya-Laguerre theory of Infinite Convolution Products to explain why Feynman's Path Integral does not exist, why Schrodinger's Propagator need not be renamed after Feynman, and why Feynman's New approach to Quantum Mechanics is Schrodinger's Wave Mechanics.