American Journal of Physics claims that a Harmonically Oscillating Mass has Zero Point Energy

H. Vic Dannon
vic0@comcast.net
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Abstract: In a 2003 article [Boyer1], the American Journal of Physics fantasized that

- A harmonically oscillating point mass has electromagnetic energy
- The mechanical energy of harmonically oscillating point is a thermodynamical internal energy
- The generalized momentum of the harmonic oscillator is a constant.
- The frequency of the lossless mechanical oscillations is a thermodynamic variable

And this leads to Planck's Radiation Law, with no need for quantum assumptions. $\hbar$ appears from nowhere after mumbling about asymptotics, and interpolation.

In 2018, this new physics was proudly recited in another seminal article, [Boyer2], by the American Journal of Physics.

Contents

0.

The Harmonic Oscillator

A point mass

$$m,$$

at the end of a spring with constant

$$k,$$

that satisfies Hook's Force Law

$$F = -kx$$

accelerating along x by Newton's Law,

$$F = m\ddot{x},$$

satisfies the equation

$$m\ddot{x} = -kx,$$

$$\ddot{x} + \underbrace{\frac{k}{m}}_{\omega^2} x = 0,$$

$$\ddot{x} + \omega^2 x = 0$$

This is the equation of harmonic motion with angular velocity ω, that has the solution

$$x(t) = A\cos\omega(t + \varphi),$$

where the amplitude A, and the phase shift φ are determined from given initial, and boundary conditions.

As well, A, and φ may be determined by the Hamilton-Jacobi Method, as discussed in [Goldstein, p.442]

0.1 The Momentum, and the Amplitude Dependence on the Energy

The Energy of the system (its Hamiltonian) is the sum of its kinetic energy,

$$\tfrac{1}{2}m\upsilon^2 = \tfrac{1}{2m}p^2$$

and potential energy

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 .$$

That is,

$$E(x,p) = \tfrac{1}{2m}p^2 + \tfrac{1}{2}m\omega^2x^2 .$$

The Action

$$S = S(x,E,t)$$

is defined by the equations

$$\partial_x S = p = \sqrt{2mE - m^2\omega^2x^2} ,$$

$$\partial_t S = -E(x,p) ,$$

Therefore,

$$S = \int p\,dx - Et ,$$

$$= \int \sqrt{2mE - m^2\omega^2x^2}\,dx - Et$$

Hence,

$$\partial_E S = \int \frac{2m}{2\sqrt{2mE - m^2\omega^2x^2}}\,dx - t$$

$$= \frac{1}{\omega}\int \frac{1}{\sqrt{1 - \frac{m\omega^2x^2}{2E}}}\,d\underbrace{\sqrt{\frac{m\omega^2x^2}{2E}}}_{\xi} - t$$

$$= \frac{1}{\omega}\int \frac{1}{\sqrt{1-\xi^2}}\,d\xi - t$$

Thus,

$$\omega(t + \partial_E S) = \int \frac{1}{\sqrt{1-\xi^2}} d\xi$$

$$= \arcsin \xi ,$$

That is,

$$\sin \omega(t + \partial_E S) = \xi$$

$$= \sqrt{\frac{m}{2E}} \omega x$$

Therefore,

$$x(t) = \sqrt{\frac{2E}{m\omega^2}} \sin \omega(t + \partial_E S) ,$$

$$p(t) = m\dot{x}(t)$$

$$= m\omega \sqrt{\frac{2E}{m\omega^2}} \cos \omega(t + \partial_E S),$$

$$= \sqrt{2mE} \cos \omega(t + \partial_E S)$$

0.2 Transformation to Generalized Momentum and Path

Put

$$x = \sqrt{\frac{2E}{m\omega^2}} \sin \theta ,$$

$$dx = \sqrt{\frac{2E}{m\omega^2}} \cos \theta d\theta ,$$

$$p = \sqrt{2mE} \cos \theta ,$$

Then, the Action variable J [Goldstein, p.462] is given by

$$J = \oint p dx ,$$

$$= \int_{\theta=0}^{\theta=2\pi} \sqrt{2mE}\cos\theta\sqrt{\frac{2E}{m\omega^2}}\cos\theta d\theta$$

$$= 2\frac{E}{\omega}\int_{\theta=0}^{\theta=2\pi} \cos^2\theta d\theta$$

$$= 2\frac{E}{\omega}\int_{\theta=0}^{\theta=2\pi} \tfrac{1}{2}(\cos 2\theta + 1)d\theta$$

$$= \frac{E}{\omega}\Big[\tfrac{1}{2}\sin 2\theta + \theta\Big]_{\theta=0}^{\theta=2\pi}$$

$$= 2\pi\frac{E}{\omega}$$

$$= \frac{E}{\nu}.$$

That is,

$$E = J\nu.$$

J is a Generalized Momentum with units of Angular Momentum

$$\mathrm{m^2 kg/sec}.$$

The Generalized Coordinate which is Canonical to J is

$$\frac{\partial E}{\partial J} = \nu.$$

Therefore,

$$x(t) = \sqrt{\frac{2E}{m\omega^2}}\sin\omega(t + \partial_E S)$$

$$= \sqrt{\frac{2J\nu}{m4\pi^2\nu^2}}\sin 2\pi\nu(t + \partial_E S)$$

$$= \frac{1}{2\pi}\sqrt{\frac{2J}{m\nu}}\sin 2\pi\nu(t + \partial_E S)$$

$$p(t) = \sqrt{2mE}\cos\omega(t + \partial_E S)$$

$$= \sqrt{2mJ\nu}\cos 2\pi(t + \partial_E S).$$

These are the transformation equations from the Canonical Variables

$$(\nu, J),$$

To the Canonical Variables

$$(x, p).$$

0.3 The Harmonic Oscillator as a Thermodynamic System

A harmonic oscillator is loss-less. It looses no energy to friction, and generates no heat.

As a thermodynamic system, it is trivial.

A Thermodynamic System is given by two differential forms for the infinitesimal change in the heat, and the infinitesimal change in the work. For the harmonic oscillator,

$$\delta Q = 0,$$

$$\delta W = 0.$$

Therefore,

$$du = \delta Q - \delta W = 0.$$

And

$$ds = \frac{\delta Q}{T} = 0.$$

The internal energy, and the entropy do not change.

1.

Oscillating Point Mass has No Zero Point Energy

Depending on the Physics involved, Zero Point Electromagnetic Energy may or may not exist.

For instance, in the derivation of his second radiation law for the Electromagnetic Radiation of a Black Body, Planck unknowingly assumed that Zero Point Energy exists, but there is no reason to believe that it does. [Dan1,6]

However, it is an electromagnetic energy with density $\frac{1}{2}\hbar\omega$.

An oscillating point mass does not radiate photons, and does not have any electromagnetic energy, including the Zero Point Electromagnetic Energy.

The American Journal of Physics article that claims the impossible opposite, refers specifically to an oscillating point mass. In [Boyer1, p.866] it says

A harmonic oscillator corresponds to nonrelativistic point mass

$$m$$

moving in one dimension in a potential

$$V = \frac{1}{2}m\omega^2 x^2.$$

It can be described by the Lagrangian

$$L(x, \dot{x}) = \tfrac{1}{2} m\dot{x}^2 - \tfrac{1}{2} m\omega^2 x^2,$$

and corresponds to the motion

$$x(t) = \sqrt{\frac{2U}{m\omega}} \cos(\omega t + \phi),$$

at energy U*, and angular frequency* ω.

So the American Journal of Physics did not know that

- Zero Point Energy is electromagnetic, and cannot belong to the mechanical oscillations of a point mass.
- the frequency of electromagnetic energy has nothing to do with the frequency of mechanical oscillations of a point mass.

2.

Harmonically Oscillating Point Mass has Trivial Internal Energy

[Boyer1, p.866] has

> *...If the harmonic oscillator is weakly coupled to heat bath at temperature*
>
> $$T,$$
>
> *then it will exchange energy with the heat bath, and will have an average energy*
>
> $$U(\omega, T)$$
>
> *Thus, the oscillator is a thermodynamic system suitable for treatment by the usual methods of thermodynamics involving (average) energy*
>
> $$U,...$$

A harmonic oscillator is lossless, and the infinitesimal change of heat is zero,

$$\delta Q = 0.$$

Similarly, an infinitesimal change of work in a heat process is zero,

$$\delta W = 0$$

Therefore,

$$du = \underbrace{\delta Q}_{0} - \underbrace{\delta W}_{0} = 0\,.$$

The internal energy does not change in a heat process, and is of no consequence to the physics of the oscillator

The American Journal of Physics did not know that for harmonic oscillator, the Internal energy

- is trivial, and of no consequence
- is not the External energy of the harmonic oscillations of a point mass.

3.

The Generalized Momentum of a Harmonic Oscillator is Not a Constant

Recall from the first section here that

$$E = J\nu .$$

where J is a Generalized Momentum with units of Angular Momentum $\mathrm{m^2 kg/sec}$.

The Generalized Coordinate which is Canonical to J is

$$\frac{\partial E}{\partial J} = \nu .$$

And

$$x(t) = \frac{1}{2\pi}\sqrt{\frac{2J}{m\nu}}\sin 2\pi\nu(t + \partial_E S)$$

$$p(t) = \sqrt{2mJ\nu}\cos 2\pi(t + \partial_E S).$$

are the transformation equations from the Canonical Variables

$$(\nu, J),$$

To the Canonical Variables

$$(x, p).$$

In [Boyer1, p.866], we read

...and so the change in the energy of the system is

$$dU = \frac{U}{\omega} d\omega \ldots.$$

This means

$$\omega dU - U d\omega = 0\,,$$

$$2\pi \frac{\omega dU - U d\omega}{\omega^2} = 0\,,$$

$$d\left(2\pi \frac{U}{\omega}\right) = 0\,,$$

$$d\left(\frac{E}{\nu}\right) = 0\,,$$

$$dJ = 0\,,$$

$$J = \text{constant}???$$

Not really.

Only for the American Journal of Physics.

4.

Harmonic Oscillator Frequency is Not a Thermodynamic Variable.

In [Boyer1,p.866], we read

> *....the oscillator is a thermodynamics system suitable for treatment by the usual methods of thermodynamics involving (average) energy*
>
> $$U,$$
>
> *parameter*
>
> $$\omega,$$
>
> *and associated work*
>
> $$dW = -\frac{U}{\omega}d\omega.$$

First, there is no

$$dW$$

in thermodynamics.

The work in thermodynamics depends on the path the process takes, and NOT on initial, and final states. So an infinitesimal change in the work is

$$\delta W.$$

Second, the infinitesimal change of work in thermodynamics is assumed to be given by

$$\delta W = pdv,$$

where

$$\upsilon$$

may be a chemical concentration, or strain, or electric or magnetic moments.

In the lossless Mechanical Harmonic oscillator, there is no such parameter υ.

The frequency of the oscillations ω is a parameter on which the External oscillations energy depends.

Heat, and work due to heat do not depend on the frequency of the oscillations, and ω is not a thermodynamic variable.

References

[Boyer1] Timothy H. Boyer, "*Thermodynamics of the harmonic oscillator: Wien's displacement Law and the Planck Spectrum*", American Journal of Physics, September 2003, Volume 71, No. 9, pp. 866-870.

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[Dan3] Vic Dannon, "*Zero Point Energy does not imply the Radiation Law*" Bulletin of the APS, December 1999, Volume 44, No.9, p.30, BB-6.

[Dan4] H. Vic Dannon, "*Zero Point Energy: Planck's Radiation Law*", Gauge Institute Journal Vol.1 No 3, August 2005.

[Dan5] H. Vic Dannon, "*Zero Point Energy: Thermodynamic Equilibrium and Planck Radiation Law*", Gauge Institute Journal Vol.1 No 4, November 2005.

[Dan6] H. Vic Dannon, "*Zero Point Energy and Electromagnetic Entropy*", Gauge Institute Journal Vol.14 No 3, August 2018.

[Goldstein] Hebert Goldstein, "*Classical Mechanics*", Second Edition, Addison Wesley, 1980.