The Casimir Effect Fallacy: Casimir's Zero Point Energy has Infinite Mass, is not Conserved, and is Falsely based on Euler's Formula. The Fallacy No Journal ever Accepted, that Resonates to No End in Journals

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August, 2015

Abstract To compute the Zero Point Energy potential, Casimir applied the Euler summation formula for the difference between a series and an integral of a function.

To that end, he **defined** the zero point energy between distant plates by integrating a double integral multiplied by $1 / a^3$, where a is an infinite length measured by zillion meters, and **defined** the zero point energy between very close plates by summing on a

double integral multiplied by $1/a^3$, where a stands for an infinitesimal length measured by microns (10^{-6}meters).

Since the a in each instance is different, the function is different, and the Euler Summation Formula does not apply. Consequently, the Casimir Zero Point Energy Potential cannot be determined, and the Casimir Force cannot be evaluated.

The experiments that confirmed the existence of the Casimir force at microns distance, could have measured the Van der Walls forces, or even Atomic forces between the two plates, or just noise.

If Casimir's Zero Point Energy exists between two uncharged conducting plates, no one has a clue how to determine it.

But it is more likely that Vacuum Zero Point Energy in the Cavity between two conductors stems from the urge to fill the Vacuum:

Like the Greeks who could not comprehend Zero, Physics could not comprehend the Vacuum, and filled it with Aether. But electro-magnetic-wave-propagation needs no Aether particles, and physics prefers waves anyway. The Zero-Point-Energy-Waves are an infinite-mass Aether.

The Force measured in the laboratories cannot be Casimir's Zero Point Energy Force. The measured Force-values cannot fit Energy-values that were obtained by error, and do not exist.

Had the measured Casimir Force values been Zero Point Energy

Force values, then the Euler-Maclaurin Formula would be wrong.

Applying Casimir's messed-up version of the Euler-Maclaurin Formula, the Casimir's Zero Point Energy Potential remains unknown, and the Force that it may generate is unknown too.

Casimir's meeting brief that qualifies for an erroneous Poster in a Physics Meeting, have produced thousands of pseudo-scientific written pages, along with experiments reporting measurements with up to 1% accuracy.

Those fraudulent publications, and reports show more about the lack of critical thinking of the proponents, than about the validity of a non-existent phenomena.

The Fallacy that No Journal ever accepted for publication, have developed a cult life of its own, and keeps resonating in the papers about it in the Journals.

Keywords: Casimir effect, Zero Point Energy, Vacuum, Hydrogen Atom, Quantized Angular Momentum, Fine Structure Constant, Radiation Energy, Charge-Radiation Equation

2000 Physics and Astronomy Classification Scheme 42.50.Pq, 42.50.Lc,

Contents

Zero Point Energy

0.1 Black-Body Radiation Zero Point Energy

In 1901, Planck showed that the radiation-energy density per unit volume at frequencies between ν, and $\nu + \delta\nu$ of an ideal radiator (black body) is

$$u(\nu, T) = \frac{8\pi}{c^3}\nu^2 \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}.$$

The assumption of discrete radiation energy, conflicted with Planck's belief in radiation of continuous waves, and he kept searching for a more believable law

To reconcile his quantum hypothesis with his conception of wave radiation, he avoided the conclusion that radiation energy must be made of particles, and postulated that radiation is a transition between the energy levels of an oscillator. Furthermore, ignoring the symmetry between emission and absorption, he maintained that the absorption of radiation energy is continuous.

Under these assumptions, Planck derived in 1912 his second radiation law in which zero point energy in the amount of $\frac{1}{2}h\nu$ is added to $\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$, of his 1901 radiation law.

In [Dan], we showed that Planck's derivation of his 1912 radiation law only recovers the Zero Point Energy of Black Body Radiation that he unknowingly assumed in his model for that derivation.

In particular, we showed that Planck's ZPE Radiation Law is equivalent to the combined three assumptions of

1) Zero Point Energy of Black Body Radiation Hypothesis,
2) the Quantum Law, and
3) the approximated Boson Statistics distribution law.

Planck's 1901 radiation law resolves the Black body radiation problem, and confirms the boson statistics distribution law.

The Quantum law holds independently, in the photoelectric effect, in Compton's scattering, and in spectroscopy.

But Planck's Zero Point Energy of Black Body Radiation remains a hypothesis. Thus, the validity of Planck's 1912 radiation law, and the existence of Planck's Zero Point Energy are doubtful.

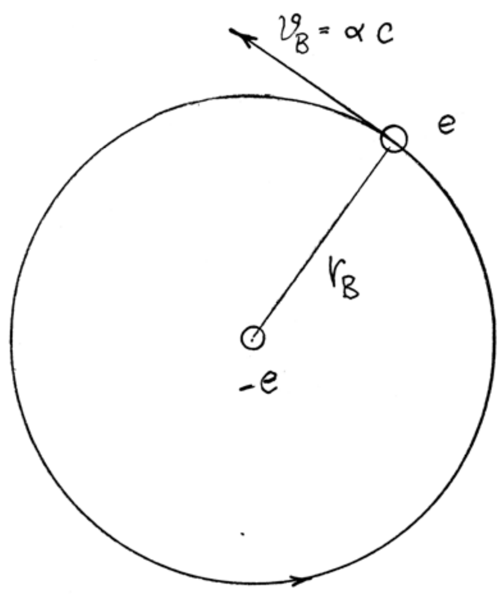
0.2 Hydrogen Atom Zero Point Energy

In [Dan2] we showed that Bohr's postulate of Quantized Angular Momentum is equivalent to the assumption of Zero Point Energy for the Hydrogen Atom.

For a Hydrogen electron with mass m_e, and charge e, in the first orbit of radius $r_1 = r_B$, at frequency $\nu_1 = \nu_B$, and speed

$$v_1 = v_B = \omega_B r_B = 2\pi\nu_B r_B,$$

encircling a proton with charge $-e$,



An electron with charge e, orbiting at frequency ν_1, and speed $v_1 = \alpha c = \omega_1 r_1 = 2\pi\nu_1 r_1$, a proton with charge $-e$ at distance r_1.

the following are equivalent

1) The <u>Angular Momentum is quantized</u>, $m_e v_B r_B = \hbar$.

2) The <u>Fine-Structure constant Formula</u>, $\alpha = \dfrac{e^2}{4\pi\varepsilon_0 c\hbar}$.

3) The <u>Charge-Radiation Equation</u>, $\dfrac{e^2}{4\pi\varepsilon_0 r_B} = h\nu_B$.

4) <u>Ground State Energy is Zero Point Energy</u> $\dfrac{-e^2}{8\pi\varepsilon_0 r_B} = -\frac{1}{2}h\nu_B$,

$$\nu_B = 6.576928348 \times 10^{15} \text{cycles/second}$$

$\alpha \approx \frac{1}{137}$ in any unit system, determines the spacing between the fine structure of spectral lines.

The binding electric energy of the Hydrogen Atom in the ground state is precisely the radiation energy of a photon with frequency that equals the frequency of the electron motion.

The Zero Point Energy of the Hydrogen Atom is the Ground state energy. Its value, $\frac{1}{2}h\nu_B$ is the lowest energy of that Atom.

The loss of that Zero Point Energy, would mean the destruction of the Atom.

Therefore, at absolute zero temperature, when all thermal motions cease, the atom still retains that amount of energy.

The existence of Zero Point Energy has been always difficult to confirm in experiments. But the confirmation of Bohr's quantization of Angular Momentum in spectroscopy, validates Zero Point Energy in the amount of $\frac{1}{2}h\nu_B$, for the Hydrogen Atom.

0.3 Casimir's Zero Point Energy between two perfectly conducting plates

It is well known that Standing Electromagnetic waves in the cavity between two plates, must vanish on the cavity walls.

Casimir observed that distant plates allow for more standing waves frequencies between them than close plates.

Consequently, work must be done in order to move a distant plate closer to the other plate.

Thus, the difference between wave energies in a cavity bounded by close plates, and wave energies in a cavity bounded by distant plates is an electromagnetic potential. The gradient of that Potential is a force pulling together the uncharged plates.

Such standing waves exist on the walls of Radar systems waveguides that carry Microwave Radiation, and in optical fibers that carry light wavelength Radiation. In either case the waves are emitted by some radiation generator.

How would electromagnetic waves appear in a cavity without a generator?

The assumption is that charged particles appearing and disappearing in the vacuum in creation, and annihilation processes, allowed by the uncertainty principle, will accelerate, and radiate that Zero Point Energy of Casimir.

But if Energy is conserved, the Mean of the Radiation Field Energy must be Zero, and there will be no Zero Point Energy Potential, or Force. If Energy is not conserved, this is not Physics, but fiction.

To compute the Zero Point Energy potential, Casimir applied the

Euler summation formula for the difference between a series and an integral of a function.

To that end, he **defined** the zero point energy between distant plates by integrating a double integral multiplied by $1/a^3$, where a is an infinite length measured by zillion meters, and **defined** the zero point energy between very close plates by summing on a double integral multiplied by $1/a^3$, where a stands for an infinitesimal length measured by microns (10^{-6}meters).

Since the a in each instance is different, the function is different, and the Euler Summation Formula does not apply. Consequently, the Casimir Zero Point Energy Potential cannot be determined, and the Casimir Force cannot be evaluated.

The experiments that confirmed the existence of the Casimir force at microns distance, could have measured the Van der Walls forces, or even Atomic forces between the two plates, or just noise.

Until now, it was never noticed that the Euler summation formula does not apply to Casimir's Zero Point Energy.

If Casimir's Zero Point Energy exists between two uncharged conducting plates, no one has a clue how to determine it.

But it is more likely that Vacuum Zero Point Energy in the Cavity between two conductors stems from the urge to fill the Vacuum: Like the Greeks who could not comprehend Zero, Physics could not comprehend the Vacuum, and filled it with Aether. But electro-magnetic-wave-propagation needs no Aether particles, and physics prefers waves anyway. The Zero-Point-Energy-Waves are an infinite-mass Aether.

1.

Electromagnetic Radiation between Two Uncharged Plates

Casimir wrote [Casimir, p. 793]

"Let us consider a cubic cavity of volume L^3 *bounded by perfectly conducting walls and let a perfectly conducting square plate with side* L *be placed in this cavity parallel to the* xy *face and let us compare*

the situation in which this plate is at a

small distance a *from the* xy *face*

and the situation in which it is at a

very large distance, say $L/2$.

In both cases, the expressions

$$\frac{1}{2}\sum \hbar\omega,$$

where the summation extends over all possible resonance frequencies of the cavities are divergent and devoid of physical meaning but the difference between these sums in the two situations

$$\frac{1}{2}\left(\sum \hbar\omega\right)_I - \frac{1}{2}\left(\sum \hbar\omega\right)_{II}$$

will be shown to have a well defined value and this value will be interpreted as the interaction between the plate and the xy face

It is clear that Casimir had in mind the infinite sum

$$\frac{1}{2}\left\{\hbar\omega_1 + \hbar\omega_2 + ... + \hbar\omega_n + ...\right\}$$

But

No physical frequencies can be infinite!

No physical vibrations can have zero period!

$\frac{1}{2}\sum \hbar\omega$ will include frequencies only up to a large number N.

And

$$\frac{1}{2}\left\{\hbar\omega_1 + \hbar\omega_2 + ... + \hbar\omega_N\right\}$$

is a finite, well-defined sum.

It is well known that any electromagnetic radiation, including the electromagnetic Zero Point Energy, is generated by accelerating charges.

These accelerating charges may appear and disappear in the vacuum due to creation and annihilation processes allowed by the uncertainty principle. But they do not radiate infinite frequencies. Infinity is an idealization that is misunderstood in mathematics, and in physics. But it is well known that infinity does not appear anywhere in physics.

2.

Casimir's Zero Point Energy between two very close Planes

Casimir wrote [Casimir, p.793]

"The possible vibrations of a cavity defined by

$$0 \le x \le L, \quad 0 \le y \le L, \quad 0 \le z \le a$$

have wave numbers

$$k_x = \frac{\pi}{L} n_x, \qquad k_y = \frac{\pi}{L} n_y, \qquad k_z = \frac{\pi}{a} n_z,$$

where n_x, n_y, n_z, *are positive integers;*

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{\kappa^2 + k_z^2}.$$

To every k_x, k_y, k_z *correspond two standing waves unless one of the* n_i *is zero, when there is only one.*

For k_x, k_y *this is without importance since for very large* L *we may regard* k_x, k_y *as continuous variables.*

Thus, we find

$$\frac{1}{2}\sum \hbar\omega = \hbar c \frac{L^2}{\pi^2} \int_0^\infty \int_0^\infty \left[\frac{1}{2}\sqrt{k_x^2 + k_y^2} + \sum_{n=1}^{\infty} \sqrt{n^2 \frac{\pi^2}{a^2} + k_x^2 + k_y^2} \right] dk_x dk_y$$

or, introducing polar coordinates in the k_x, k_y *plane,*

$$\frac{1}{2}\sum \hbar\omega = \hbar c \frac{L^2}{\pi^2}\frac{\pi}{2}\sum_{(0)1}^{\infty}\int_0^{\infty}\sqrt{\left(n^2\frac{\pi^2}{a^2}+\kappa^2\right)}\kappa d\kappa,$$

where the notation $(0)1$ *is meant to indicate that the term*

$n = 0$ *has to be multiplied by* $\frac{1}{2}$.

First ***Due to no physical infinite frequencies,***

the summation is finite, say up to a large number N**.**

Second, ***Casimir's Cavity Zero Point Energy***

$$\hbar c \frac{L^2}{\pi^2}\frac{\pi}{2}\sum_{(0)1}^{n=N}\sum_{\kappa=0}^{\kappa=N}\sqrt{\left(n^2\frac{\pi^2}{a^2}+\kappa^2\right)}\kappa d\kappa,$$

involves the summation of infinitesimals,

unlike Planck's Black-Body Radiation ZPE

$$\frac{1}{2}\sum_{n=0}^{n=N}\hbar\omega_n,$$

that involves no summation of infinitesimals.

Summation with no infinitesimals is different from summation with infinitesimals. For instance, the example

$$\sum_{n=1}^{n=\infty}\frac{1}{n(n+1)} = \sum_{n=1}^{n=\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + ...\frac{1}{n} - \frac{1}{n+1} + ... = \boxed{1}.$$

$$\sum_{x=1}^{x=\infty} \frac{1}{x(x+1)} dx = \sum_{x=1}^{x=\infty} \left(\frac{1}{x} - \frac{1}{x+1} \right) dx$$

$$= \lim_{N \to \infty} \left\{ \int_{x=1}^{x=N} \frac{1}{x} dx - \int_{x=1}^{x=N} \frac{1}{x+1} dx \right\}$$

$$= \lim_{N \to \infty} \left\{ \int_{x=1}^{x=N} \frac{1}{x} dx - \int_{u=2}^{u=N+1} \frac{1}{u} du \right\}$$

$$= \lim_{N \to \infty} \left\{ \int_{x=1}^{x=N} \frac{1}{x} dx - \int_{x=2}^{x=N+1} \frac{1}{x} dx \right\}$$

$$= \lim_{N \to \infty} \left\{ \int_{x=1}^{x=N} \frac{1}{x} dx - \left(\int_{x=2}^{x=N} \frac{1}{x} dx + \int_{x=N}^{x=N+1} \frac{1}{x} dx \right) \right\}$$

$$= \lim_{N \to \infty} \left\{ \int_{x=1}^{x=2} \frac{1}{x} dx - \int_{x=N}^{x=N+1} \frac{1}{x} dx \right\}$$

$$= \int_{x=1}^{x=2} \frac{1}{x} dx - \lim_{N \to \infty} \left\{ \int_{x=N}^{x=N+1} \frac{1}{x} dx \right\}$$

$$= \log 2 - \lim_{N \to \infty} \log \frac{N+1}{N}$$

$$= \log 2 - \log \underbrace{\lim_{N \to \infty} \frac{N+1}{N}}_{1} = \boxed{\log 2}$$

To define the Zero Point Energy between close plates, start with

$$\frac{1}{2}\sum_{n=0}^{n=N}\hbar\omega_n = \frac{1}{2}\hbar c\sum_{n=0}^{n=N}k_n$$

$$= \hbar c\sum_{n_y=0}^{n_y=N}\sum_{n_x=0}^{n_x=N}\left[\frac{1}{2}\sqrt{n_x^2\frac{\pi^2}{L^2}+n_y^2\frac{\pi^2}{L^2}}+\sum_{n_z=1}^{n_z=N}\sqrt{n_x^2\frac{\pi^2}{L^2}+n_y^2\frac{\pi^2}{L^2}+n_z^2\frac{\pi^2}{a^2}}\right]$$

Since $\Delta n_x = 1$, and $\Delta n_y = 1$,

$$= \hbar c\frac{L^2}{\pi^2}\sum_{n_y=0}^{n_y=N}\sum_{n_x=0}^{n_x=N}\left[\frac{1}{2}\sqrt{n_x^2\frac{\pi^2}{L^2}+n_y^2\frac{\pi^2}{L^2}}+\sum_{n=1}^{N}\sqrt{n_x^2\frac{\pi^2}{L^2}+n_y^2\frac{\pi^2}{L^2}+n_z^2\frac{\pi^2}{a^2}}\right]\underbrace{\frac{\pi}{L}\Delta n_x}_{\Delta k_x}\underbrace{\frac{\pi}{L}\Delta n_y}_{\Delta k_y}.$$

For very large L, (say, $L = \sqrt{N}$)

$$\Delta k_x = \frac{\pi}{L}\Delta n_x, \text{ and } \Delta k_y = \frac{\pi}{L}\Delta n_y$$

suggest infinitesimals. Thus, for a Large number N, we **define** the Zero Point Energy between the close plates by

$$\hbar c\frac{L^2}{\pi^2}\int_{k_y=0}^{k_y=N}\int_{k_x=0}^{k_x=N}\left[\frac{1}{2}\sqrt{k_x^2+k_y^2}+\sum_{n_z=1}^{n_z=N}\sqrt{k_x^2+k_y^2+n_z^2\frac{\pi^2}{a^2}}\right]dk_x dk_y$$

In polar coordinates, $dk_x dk_y = \kappa d\kappa d\phi$, the ZPE is

$$= \hbar c\frac{L^2}{\pi^2}\int_{\kappa=0}^{\kappa=N}\left[\frac{1}{2}\kappa+\sum_{n_z=1}^{n_z=N}\sqrt{\kappa^2+n_z^2\frac{\pi^2}{a^2}}\right]\kappa d\kappa\int_{\phi=0}^{\phi=\frac{\pi}{2}}d\phi$$

$$= \hbar c\frac{L^2}{\pi^2}\frac{\pi}{2}\int_{\kappa=0}^{\kappa=N}\left[\frac{1}{2}\kappa+\sum_{n_z=1}^{n_z=N}\sqrt{\kappa^2+n_z^2\frac{\pi^2}{a^2}}\right]\kappa d\kappa$$

Put $u = \frac{a^2}{\pi^2}\kappa^2$. Then, $\kappa d\kappa = \frac{1}{2}\frac{\pi^2}{a^2}du$, and the ZPE is

$$= \hbar c \frac{L^2}{\pi^2}\frac{\pi}{2}\int_{u=0}^{u=\frac{a^2}{\pi^2}N^2}\left[\frac{1}{2}\sqrt{u}\,\frac{\pi}{a}+\sum_{n_z=1}^{n_z=N}\sqrt{u\frac{\pi^2}{a^2}+n_z^2\frac{\pi^2}{a^2}}\right]\frac{1}{2}\frac{\pi^2}{a^2}du$$

$$= \hbar c \frac{L^2}{a^3}\frac{\pi^2}{4}\int_{u=0}^{u=\frac{a^2}{\pi^2}N^2}\left[\frac{1}{2}\sqrt{u}+\sum_{n_z=1}^{n_z=N}\sqrt{u+n_z^2}\right]du$$

$$\boxed{= L^2\hbar c \frac{\pi^2}{4}\left\{\frac{1}{2}\frac{1}{a^3}\int_{u=0}^{u=\frac{a^2}{\pi^2}N^2}\sqrt{u}du+\sum_{n_z=1}^{n_z=N}\frac{1}{a^3}\int_{u=0}^{u=\frac{a^2}{\pi^2}N^2}\sqrt{u+n_z^2}du\right\}}$$

3.

Casimir's Zero Point Energy between two distant Planes

To define the Zero Point Energy between distant plates, start with

$$\frac{1}{2}\sum_{n=0}^{n=N}\hbar\omega_n = \frac{1}{2}\hbar c\sum_{n=0}^{n=N}k_n$$

$$= \hbar c\sum_{n_y=0}^{n_y=N}\sum_{n_x=0}^{n_x=N}\sum_{n_z=1}^{n_z=N}\sqrt{n_x^2\frac{\pi^2}{L^2}+n_y^2\frac{\pi^2}{L^2}+n_z^2\frac{\pi^2}{L^2}}$$

Since $\Delta n_x = 1$, $\Delta n_y = 1$, and $\Delta n_z = 1$,

$$= \hbar c\frac{L^3}{\pi^3}\sum_{n_y=0}^{n_y=N}\sum_{n_x=0}^{n_x=N}\sum_{n=1}^{N}\sqrt{n_x^2\frac{\pi^2}{L^2}+n_y^2\frac{\pi^2}{L^2}+n_z^2\frac{\pi^2}{L^2}}\underbrace{\frac{\pi}{L}\Delta n_x}_{\Delta k_x}\underbrace{\frac{\pi}{L}\Delta n_y}_{\Delta k_y}\underbrace{\frac{\pi}{L}\Delta n_z}_{\Delta k_z}.$$

For very large L, (say, $L = \sqrt{N}$)

$$\Delta k_x = \frac{\pi}{L}\Delta n_x, \quad \Delta k_y = \frac{\pi}{L}\Delta n_y, \quad \text{and } \Delta k_z = \frac{\pi}{L}\Delta n_z$$

suggest infinitesimals. Thus, for a Large number N, we **define** the Zero Point Energy between the distant plates by

$$\hbar c\frac{L^3}{\pi^3}\int_{k_z=0}^{k_z=\frac{\pi}{L}N}\int_{k_y=0}^{k_y=N}\int_{k_x=0}^{k_x=N}\sqrt{k_x^2+k_y^2+k_z^2}\,dk_x dk_y dk_z$$

In polar coordinates, $dk_x dk_y = \kappa d\kappa d\phi$, and the ZPE is

$$= \hbar c \frac{L^3}{\pi^3} \int_{\kappa=0}^{\kappa=N} \int_{k_z=0}^{k_z=\frac{\pi}{L}N} \sqrt{\kappa^2 + k_z^2}\,\kappa d\kappa dk_z \int_{\phi=0}^{\phi=\frac{\pi}{2}} d\phi$$

$$= \hbar c \frac{L^3}{\pi^3} \frac{\pi}{2} \int_{\kappa=0}^{\kappa=N} \int_{k_z=0}^{k_z=\frac{\pi}{L}N} \sqrt{\kappa^2 + k_z^2}\,\kappa d\kappa dk_z$$

Put $u = \frac{L^2}{\pi^2}\kappa^2$. Then, $\kappa d\kappa = \frac{1}{2}\frac{\pi^2}{L^2} du$, and the ZPE is

$$= \hbar c \frac{L^3}{\pi^3} \frac{\pi}{2} \int_{u=0}^{u=\frac{L^2}{\pi^2}N^2} \int_{n_z=0}^{n_z=N} \sqrt{u\frac{\pi^2}{L^2} + n_z^2 \frac{\pi^2}{L^2}} \frac{1}{2}\frac{\pi^2}{L^2} du \frac{\pi}{L} dn_z$$

$$= \hbar c \frac{\pi^2}{4L} \int_{u=0}^{u=\frac{L^2}{\pi^2}N^2} \int_{n_z=0}^{n_z=N} \sqrt{u + n_z^2}\, du dn_z$$

$$\boxed{= L^2 \hbar c \frac{\pi^2}{4} \int_{n_z=0}^{n_z=N} \left\{ \frac{1}{L^3} \int_{u=0}^{u=\frac{L^2}{\pi^2}N^2} \sqrt{u + n_z^2}\, du dn_z \right\}}$$

4.

Casimir's Zero Point Energy Potential between two plates

Casimir wrote

"introducing polar coordinates in the k_x, k_y *plane,*

$$\frac{1}{2}\sum \hbar\omega = \hbar c \frac{L^2}{\pi^2}\frac{\pi}{2}\sum_{(0)1}^{\infty}\int_0^{\infty}\sqrt{\left(n^2\frac{\pi^2}{a^2}+\kappa^2\right)}\kappa d\kappa,$$

where the notation $(0)1$ *is meant to indicate that the term* $n = 0$ *has to be multiplied by* $\frac{1}{2}$.

For very large a *also this last summation may be replaced by an integral and it is therefore easily seen that our interaction energy is given by*

$$\delta E = \hbar c \frac{L^2}{\pi^2}\frac{\pi}{2}\left\{\sum_{(0)1}^{\infty}\int_0^{\infty}\sqrt{\left(n^2\frac{\pi^2}{a^2}+\kappa^2\right)}\kappa d\kappa - \int_0^{\infty}\int_0^{\infty}\sqrt{\left(k_z^2+\kappa^2\right)}\kappa d\kappa \frac{a}{\pi}dk_z\right\}$$

In order to obtain a finite result it is necessary to multiply the integrands by a function $f(k / k_m)$ *which is unity for* $k \ll k_m$ *but tends to zero sufficiently rapidly for* $k / k_m \to \infty$, *where* k_m *may be defined by* $f(1) = \frac{1}{2}$.

The physical meaning is obvious: For very short waves (X-rays e.g.) our plate is hardly an obstacle at all and therefore the zero point energy of these waves will not be influenced by the position of this plate.

Introducing the variable $u = \frac{L^2}{\pi^2}\kappa^2$,

$$\delta E = L^2 \hbar c \frac{\pi^2}{4a^3}\left\{ \sum_{(0)1}^{\infty} \int_0^{\infty} \sqrt{\left(n^2 + u\right)} f\left(\pi\sqrt{\left(n^2 + u\right)} \,/\, ak_m\right) du \right.$$

$$\left. - \int_0^{\infty}\int_0^{\infty} \sqrt{\left(k_z^2 + \kappa^2\right)} f\left(\pi\sqrt{\left(n^2 + u\right)} \,/\, ak_m\right) du dn \right\}$$

We apply the Euler-Maclaurin Formula"

In Casimir's formula for δE, a is assumed to be very small, But in fact, it is small only in the first half of the formula, and it is very large in the second part in the formula.

From **2**, the Zero Point Energy between close plates is

$$L^2 \hbar c \frac{\pi^2}{4}\left\{ \frac{1}{2}\frac{1}{a^3} \int_{u=0}^{u=\frac{a^2}{\pi^2}N^2} \sqrt{u} du + \sum_{n_z=1}^{n_z=N} \frac{1}{a^3} \int_{u=0}^{u=\frac{a^2}{\pi^2}N^2} \sqrt{u + n_z^2} du \right\}$$

From **3**, the Zero Point Energy between distant plates is

$$L^2 \hbar c \frac{\pi^2}{4} \int_{n_z=0}^{n_z=N} \left\{ \frac{1}{L^3} \int_{u=0}^{u=\frac{L^2}{\pi^2}N^2} \sqrt{u + n_z^2} du dn_z \right\}$$

Hence, the difference of the Zero Point Energies is

$$L^2\hbar c\frac{\pi^2}{4}\left\{\frac{1}{2}\frac{1}{a^3}\int_{u=0}^{u=\frac{a^2}{\pi^2}N^2}\sqrt{u}du+\sum_{n_z=1}^{n_z=N}\frac{1}{a^3}\int_{u=0}^{u=\frac{a^2}{\pi^2}N^2}\sqrt{u+n_z^2}du\right.$$

$$\left.-\int_{n_z=0}^{n_z=N}\frac{1}{L^3}\int_{u=0}^{u=\frac{L^2}{\pi^2}N^2}\sqrt{u+n_z^2}dudn_z\right\}$$

Since

$$\frac{1}{a^3}\int_{u=0}^{u=\frac{a^2}{\pi^2}N^2}\sqrt{u+n_z^2}du$$

is a function different from the function

$$\frac{1}{L^3}\int_{u=0}^{u=\frac{L^2}{\pi^2}N^2}\sqrt{u+n_z^2}dudn_z,$$

the Euler Summation Formula does not apply to the Casimir potential δE.

In our formulas, a cutoff function is not necessary because the summation is up to a large number N.

5.

Euler's Summation Formula

Euler defined the zeta function as the infinite series

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + ...$$

and found a Formula for $\zeta(2k)$, $k = 1, 2, 3, ...$ For instance,

$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^s} + ... = \frac{\pi^2}{6}$$

$$\zeta(4) = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + ... = \frac{\pi^4}{90}$$

Euler's Formula did not apply to $\zeta(2k+1)$, $k = 1, 2, 3, ...$and he approximated them with his Summation Formula:

For instance, for $\zeta(3)$,

$$f(x) = \frac{1}{x^3},$$

$$f'(x) = -3x^{-4},$$

$$f''(x) = -3(-4)x^{-5},$$

$$f'''(x) = -3(-4)(-5)x^{-6},$$

$$f''''(x) = -3(-4)(-5)(-6)x^{-7},$$

$$f^{(5)}(x) = -3(-4)(-5)(-6)(-7)x^{-8}$$

....

$$\zeta(3) = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + ... = \underbrace{\int_{x=1}^{x=\infty} \frac{1}{x^3}dx}_{\frac{1}{2x^2}\Big|_{x=\infty}^{x=1} = \frac{1}{2}} + \frac{1}{2}\underbrace{\left[\frac{1}{x^3}\Big|_{x=1} + \frac{1}{x^3}\Big|_{x=\infty} \right]}_{1} +$$

$$+\frac{1}{12}\underbrace{[-3x^{-4}]_{x=1}^{x=\infty}}_{3} - \underbrace{\frac{1}{720}\underbrace{[-3(-4)(-5)x^{-6}]_{x=1}^{x=\infty}}_{60}}_{\frac{1}{12}} +$$

$$+\underbrace{\frac{1}{30240}\underbrace{[-3(-4)(-5)(-6)(-7)x^{-8}]_{x=1}^{x=\infty}}_{2520}}_{\frac{1}{12}} - ...$$

That is, up to the 5^{th} Derivative, $\zeta(3) \sim 1.25$.

In general, for $m < x < N$, Euler's Summation Formula is

$$f(m) + f(1) + f(2) + ...f(N) = \int_{x=m}^{x=N} f(x)dx + \frac{1}{2}\left[f(m) + f(N) \right] +$$

$$+\frac{1}{12}\left[f'(N) - f'(m) \right] - \frac{1}{720}\left[f^{(3)}(N) - f^{(3)}(m) \right] + \frac{1}{30240}\left[f^{(5)}(N) - f^{(5)}(m) \right] - ..$$

The coefficients in the formula are

$$\frac{B_{2k}}{(2k)!},$$

where B_{2k} are Bernoulli numbers [Zeidler, p.34]

$$B_2 = \frac{1}{6}; \quad B_4 = -\frac{1}{30}; \quad B_6 = \frac{1}{42};.....$$

Casimir wished to use Euler's Formula to evaluate the difference between the series and the integral. But he had summed one function, and integrated another function.

Clearly, the Formula applies only to one single function. the same function must appear in both the series summation, and under the integral summation sign.

6.

The Force from Casimir's Messed-Up Version of Euler's Formula

In [Casimir, p.794], Casimir claims

"We apply the EULER-MACLAURIN formula

$$\sum_{(0)1}^{\infty} F(n) - \int_{0}^{\infty} F(n)dn = -\frac{1}{12}F'(0) + \frac{1}{24\times 30}F'''(0) + \ldots$$

Introducing $w = u + n^2$ *we have*

$$F(n) = \int_{n^2}^{\infty} w^{1/2} f(w\pi / ak_m)dw\,,$$

whence

$$F'(n) = -2n^2 f(n^2\pi / ak_m)$$

$$F'(0) = 0$$

$$F'''(0) = -4$$

The higher derivatives will contain powers of (π / ak_m).

Thus we find

$$\frac{\delta E}{L^2} = -\hbar c \frac{\pi^2}{24\times 30}\frac{1}{a^3}.$$

a formula which holds as long as $ak_m \gg 1$.

For the force per cm^2 *we find*

$$F = \hbar c \frac{\pi^2}{240} \frac{1}{a_\mu^4} \text{ dyne/cm}^2$$

where a_μ^4 *is the distance measured in microns.*

We are thus led to the following conclusions.

There exists an attractive force between two metal plates which is independent of the material of the plates as long as the distance is so large that for wave lengths comparable with that distance the penetration depth is small compared with the distance.

This force may be interpreted as a Zero point pressure of electromagnetic waves"

Casimir's messed-up version of the Euler-Maclaurin Formula, leads to

$$F = \hbar c \frac{\pi^2}{240} \frac{1}{a_\mu^4} \text{ dyne/cm}^2.$$

It is approximate due to the cut-off function that does affect his final result, as Casimir admits above

The higher derivatives will contain powers of (π / ak_m).

Summing till a large number N, eliminates the cut-off function.

It is likely that measurements of the Casimir force above have measured the intermolecular van-der-Walls forces

And with great uncertainty affected by the Temperature:

7.

The Critical effect of Temperature on the Meaningless Casimir Force

By [Pena, p. 172], accounting for the temperature, the Casimir Force is approximately

$$-\hbar c\frac{\pi^2}{240}\frac{1}{a^4}-\frac{1.202}{4\pi}\frac{kT}{a^3}.$$

At room temperature $T=300K$, and for a in $10^{-6}\mathrm{m}$

$$\frac{\frac{1.202}{4\pi}\frac{kT}{a^3}}{\hbar c\frac{\pi^2}{240}\frac{1}{a^4}}=\frac{60(1.202)}{\pi^3}\frac{k(300)}{\hbar c}a(10^{-6})$$

$$=\frac{60(1.202)}{31.00627668}\frac{(1.380658\times10^{-23})(300)}{(1.05457266\times10^{-34})3\cdot10^8}a(10^{-6})$$

$$\approx(0.30452)a$$

$a=1\mu\mathrm{m}\Rightarrow$ the temperature term is ~ 0.3 the ZPE term

$a=3.3\mu\mathrm{m}\Rightarrow$ the temperature term is $\sim$ the ZPE term

$a=5\mu\mathrm{m}\Rightarrow$ the temperature term is ~ 1.5 the ZPE term

That is, the temperature term implies critical dependence on the distance between the plates, and makes the meaningless Casimir Force Formula even more meaningless.

One can claim any degree of accuracy of his measurements by a little adjustment of the distance between the plates.

And it is not at all clear how the same critical distance is kept uniformly between the plates at any point on them

8.

The Critical effect of Models and Corrections in the Casimir Force Measurements

The uncertainty in the Casimir Force measurements goes further according to [Lamoreaux, p. 43]

> *"In 2000 Mathias Bostrom and Bo Sernelius showed that the form of the electric permittivity has a critical effect when one calculates the finite-conductivity correction in combination with the nonzero finite temperature correction…*
>
> *Bostrom and Sernelius assumed the material to have an electric permittivity to be given by the so called Drude model and found that the contribution of the* $n = 0$ *transverse electric mode vanishes.*
>
> *So, compared with previous calculations in which the transverse electric and the transverse magnetic modes had roughly equal contributions, their result shows a large correction to the force at distances of order* $1\mu\mathrm{m}$ *and*

demonstrates that the force is reduced by a factor of two for large separation.

However, the theoretical story is far from simple. Bostrom and Sernelius result apparently disagrees with a number of experiments and most significantly with my own. Their correction is about as large as the plasma model corrections that I ruled out.

A study of the spectrum of the finite-temperature correction suggests that low frequencies make the dominant contribution; the frequencies are low enough that it might not be possible to describe the response of the metal in terms of a simple Drude model electrical permittivity.

This experimenters' talk goes on and on…

Nevertheless, it is clear that the validity of Casimir's ZPE Force is secondary to the corrections, the models, and the experimenters.

It is likely that for experimenters, the verification of the Casimir ZPE force formula, involves more models and corrections that are unknown to the rest of us.

Consequently, experimenters claims about confirming Casimir's force at 1% accuracy without mentioning the models, and the corrections cannot be taken seriously.

9.

The Zero-Point-Energy Waves is an Infinite Mass Aether

The existence of a Non-zero Mean of energy with no source, simply from nowhere, that violates Energy Conservation is unlikely.

Physics was saved from Bohr's disregard for energy conservation in radioactive decay, by Pauli's hypothesis of the Neutrino.

Casimir picked the idea of source-less Zero Point Energy from Bohr, and planted it in the uncertain zone of intermolecular van der Walls forces.

Since 1948, the only proof for the Casimir Zero Point Energy was the Casimir Force.

But what was measured in the laboratories cannot be the Zero Point Energy Force. The measured Force-values cannot fit Energy-values that were obtained by error, and do not exist.

Had the measured Casimir Force values been Zero Point Energy Force values, then the Euler-Maclaurin Formula would be wrong.

Applying Casimir's messed-up version of the Euler-Maclaurin Formula, the Casimir's Zero Point Energy remains Unknown, and the Force that it may generate is unknown too.

To date, the only argument for Vacuum Zero Point Energy is Casimir's 1948 meeting brief, with its messed-up version of the Euler-Maclaurin Formula.

There are two works on Vacuum Zero Point Energy:

The book by [Pena] claims to establish Vacuum Zero Point Energy as part of what it calls "Stochastic Electrodynamics".

The book by [Milonni] presents Quantum Electrodynamics through Vacuum Zero Point Energy.

Both works, at over 500 pages each, leave the substantiation of Vacuum Zero Point Energy to Casimir's messed-up version of the Euler-Maclaurin Formula.

If Casimir's Zero Point Energy exists between two uncharged conducting plates, no one has a clue how to compute it.

But it is more likely that Vacuum Zero Point Energy in the Cavity between two conductors stems from the urge to fill the Vacuum

Like the Greeks who could not comprehend Zero, Physics could not comprehend the Vacuum, and filled it with Aether. But electro-magnetic-wave-propagation needs no Aether particles, and Physics prefers Aether waves. Filling the Vacuum with Zero Point Energy Waves, eliminates emptiness, and allows "the Vacuum Effect…"

The Vacuum Zero-Point-Energy-Waves replaced the Old Aether-

Particles, to become the New Aether-Waves.

That Aether is postulated as carrying Infinite Energy.

That is,

energy being mass through $m = E / c^2$,

the new Vacuum contains an infinite amount of mass.

Never mind that a Vacuum filled with infinite mass is not a Vacuum.

10.

The Fallacy No Journal Accepted, that Resonates to No End in Journals

Pauli rejected Bohr's disregard for Energy Conservation, that lead to the Vacuum Zero Point Energy, but Casimir did not get it:

According to Casimir, [Casimir1, p. 247]

> "*…In 1951..Pauli and I attended the Bothe Conference in Heidelberg…I explained to him my results on the Van der Walls forces and their relation to field fluctuation in empty space.*
>
> *He began by **bluntly telling me it was all nonsense**, but was obviously amused when I did not give in. Finally, after I have countered all his arguments he agreed, and called me a real …tumbler,…defined in my dictionary as "a little doll with a sphere of lead instead of legs, which always gets back to the vertical position"….*"

So **Casimir figured that Pauli agreed with a real tumbler…**

So far, with that kind of reference, by Pauli, the Casimir Fallacy could not be published in any Journal.

Casimir's meeting brief that qualifies for an erroneous Poster in a Physics Meeting, have produced thousands of pseudo-scientific written pages, along with experiments reporting measurements with up to 1% accuracy.

Those fraudulent publications, and reports show more about the lack of critical thinking of the proponents, than about the validity of a non-existent phenomena.

The Fallacy that No Journal ever accepted for publication, have developed a cult life of its own, and keeps resonating in the papers about it in the Journals.

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