Zero Point Energy Does Not Generate Gravitation

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Abstract. The Casimir-Polder Zero Point Energy Potential at distance R is

$$U(R) = -\frac{\hbar c^3}{\pi}\Gamma^2 \operatorname{Re}\left\{\int_0^{u_c} e^{-2uR}\left(\frac{1}{R^2}+\frac{2}{uR^3}+\frac{5}{u^2R^4}+\frac{6}{u^3R^5}+\frac{3}{u^4R^6}\right)du\right\}$$

Where

$$\Gamma = G\frac{m}{c^3} = \frac{m\pi c^2}{\hbar\omega_c^2},$$

$$u_c = -i\frac{\omega_c}{c},$$

and

$$\omega_c = \sqrt{\frac{\pi c^5}{\hbar G}} = \text{Sakharov cutoff frequency.}$$

In "Zero Point Energy may not yield gravitation" (APS/Minneapolis 03/00), we noted that the term that depends on $1/R^2$, does not imply Newtonian gravitation.

Here, we extend the same to U. We show that

$$U_{ZPE} = \gamma\left(\frac{\sin 2\Re}{\Re^6} - 2\frac{\cos 2\Re}{\Re^5} - \frac{\sin 2\Re}{\Re^4} + \frac{3\cos 2\Re - 7}{6\Re^3}\right),$$

where

$$\gamma \equiv \frac{\hbar\omega_c^3}{\pi}\Gamma^2,$$

and

$$\Re \equiv R\frac{\omega_c}{c}.$$

This yields a dominating force

$$\left(\frac{\hbar G}{\pi c^3}\right)\left(\frac{1}{R}\right)\left(G\frac{m^2}{R^2}\right)$$

Negligible, due to its dependence on

$$1/R^3,$$

and since

$$\frac{\hbar G}{\pi c^3} < \frac{1}{10^{70}}.$$

Thus, the Casimir-Polder Zero Point Energy Potential does not generate Newtonian Gravitation.

Keywords: Van der Walls Forces, Vacuum interaction.

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Contents

0.

The Dubious Casimir-Polder Zero Point Energy Potential.

In [Casimir-Polder], the authors claimed that using perturbation methods, the Potential function for Retarded Van der Walls forces at distance R is

$$U = -\frac{\hbar c^3}{\pi}\Gamma^2 \operatorname{Re}\left\{\int_0^{u_c} e^{-2uR}\left(\frac{1}{R^2}+\frac{2}{uR^3}+\frac{5}{u^2R^4}+\frac{6}{u^3R^5}+\frac{3}{u^4R^6}\right)du\right\}$$

Where

$$\Gamma = G\frac{m}{c^3} = \frac{m\pi c^2}{\hbar\omega_c^2},$$

$$u_c = -i\frac{\omega_c}{c},$$

and

$$\omega_c = \sqrt{\frac{\pi c^5}{\hbar G}}$$ = Sakharov cutoff frequency.

In [Renne], that formula was rederived with an oscillator model, and in [Boyer] it was confirmed

<u>We do Not know whether these claims are true</u>

because in the past, Boyer, and Casimir made bizarre claims.

0.1 Casimir Claim that Two Uncharged Plates Attract

Casimir is known for his 1948 meeting poster [Casimir]

"*On the attraction between two perfectly conducting plates*"

in which he claimed the existence of an attractive force between two uncharged conducting plates

In [Dan1], we showed that he applied Euler's summation formula for the difference between a series and an integral of a function.

To that end, he defined the zero point energy between distant plates by integrating a double integral multiplied by $1/a^3$, where a is an infinite length measured by zillion meters, and defined the zero point energy between very close plates by summing on a double integral multiplied by $1/a^3$, where a stands for an infinitesimal length measured by microns (10^{-6}meters).

Since the a in each instance is different, the function is different, and Euler's Summation Formula does not apply.

Consequently, the Casimir Zero Point Energy Potential cannot be determined, and the Casimir Force cannot be evaluated.

No journal ever accepted Casimir's claims, but it kept resonating in Journals, and led to the publication of the [Casimir-Polder] paper.

0.2 Boyer's Defiance of Quantum Assumptions

In [Boyer1], Boyer shared with the Physical Review his

"*Derivation of the Blackbody Radiation Spectrum without Quantum Assumptions*", We disputed that in a meeting [Dan2],

and in a paper [Dan3]

"*Zero Point Energy Does not Imply the Radiation Law"*

We showed that he never supplied any derivation of the radiation law, with or without quantum assumptions. And his model yields a Ricatti differential equation, with infinitely many solutions, that are inconsistent with the radiation law.

0.3 Boyer's Confusion about Mechanics, Electromagnetics, and Thermodynamics

In [Boyer2], he fantasized that

- A harmonically oscillating point mass has electromagnetic energy
- The mechanical energy of harmonically oscillating point is a thermodynamical internal energy
- The generalized momentum of the harmonic oscillator is a constant.
- The frequency of the lossless mechanical oscillations is a thermodynamic variable

And this lead him to Planck's Radiation Law, with no need for quantum assumptions. $\hbar$ appears from nowhere after mumbling about asymptotics, and interpolation.

In 2018, this new physics was proudly recited in another seminal article, [Boyer3], by the American Journal of Physics.

We disproved these fantasies in [Dan4],

American Journal of Physics Claims that a Harmonically Oscillating mass has Zero Point Energy

We aim to invalidate the claim that the dubious Casimir-Polder Zero Point Energy Potential generates gravitation.

The potential's validity does not concern us. Just that it is irrelevant to gravitation. Then, it will not matter if the Casimir-Polder potential is correct, or incorrect.

1.

Sakharov's, and Puthoff's Zero Point Energy, and Gravitation

1.1 [Puthoff1] Claim that Zero Point Energy Generates Gravitation

[Puthoff1,App.A] claimed that for a dipole oscillating in the plane, the Casimir-Polder Zero Point Energy Potential is multiplied by $4/9$,

$$U_{ZPE} = -\frac{\hbar c^3}{\pi}\Gamma^2 \operatorname{Re}\left\{\int_0^{u_c} e^{-2uR}\left(\frac{1}{R^2}+\frac{2}{uR^3}+\frac{5}{u^2R^4}+\frac{6}{u^3R^5}+\frac{3}{u^4R^6}\right)du\right\},$$

where

$$\Gamma = G\frac{m}{c^3} = \frac{m\pi c^2}{\hbar\omega_c^2},$$

$$u_c = -i\frac{\omega_c}{c},$$

Sakharov's cutoff frequency ω_c is given by

$$\omega_c^2 = \frac{\pi c^5}{\hbar G},$$

$G =$ the gravitation constant,

$m =$ the mass of each of the particles,

$c =$ the light speed,

and

$R =$ the distance between the masses.

Sakharov proposed that gravitation is a manifestation of zero point energy fluctuations, [Sakharov].

Following that, [Puthoff1,App.B] used [Boyer] to show that the first term of the Van der Walls potential

$$U_2 = -\frac{\hbar c^3}{\pi}\Gamma^2 \operatorname{Re}\int_0^{u_c} e^{-2uR}\frac{1}{R^2}du$$

$$= -\frac{\hbar c^2}{\pi R^2}\Gamma^2 \int_0^{\omega_c} \sin\left(2\frac{R}{c}\omega\right)d\omega$$

yields Newton's gravitation force.

1.2 [Carlip] Disproof of [Puthoff1]

Using asymptotic expansions, [Carlip] disputed Puthoff's derivation, and showed that U_2 leads to a non-gravitational force.

[Puthoff2] replied that the range of the integration has to be restricted to the low frequencies that are relevant to Van Der Walls forces interaction.

To that end, [Puthoff2] wrote U_2 in the form referenced in [Boyer] with a resonance denominator as

$$U_2 = -\frac{\hbar c^2}{\pi R^2}\Gamma^2 \int_{\omega=0}^{\omega=\omega_i} \sin\left(2\frac{R}{c}\omega\right)\frac{\omega^4}{\left(\omega_0^2 - \omega^2\right)^2}d\omega,$$

where

$$\omega_i \approx 0$$

corresponds to the "low frequency cutoff of meaningful non-phase canceling **i**nteractions, at large separations"

and

$$\omega_0 \approx 0$$

corresponds to the "vanishing binding energy for an asymptotically free quark".

ω_i is constrained to be smaller than ω_0

$$\omega_i < \omega_0$$

We will show that this U_2 leads to a negligible nongravitational force.

1.3 [Puthoff2] Force is Negligible and Nongravitational.

Since ω_i is close to zero, ω in the integration range is small, and

$$\sin\left(2\frac{R}{c}\omega\right) \approx 2\frac{R}{c}\omega,$$

Hence,

$$U_2 \approx -\frac{\hbar c^2}{\pi R^2}\Gamma^2 \int_0^{\omega_i} 2\frac{R}{c}\omega \frac{\omega^4}{(\omega_0^2-\omega^2)^2}d\omega$$

$$= -\frac{\hbar c}{\pi R}\Gamma^2 \int_0^{\omega_i} \frac{\omega^4}{(\omega_0^2-\omega^2)^2}d\left(\omega^2\right)$$

Denoting

$$\omega^2 = \sigma,$$

$$U_2 = -\frac{\hbar c}{\pi R}\Gamma^2 \int_{\sigma=0}^{\sigma=\omega_i^2} \frac{1}{(\omega_0^2-\sigma)^2}\sigma^2 d\sigma$$

Integrating by parts,

$$U_2 = -\frac{\hbar c}{\pi R}\Gamma^2 \left\{ \frac{1}{\omega_0^2-\sigma}\sigma^2 \Bigg|_{\sigma=0}^{\sigma=\omega_i^2} - \int_{\sigma=0}^{\sigma=\omega_i^2} \frac{1}{\omega_0^2-\sigma} 2\sigma d\sigma \right\}$$

The integral vanishes because

$$\int_{\sigma=0}^{\sigma=\omega_i^2} \frac{-\sigma}{\omega_0^2-\sigma} d\sigma = \int_{\sigma=0}^{\sigma=\omega_i^2} \left(1 - \frac{\omega_0^2}{\omega_0^2-\sigma}\right) d\sigma$$

$$= \underbrace{\int_{\sigma=0}^{\sigma=\omega_i^2} d\sigma}_{\omega_i^2} + \omega_0^2 \underbrace{\int_{\sigma=0}^{\sigma=\omega_i^2} -\frac{1}{\omega_0^2-\sigma} d\sigma}_{\log(\omega_0^2-\sigma)\big|_{\sigma=0}^{\sigma=\omega_i^2}}$$

$$= \omega_i^2 + \omega_0^2 \left[\log(\omega_0^2-\omega_i^2) - \log(\omega_0^2)\right]$$

$$= \underbrace{\omega_i^2}_{\approx 0} + \underbrace{\frac{\log(\omega_0^2-\omega_i^2)}{\omega_0^{-2}}}_{\frac{\infty}{\infty}} - \underbrace{\frac{\log(\omega_0^2)}{\omega_0^{-2}}}_{\frac{\infty}{\infty}}$$

By L'hospital,

$$\underbrace{\frac{\log(\omega_0^2-\omega_i^2)}{\omega_0^{-2}}}_{\frac{\infty}{\infty}} = \frac{\partial_{\omega_0} \log(\omega_0^2-\omega_i^2)}{\partial_{\omega_0} \omega_0^{-2}}$$

$$= \frac{\frac{1}{(\omega_0^2-\omega_i^2)} 2\omega_0}{-2\omega_0^{-3}}$$

$$= -\frac{\omega_0^4}{\underbrace{(\omega_0^2-\omega_i^2)}_{\frac{0}{0}}}$$

$$= -\frac{\partial_{\omega_0} \omega_0^4}{\partial_{\omega_0} (\omega_0^2-\omega_i^2)}$$

$$= -\frac{4\omega_0^3}{2\omega_0}$$

$$= -2\omega_0^2 \approx 0$$

And

$$\underbrace{\frac{\log \omega_0^2}{\omega_0^{-2}}}_{\frac{\infty}{\infty}} = \frac{2\log \omega_0}{\omega_0^{-2}}$$

$$= \frac{\partial_{\omega_0}(2\log \omega_0)}{\partial_{\omega_0}\omega_0^{-2}}$$

$$= \frac{\frac{2}{\omega_0}}{-2\omega_0^{-3}}$$

$$= -\omega_0^2 \approx 0$$

Therefore,

$$U_2 \approx -\frac{\hbar c}{\pi R}\Gamma^2 \underbrace{\frac{\omega_i^4}{\omega_0^2 - \omega_i^2}}_{\frac{0}{0}}$$

By L'hospital,

$$\underbrace{\frac{\omega_i^4}{\omega_0^2 - \omega_i^2}}_{\frac{0}{0}} = \frac{\partial_{\omega_i}\omega_i^4}{\partial_{\omega_i}(\omega_0^2 - \omega_i^2)}$$

$$= \frac{4\omega_i^3}{-2\omega_i}$$

$$= -2\omega_i^2 .$$

Hence,

$$U_2 \approx \left(-\frac{\hbar c}{\pi R}\Gamma^2\omega_c^2\right)\left(\frac{-2\omega_i^2}{\omega_c^2}\right)$$

$$= \left(\frac{\hbar c}{\pi R} \left[G \frac{m}{c^3} \right] \left[\frac{m \pi c^2}{\hbar \omega_c^2} \right] \omega_c^2 \right) \left(\frac{2\omega_i^2}{\omega_c^2} \right)$$

$$= 2 \frac{\omega_i^2}{\omega_c^2} \left(G \frac{m^2}{R} \right).$$

This is a negligible potential.

$$-U_2 = -2 \frac{\omega_i^2}{\omega_c^2} \left(G \frac{m^2}{R} \right)$$

Its derivative

$$-\frac{dU_2}{dR} \approx 2 \frac{\omega_i^2}{\omega_c^2} \left(G \frac{m^2}{R^2} \right)$$

is a nongravitational force. It is negligible because

$$\frac{\omega_i^2}{\omega_c^2} \ll \omega_i^2 \approx 0 .$$

Next, we will show that No part of the Casimir-Polder Zero Point Energy Potential generates Newtonian Gravitation.

That is, gravitation is not a result of a zero point energy potential.

2.

The Distance Squared Term

The first term in the Van der Walls potential is

$$U_2 = -\frac{\hbar c^3}{\pi}\Gamma^2 \operatorname{Re}\left\{\int_{u=0}^{u=u_c} e^{-2uR}\frac{1}{R^2}du\right\}$$

$$= -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{1}{R^2}\operatorname{Re}\left\{\int_{u=0}^{u=u_c} e^{-2uR}du\right\}$$

Substitute

$$u = -i\frac{\omega}{c},$$

$$du = -\frac{i}{c}d\omega$$

Then,

$$U_2 = -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{1}{R^2}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} e^{2i\frac{\omega}{c}R}\left(-\frac{i}{c}\right)d\omega\right\}$$

Denoting

$$\frac{R}{c} \equiv \frac{\Re}{\omega_c},$$

$$U_2 = -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{\omega_c^2}{c^2\Re^2}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} e^{i2\frac{\Re}{\omega_c}\omega}\left(-\frac{i}{c}\right)d\omega\right\}$$

$$= -\underbrace{\frac{\hbar \omega_c^3}{\pi}\Gamma^2}_{\gamma} \frac{1}{\Re^2}\frac{1}{\omega_c}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} -i\left(\cos 2\frac{\Re}{\omega_c}\omega + i\sin 2\frac{\Re}{\omega_c}\omega\right)d\omega\right\}$$

Denoting

$$\frac{\hbar \omega_c^3}{\pi}\Gamma^2 \equiv \gamma\,,$$

$$U_2 = \gamma \frac{1}{\Re^2} \frac{1}{\omega_c} \int_{\omega=0}^{\omega=\omega_c} \left(-\sin 2 \frac{\Re}{\omega_c} \omega \right) d\omega$$

$$= \gamma \frac{1}{\Re^2} \frac{1}{\omega_c} \frac{1}{2 \frac{\Re}{\omega_c}} \cos 2 \frac{\Re}{\omega_c} \omega \Bigg|_{\omega=0}^{\omega=\omega_c}$$

$$= \gamma \frac{\cos 2\Re - 1}{2\Re^3}$$

3.

The Distance Cubed Term

The second term in the van der Walls potential is

$$U_3 = -\frac{\hbar c^3}{\pi}\Gamma^2 \operatorname{Re}\left\{\int_{u=0}^{u=u_c} e^{-2uR}\frac{2}{uR^3}du\right\}$$

$$= -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{2}{R^3}\operatorname{Re}\left\{\int_{u=0}^{u=u_c} e^{-2uR}\frac{1}{u}du\right\}$$

Substitute

$$u = -i\frac{\omega}{c},$$

$$du = -\frac{i}{c}d\omega .$$

Then,

$$U_3 = -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{2}{R^3}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} e^{i2\frac{\omega}{c}R}\frac{1}{-i\frac{\omega}{c}}\left(-\frac{i}{c}\right)d\omega\right\}$$

Denoting

$$\frac{R}{c} \equiv \frac{\Re}{\omega_c},$$

$$U_3 = -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{2\omega_c^3}{\Re^3 c^3}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} e^{i2\frac{\Re}{\omega_c}\omega}\frac{1}{\omega}d\omega\right\}$$

$$= -\underbrace{\frac{\hbar\omega_c^3}{\pi}}_{\gamma}\Gamma^2 \frac{2}{\Re^3}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c}\left(\cos 2\frac{\Re}{\omega_c}\omega + i\sin 2\frac{\Re}{\omega_c}\omega\right)\frac{d\omega}{\omega}\right\}$$

$$= -2\gamma \frac{1}{\Re^3} \int_{\omega=0}^{\omega=\omega_c} \frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right) d\omega$$

4.

The Distance Quadrupled Term

The third term in the van der Walls potential is

$$U_4 = -\frac{\hbar c^3}{\pi}\Gamma^2 \operatorname{Re}\left\{\int_{u=0}^{u=u_c} e^{-2uR}\frac{5}{u^2R^4}du\right\}$$

$$= -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{5}{R^4}\operatorname{Re}\left\{\int_{u=0}^{u=u_c} e^{-2uR}\frac{1}{u^2}du\right\}$$

Substitute

$$u = -i\frac{\omega}{c},$$

$$du = -\frac{i}{c}d\omega.$$

Then,

$$U_4 = -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{5}{R^4}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} e^{i2\frac{\omega}{c}R}\frac{1}{\left(-i\frac{\omega}{c}\right)^2}\left(-\frac{i}{c}\right)d\omega\right\}$$

Denoting

$$\frac{R}{c} \equiv \frac{\Re}{\omega_c},$$

$$U_4 = -\frac{\hbar c^3}{\pi}\Gamma^2 5c\frac{\omega_c^4}{\Re^4 c^4}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} ie^{i2\frac{\Re}{\omega_c}\omega}\frac{1}{\omega^2}d\omega\right\}$$

$$= \underbrace{-\frac{\hbar\omega_c^3}{\pi}\Gamma^2}_{\gamma}\frac{5\omega_c}{\Re^4}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} i\left(\cos 2\frac{\Re}{\omega_c}\omega + i\sin 2\frac{\Re}{\omega_c}\omega\right)\frac{d\omega}{\omega^2}\right\}$$

$$= \gamma\frac{5\omega_c}{\Re^4}\int_{\omega=0}^{\omega=\omega_c}\frac{1}{\omega^2}\left(\sin 2\frac{\Re}{\omega_c}\omega\right)d\omega$$

Integrating by parts,

$$= \gamma \frac{5\omega_c}{\Re^4}\left\{ -\frac{1}{\omega}\sin 2\frac{\Re}{\omega_c}\omega \Big|_{\omega=0}^{\omega=\omega_c} + \int_{\omega=0}^{\omega=\omega_c} \frac{1}{\omega}\left(2\frac{\Re}{\omega_c}\cos 2\frac{\Re}{\omega_c}\omega \right) d\omega \right\}$$

By L'hospital, for $\omega \approx 0$,

$$\left. \frac{\sin 2\frac{\Re}{\omega_c}\omega}{\omega} \right|_{\omega\approx 0} = \left. \frac{\partial_\omega \sin 2\frac{\Re}{\omega_c}\omega}{\partial_\omega \omega} \right|_{\omega\approx 0}$$

$$= 2\frac{\Re}{\omega_c}\cos 2\frac{\Re}{\omega_c}\omega \Big|_{\omega\approx 0}$$

$$= 2\frac{\Re}{\omega_c}$$

Therefore,

$$U_4 = \gamma \frac{5\omega_c}{\Re^4}\left\{ -\frac{\sin 2\Re}{\omega_c} + 2\frac{\Re}{\omega_c} + 2\frac{\Re}{\omega_c}\int_{\omega=0}^{\omega=\omega_c} \frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega \right) d\omega \right\}$$

$$= \gamma \left\{ -\frac{5\sin 2\Re}{\Re^4} + 10\frac{1}{\Re^3} + \frac{10}{\Re^3}\int_{\omega=0}^{\omega=\omega_c} \frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega \right) d\omega \right\}$$

5.

The Distance Quintupled Term

The fourth van der Walls potential is

$$U_5 = -\frac{\hbar c^3}{\pi}\Gamma^2 \operatorname{Re}\left\{\int_{u=0}^{u=u_c} e^{-2uR}\frac{6}{u^3R^5}du\right\}$$

$$= -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{6}{R^5}\operatorname{Re}\left\{\int_{u=0}^{u=u_c} e^{-2uR}\frac{1}{u^3}du\right\}$$

Substitute

$$u = -i\frac{\omega}{c},$$

$$du = -\frac{i}{c}d\omega.$$

Then,

$$U_5 = -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{6}{R^5}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} e^{i2\frac{\omega}{c}R}\frac{1}{\left(-i\frac{\omega}{c}\right)^3}\left(-\frac{i}{c}\right)d\omega\right\}$$

$$= \frac{\hbar c^3}{\pi}\Gamma^2 \frac{6c^2}{R^5}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} e^{i2\frac{\omega}{c}R}\frac{1}{\omega^3}d\omega\right\}$$

Denoting

$$\frac{R}{c} \equiv \frac{\Re}{\omega_c},$$

$$U_5 = \frac{\hbar c^3}{\pi}\Gamma^2 6c^2\frac{\omega_c^5}{\Re^5 c^5}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} e^{i2\frac{\Re}{\omega_c}\omega}\frac{1}{\omega^3}d\omega\right\}$$

$$= \underbrace{\frac{\hbar\omega_c^3}{\pi}\Gamma^2}_{\gamma} \frac{6}{\mathfrak{R}^5}\omega_c^2 \,\mathrm{Re}\left\{ \int_{\omega=0}^{\omega=\omega_c} \frac{1}{\omega^3}\left(\cos 2\frac{\mathfrak{R}}{\omega_c}\omega + i\sin 2\frac{\mathfrak{R}}{\omega_c}\omega \right) d\omega \right\}$$

$$= \gamma \frac{3}{\mathfrak{R}^5}\omega_c^2 \int_{\omega=0}^{\omega=\omega_c} \frac{2}{\omega^3}\left(\cos 2\frac{\mathfrak{R}}{\omega_c}\omega \right) d\omega$$

$$= \gamma \frac{3}{\mathfrak{R}^5}\omega_c^2 \left\{ \left(-\frac{1}{\omega^2}\right)\cos 2\frac{\mathfrak{R}}{\omega_c}\omega \Big|_{\omega=0}^{\omega=\omega_c} - \int_{\omega=0}^{\omega=\omega_c} \left(-\frac{1}{\omega^2}\right)\left(-2\frac{\mathfrak{R}}{\omega_c}\sin 2\frac{\mathfrak{R}}{\omega_c}\omega\right) d\omega \right\}$$

By L'hospital, for $\omega \approx 0$,

$$\left.\frac{\cos 2\frac{\mathfrak{R}}{\omega_c}\omega}{\omega^2}\right|_{\omega\approx 0} = \left.\frac{\partial_\omega \cos 2\frac{\mathfrak{R}}{\omega_c}\omega}{\partial_\omega \omega^2}\right|_{\omega\approx 0}$$

$$= \left.\frac{2\frac{\mathfrak{R}}{\omega_c}\left(-\sin 2\frac{\mathfrak{R}}{\omega_c}\omega\right)}{2\omega}\right|_{\omega\approx 0}$$

$$= -\frac{\mathfrak{R}}{\omega_c}\left.\frac{\partial_\omega\left(\sin 2\frac{\mathfrak{R}}{\omega_c}\omega\right)}{\partial_\omega \omega}\right|_{\omega\approx 0}$$

$$= -2\left(\frac{\mathfrak{R}}{\omega_c}\right)^2 \cos 2\frac{\mathfrak{R}}{\omega_c}\omega \Big|_{\omega\approx 0}$$

$$= -2\frac{\mathfrak{R}^2}{\omega_c^2}$$

Therefore,

$$\left(-\frac{1}{\omega^2}\right)\cos 2\frac{\mathfrak{R}}{\omega_c}\omega \Big|_{\omega=0}^{\omega=\omega_c} = -\frac{\cos 2\mathfrak{R}}{\omega_c^2} - 2\frac{\mathfrak{R}^2}{\omega_c^2}$$

For the integral

$$-\int_{\omega=0}^{\omega=\omega_c}\left(-\frac{1}{\omega^2}\right)\left(-2\frac{\Re}{\omega_c}\sin 2\frac{\Re}{\omega_c}\omega\right)d\omega = 2\frac{\Re}{\omega_c}\int_{\omega=0}^{\omega=\omega_c}\left(-\frac{1}{\omega^2}\right)\left(\sin 2\frac{\Re}{\omega_c}\omega\right)d\omega$$

Integrating by parts

$$= 2\frac{\Re}{\omega_c}\left[\frac{1}{\omega}\sin 2\frac{\Re}{\omega_c}\omega\Bigg|_{\omega=0}^{\omega=\omega_c} - \int_{\omega=0}^{\omega=\omega_c}\frac{1}{\omega}\left(2\frac{\Re}{\omega_c}\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega\right]$$

From section 4,

$$\frac{1}{\omega}\sin 2\frac{\Re}{\omega_c}\omega\Bigg|_{\omega=0}^{\omega=\omega_c} = \frac{\sin 2\Re}{\omega_c} - 2\frac{\Re}{\omega_c}$$

Therefore, the integral is

$$= 2\frac{\Re}{\omega_c}\left[\frac{\sin 2\Re}{\omega_c} - 2\frac{\Re}{\omega_c} - 2\frac{\Re}{\omega_c}\int_{\omega=0}^{\omega=\omega_c}\frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega\right]$$

Hence,

$$U_5 = \gamma\frac{3}{\Re^5}\omega_c^2 \times$$

$$\times\left\{-\frac{\cos 2\Re}{\omega_c^2} - 2\frac{\Re^2}{\omega_c^2} + 2\frac{\Re}{\omega_c}\left[\frac{\sin 2\Re}{\omega_c} - 2\frac{\Re}{\omega_c} - 2\frac{\Re}{\omega_c}\int_{\omega=0}^{\omega=\omega_c}\frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega\right]\right\}$$

$$= \gamma\frac{3}{\Re^5}\left\{-\cos 2\Re - 2\Re^2 + 2\Re\left[\sin 2\Re - 2\Re - 2\Re\int_{\omega=0}^{\omega=\omega_c}\frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega\right]\right\}$$

$$= \gamma\left\{-\frac{3\cos 2\Re}{\Re^5} - \frac{6}{\Re^3} + \frac{6\sin 2\Re}{\Re^4} - \frac{12}{\Re^3} - \frac{12}{\Re^3}\int_{\omega=0}^{\omega=\omega_c}\frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega\right\}.$$

6.

The Distance Sextupled Term

The fifth term of the van der Walls potential is

$$U_6 = -\frac{\hbar c^3}{\pi}\Gamma^2 \operatorname{Re}\left\{\int_{u=0}^{u=u_c} e^{-2uR}\frac{3}{u^4 R^6}du\right\}$$

$$= -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{3}{R^6}\operatorname{Re}\left\{\int_{u=0}^{u=u_c} e^{-2uR}\frac{1}{u^4}du\right\}$$

Substitute

$$u = -i\frac{\omega}{c},$$

$$du = -\frac{i}{c}d\omega .$$

Then,

$$U_6 = -\frac{\hbar c^3}{\pi}\Gamma^2 \frac{3}{R^6}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} e^{i2\frac{\omega}{c}R}\frac{3}{\left(-i\frac{\omega}{c}\right)^4}\left(-\frac{i}{c}\right)d\omega\right\}$$

$$= \frac{\hbar c^3}{\pi}\Gamma^2 \frac{3c^3}{R^6}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} ie^{i2\frac{\omega}{c}R}\frac{1}{\omega^4}d\omega\right\}$$

Denoting

$$\frac{R}{c} \equiv \frac{\Re}{\omega_c},$$

$$U_6 = \frac{\hbar c^3}{\pi}\Gamma^2 \frac{3c^3\omega_c^6}{\Re^6 c^6}\operatorname{Re}\left\{\int_{\omega=0}^{\omega=\omega_c} ie^{i2\frac{\Re}{\omega_c}\omega}\frac{1}{\omega^4}d\omega\right\}$$

$$= \underbrace{\frac{\hbar\omega_c^3}{\pi}\Gamma^2}_{\gamma} \frac{3\omega_c^6}{\Re^6} \mathrm{Re}\left\{ \int_{\omega=0}^{\omega=\omega_c} i\left(\cos 2\frac{\Re}{\omega_c}\omega + i\sin 2\frac{\Re}{\omega_c}\omega \right)\frac{1}{\omega^4}d\omega \right\}$$

$$= \gamma \frac{\omega_c^3}{\Re^6} \int_{\omega=0}^{\omega=\omega_c} \left(-\frac{3}{\omega^4} \right)\left(\sin 2\frac{\Re}{\omega_c}\omega \right) d\omega$$

Integrating by parts,

$$U_6 = \gamma \left\{ \frac{\omega_c^3}{\Re^6} \left. \frac{\sin 2\frac{\Re}{\omega_c}\omega}{\omega^3} \right|_{\omega=0}^{\omega=\omega_c} - \frac{\omega_c^3}{\Re^6} \int_0^{\omega_c} \frac{1}{\omega^3}\left(2\frac{\Re}{\omega_c}\cos 2\frac{\Re}{\omega_c}\omega \right) d\omega \right\}$$

By L'hospital, for $\omega \approx 0$,

$$\left. \frac{\sin 2\frac{\Re}{\omega_c}\omega}{\omega^3} \right|_{\omega\approx 0} = \left. \frac{\partial_\omega\left(\sin 2\frac{\Re}{\omega_c}\omega \right)}{\partial_\omega\left(\omega^3\right)} \right|_{\omega\approx 0}$$

$$= 2\frac{\Re}{\omega_c} \left. \frac{\cos 2\frac{\Re}{\omega_c}\omega}{3\omega^2} \right|_{\omega\approx 0}$$

$$= 2\frac{\Re}{\omega_c} \left. \frac{\partial_\omega\left(\cos 2\frac{\Re}{\omega_c}\omega \right)}{\partial_\omega\left(3\omega^2\right)} \right|_{\omega\approx 0}$$

$$= \left(2\frac{\Re}{\omega_c} \right)^2 \left. \frac{\left(-\sin 2\frac{\Re}{\omega_c}\omega \right)}{6\omega} \right|_{\omega\approx 0}$$

$$= \left(2\frac{\Re}{\omega_c}\right)^2 \frac{\partial_\omega\left(-\sin 2\frac{\Re}{\omega_c}\omega\right)}{\partial_\omega(6\omega)}\Bigg|_{\omega\approx 0}$$

$$= \left(2\frac{\Re}{\omega_c}\right)^2 \frac{(-2\frac{\Re}{\omega_c}\cos 2\frac{\Re}{\omega_c}\omega)\Big|_{\omega\approx 0}}{6}$$

$$= -\frac{8}{6}\frac{\Re^3}{\omega_c^3}$$

Therefore,

$$U_6 = \gamma\left\{\frac{\sin 2\Re}{\Re^6} + \frac{8}{6\Re^3} + \frac{\omega_c^2}{\Re^5}\int_0^{\omega_c}\left(-\frac{2}{\omega^3}\right)\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega\right\}$$

Integrating by parts,

$$\frac{\omega_c^2}{\Re^5}\int_{\omega=0}^{\omega=\omega_c}\left(-\frac{2}{\omega^3}\right)\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega =$$

$$= \frac{\omega_c^2}{\Re^5}\left[\frac{1}{\omega^2}\cos 2\frac{\Re}{\omega_c}\omega\Big|_{\omega=0}^{\omega=\omega_c} - \int_{\omega=0}^{\omega=\omega_c}\left(\frac{1}{\omega^2}\right)\left(-2\frac{\Re}{\omega_c}\sin 2\frac{\Re}{\omega_c}\omega\right)d\omega\right]$$

$$= \frac{\omega_c^2}{\Re^5}\left[\frac{1}{\omega^2}\cos 2\frac{\Re}{\omega_c}\omega\Big|_{\omega=0}^{\omega=\omega_c} + 2\frac{\Re}{\omega_c}\int_{\omega=0}^{\omega=\omega_c}\left(\frac{1}{\omega^2}\right)\left(\sin 2\frac{\Re}{\omega_c}\omega\right)d\omega\right]$$

From section 5,

$$\frac{1}{\omega^2}\cos 2\frac{\Re}{\omega_c}\omega\Big|_{\omega=0}^{\omega=\omega_c} = \frac{\cos 2\Re}{\omega_c^2} + 2\frac{\Re^2}{\omega_c^2},$$

$$\frac{\omega_c^2}{\Re^5}\frac{1}{\omega^2}\cos 2\frac{\Re}{\omega_c}\omega\Big|_{\omega=0}^{\omega=\omega_c} = \frac{\cos 2\Re}{\Re^5} + \frac{2}{\Re^3}.$$

From Section 4,

$$\int_{\omega=0}^{\omega=\omega_c} \frac{1}{\omega^2}\left(\sin 2\frac{\Re}{\omega_c}\omega\right)d\omega = -\frac{\sin 2\Re}{\omega_c} + 2\frac{\Re}{\omega_c} + 2\frac{\Re}{\omega_c}\int_{\omega=0}^{\omega=\omega_c} \frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega,$$

$$\frac{\omega_c^2}{\Re^5}2\frac{\Re}{\omega_c}\int_{\omega=0}^{\omega=\omega_c}\left(\frac{1}{\omega^2}\right)\left(\sin 2\frac{\Re}{\omega_c}\omega\right)d\omega =$$

$$= 2\frac{\omega_c}{\Re^4}\left(-\frac{\sin 2\Re}{\omega_c} + 2\frac{\Re}{\omega_c} + 2\frac{\Re}{\omega_c}\int_{\omega=0}^{\omega=\omega_c} \frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega\right)$$

$$= -\frac{2\sin 2\Re}{\Re^4} + \frac{4}{\Re^3} + \frac{4}{\Re^3}\int_{\omega=0}^{\omega=\omega_c} \frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega$$

Hence,

$$U_6 = \gamma\left\{\frac{\sin 2\Re}{\Re^6} + \frac{8}{6\Re^3} + \frac{\cos 2\Re}{\Re^5} + \frac{2}{\Re^3} + \right.$$

$$\left. -\frac{2\sin 2\Re}{\Re^4} + \frac{4}{\Re^3} + \frac{4}{\Re^3}\int_{\omega=0}^{\omega=\omega_c} \frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega\right\}$$

7.

The Casimir-Polder Zero Point Energy Potential

The Casimir-Polder Zero Point Energy potential is

$$U_{ZPE} = -\frac{\hbar c^3}{\pi}\Gamma^2 \operatorname{Re}\left\{\int_0^{u_c} e^{-2uR}\left(\frac{1}{R^2}+\frac{2}{uR^3}+\frac{5}{u^2R^4}+\frac{6}{u^3R^5}+\frac{3}{u^4R^6}\right)du\right\}$$

$$= U_2 + U_3 + U_4 + U_5 + U_6$$

$$= \gamma\frac{\cos 2\Re - 1}{2\Re^3}$$

$$-2\frac{\gamma}{\Re^3}\int_0^{\omega_c}\frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega$$

$$-5\frac{\gamma}{\Re^4}\sin 2\Re + 10\frac{\gamma}{\Re^3} + 10\frac{\gamma}{\Re^3}\int_{\omega=0}^{\omega=\omega_c}\frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega$$

$$-3\frac{\gamma}{\Re^5}\cos 2\Re - 6\frac{\gamma}{\Re^3} + 6\frac{\gamma}{\Re^4}\sin 2\Re - 12\frac{\gamma}{\Re^3}$$

$$-12\frac{\gamma}{\Re^3}\int_{\omega=0}^{\omega=\omega_c}\frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega$$

$$+\left\{\frac{\gamma}{\Re^6}\sin 2\Re + \frac{8}{6}\frac{\gamma}{\Re^3} + \frac{\gamma}{\Re^5}\cos 2\Re + 2\frac{\gamma}{\Re^3} + \right.$$

$$\left. -2\frac{\gamma}{\Re^4}\sin 2\Re + 4\frac{\gamma}{\Re^3} + 4\frac{\gamma}{\Re^3}\int_{\omega=0}^{\omega=\omega_c}\frac{1}{\omega}\left(\cos 2\frac{\Re}{\omega_c}\omega\right)d\omega\right\}$$

$$= \frac{\gamma}{\Re^6}\sin 2\Re$$

$$+\frac{\gamma}{\Re^5}\cos 2\Re\underbrace{\left[-3+1\right]}_{-2}$$

$$+\frac{\gamma}{\Re^4}\sin 2\Re\underbrace{\left[-5+6-2\right]}_{-1}$$

$$+\frac{\gamma}{\Re^3}\cos 2\Re\left[\frac{1}{2}\right]$$

$$+\frac{\gamma}{\Re^3}\underbrace{\left[-\frac{1}{2}+10-6-12+\frac{8}{6}+2+4\right]}_{-\frac{7}{6}}$$

That is,

$$U_{ZPE}=\gamma\left(\frac{\sin 2\Re}{\Re^6}-2\frac{\cos 2\Re}{\Re^5}-\frac{\sin 2\Re}{\Re^4}+\frac{3\cos 2\Re-7}{6\Re^3}\right)$$

8.

The Zero Point Energy Potential has No Gravitational Effect.

$$U_{ZPE}=\gamma\left(\frac{\sin 2\Re}{\Re^6}-2\frac{\cos 2\Re}{\Re^5}-\frac{\sin 2\Re}{\Re^4}+\frac{3\cos 2\Re-7}{6\Re^3}\right)$$

$$-U_{ZPE}=-\gamma\frac{\sin 2\Re}{\Re^6}+2\gamma\frac{\cos 2\Re}{\Re^5}+\gamma\frac{\sin 2\Re}{\Re^4}+\gamma\frac{7}{6\Re^3}-\gamma\frac{\cos 2\Re}{2\Re^3}$$

The force derived from this potential is the derivative

$$-\frac{dU_{ZPE}}{d\Re}=6\gamma\frac{\sin 2\Re}{\Re^7}-2\gamma\frac{\cos 2\Re}{\Re^6}$$

$$-10\gamma\frac{\cos 2\Re}{\Re^6}-4\gamma\frac{\sin 2\Re}{\Re^5}$$

$$-4\gamma\frac{\sin 2\Re}{\Re^5}+2\gamma\frac{\cos 2\Re}{\Re^4}$$

$$-3\gamma\frac{7}{6\Re^4}$$

$$+3\gamma\frac{\cos 2\Re}{2\Re^4}+2\gamma\frac{\sin 2\Re}{2\Re^3}$$

$$-\frac{dU_{ZPE}}{d\Re}=6\gamma\frac{\sin 2\Re}{\Re^7}-12\gamma\frac{\cos 2\Re}{\Re^6}-8\gamma\frac{\sin 2\Re}{\Re^5}-\frac{7}{2}\gamma\frac{1-\cos 2\Re}{\Re^4}+\gamma\frac{\sin 2\Re}{\Re^3}$$

The dominant term is the last, the force

$$\gamma\frac{\sin 2\Re}{\Re^3}$$

that depends on the cubed distance.

It is extremely negligible because it is at most

$$\gamma \frac{1}{\Re^3} = \frac{\hbar \omega_c^3}{\pi} \Gamma^2 \frac{1}{\left(\frac{R^3 \omega_c^3}{c^3} \right)}$$

$$= \frac{\hbar c^3}{\pi} \Gamma^2 \frac{1}{R^3}$$

$$= \frac{\hbar c^3}{\pi} \left(\frac{Gm}{c^3} \right)^2 \frac{1}{R^3}$$

$$= \left(\frac{\hbar G}{\pi c^3} \right) \left(\frac{1}{R} \right) \left(G \frac{m^2}{R^2} \right)$$

It is negligible compared with the Newtonian squared distance due to the cubed distance. And

$$\frac{\hbar G}{\pi c^3} \approx \frac{(10^{-34})([6.7] \cdot 10^{-11})}{(\frac{22}{7})(3 \cdot 10^8)^3}$$

$$\approx (0.078) 10^{-69}$$

$$< \frac{1}{10^{70}}.$$

Consequently, the Zero Point Energy Potential does not generate Newtonian gravitation, and has no gravitational effect.

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