Zero Point Energy and Electromagnetic Entropy

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Abstract. In [Dan1], we proved that a Thermodynamic System with concentration υ, absolute temperature T, pressure $p = p(T, \upsilon)$, and latent heat $\nu = \nu(T, \upsilon)$, has entropy if and only if

$$\frac{\partial_T p}{\nu} \text{ is a function of } T \text{ alone}^{1}$$

Here, considering electromagnetic radiation as a frictionless photon gas, we apply our condition to establish that electromagnetic radiation has an entropy function.

Thus, for electromagnetic radiation we prove the Second Law of Thermodynamics, rather than postulate it.

The quantum of electromagnetic energy, $\varepsilon_\omega = \hbar\omega$, implies a quantum of electromagnetic entropy $\sigma_\omega(T) = \hbar\omega / T$.

Thus, at $T \approx 0$, the entropy quantum is indefinitely large.

The entropy MKS units are Joule/Kelvin.

[1] To date, the only condition ever given for the existence of entropy. The question how to establish the existence of entropy without Carnot cycles, or Caratheodory surfaces, was raised by Professor James Serrin in discussions that we had. He carefully read, and critiqued my writings till its publication.

We show that at fixed concentration υ, Planck's Radiation Law of 1901, [Planck2], [Planck3], is equivalent to an Entropy Law. Namely, The average electromagnetic radiation energy per mode in $[\omega, \omega + dw]$ is

$$u_{\omega}(T) = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1},$$

$\Leftrightarrow$ the average electromagnetic entropy at fixed υ, per mode in $[\omega, \omega + dw]$ is

$$s_{\omega}(T) = \frac{\varepsilon_{\omega}}{T}\left(\frac{1}{e^{\frac{\varepsilon_{\omega}}{kT}} - 1} + 1\right) + k\log\frac{1}{e^{\frac{\varepsilon_{\omega}}{kT}} - 1}$$

The same Entropy Law holds when we use Planck's second radiation law of 1912 [Planck1] with Zero Point Energy.

Namely, the average electromagnetic radiation energy per mode in $[\omega, \omega + dw]$ is

$$u_{\omega}(T) = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1} + \tfrac{1}{2}\hbar\omega$$

$\Leftrightarrow$ the average electromagnetic entropy at fixed υ, per mode in $[\omega, \omega + dw]$ is

$$s_{\omega}(T) = \frac{\varepsilon_{\omega}}{T}\left(\frac{1}{e^{\frac{\varepsilon_{\omega}}{kT}} - 1} + 1\right) + k\log\frac{1}{e^{\frac{\varepsilon_{\omega}}{kT}} - 1}$$

Furthermore, we prove that Zero Point Entropy is zero. That is,

$$T \approx 0 \Rightarrow s(T) \approx 0$$

Thus, for electromagnetic entropy, the 3rd Law of Thermodynamics hold

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Contents

Thermodynamic Entropy

The heat absorbed by a system undergoing a thermodynamic process depends on the values of the temperature and the density during the process. Any path starting at a thermodynamic state A and ending at a state B in the temperature-density plane results in a different amount of heat.

Therefore, an infinitesimal change in the heat absorbed during a process is not an exact differential, and is denoted by

$$\delta Q .$$

For a system characterized by temperature

$$T ,$$

concentration

$$\upsilon ,$$

specific heat

$$C(T,\upsilon) ,$$

and latent heat

$$\nu(T,\upsilon) ,$$

we assume that

$$\delta Q = C(T,\upsilon)dT + \nu(T,\upsilon)d\upsilon .$$

If for a loss-less process, δQ has an integrating factor $\frac{1}{\theta}$, so that

$$\frac{\delta Q}{\theta}$$

is an exact differential

$$ds(T,\upsilon) ,$$

then the heat function

$$s(T,\upsilon)$$

that depends only on the initial and final states is the **Entropy** of the thermodynamic system.

Such integrating factor may not exist. For instance, the form

$$xdy + dz$$

has no integrating factor [Pippard].

It is common to postulate for a thermodynamical system the Second Law of Thermodynamics

$$ds = \frac{\delta Q}{T}.$$

That is, postulate that entropy exists with

$$\theta = T\,.$$

Then, a system that does not have an entropy function is not a Thermodynamic System.

But

How do we check if a system has an entropy function?

Is there a condition for the existence of entropy?

In [Dan1], we gave the only known to date necessary and sufficient condition for the existence of entropy for a frictionless thermodynamic system.

We proceed to state it here.

We denote the infinitesimal work done in any process by

$$\delta W\,,$$

and the pressure by

$$p(T,v)\,.$$

We assume that

$$\delta W = p(T,v)dv\,.$$

We assume the First Law of Thermodynamics. That is, in any cyclic process, represented by a closed curve $\gamma(T,\upsilon)$ in the Thermodynamic state space, energy is conserved. That is,

$$\oint_{\gamma(T,\upsilon)} \delta Q = \oint_{\gamma(T,\upsilon)} \delta W$$

This means that

$$\delta Q - \delta W$$

is an exact differential

$$du(T,\upsilon).$$

Then the heat function

$$u(T,\upsilon)$$

that depends only on the initial and final states is the **Internal Energy** of the thermodynamic system.

In [Dan1], we proved that entropy exists if and only if

$$\boxed{\frac{\partial_T p}{\nu} \text{ is a function of } T \text{ alone}}$$

Furthermore, we have shown that then

$$\boxed{\theta = T}.$$

In [Dan1], we have shown that the system with

$$C = T + \upsilon,$$

$$\nu = 2(T + \upsilon),$$

$$p = T,$$

has internal energy $u = \frac{1}{2}T^2 + \upsilon T + \upsilon^2$, but No entropy because

$$\frac{\partial_T p}{\nu} = \frac{1}{2(T+\upsilon)} \text{ is Not a function of } T \text{ alone}.$$

By the First Law, we have

$$du + \delta W = \delta Q$$

$$du + pdv = CdT + \nu dv$$

If the system has an entropy function s

$$\frac{du + pdv}{T} = \frac{CdT + \nu dv}{T} = ds$$

$$ds = \frac{1}{T}du + \frac{p}{T}dv$$

Therefore,

$$\frac{\partial s}{\partial u} = \frac{1}{T}$$

$$\frac{\partial s}{\partial v} = \frac{p}{T}$$

The MKS units of the internal energy are

Joule.

The MKS units of the entropy are

$$\frac{\text{Joule}}{\text{Absolute Temperature Degree}} = \frac{\text{Joule}}{\text{Kelvin}}$$

1.

Electromagnetic Radiation has Entropy, Satisfying the Second Law of Thermodynamics

Electromagnetic radiation is a frictionless (=ideal) photon gas where

$$\nu = p = \frac{T}{\upsilon}.$$

Therefore,

$$\frac{\partial_T p}{\nu} = \frac{\partial_T \left(\frac{T}{\upsilon}\right)}{\frac{T}{\upsilon}}$$

$$= \frac{1}{T}$$

That is,

$$\frac{\partial_T p}{\nu} \text{ is a function of } T \text{ alone}.$$

Thus, by our necessary and sufficient condition for entropy existence, the electromagnetic radiation has an entropy function.

By the First Law, we have

$$du + \delta W = \delta Q$$

$$du + pd\upsilon = CdT + \nu d\upsilon$$

Since $p = \nu$

$$du = CdT .$$

Hence,

$$u \text{ is a function of } T \text{ alone},$$

$$u = u(T).$$

and

$$C \text{ is a function of } T \text{ alone},$$

$$C(T) = \frac{du}{dT}.$$

Hence,

$$ds = \frac{du + pdv}{T}$$

$$= \frac{1}{T}du + \frac{1}{T}\frac{T}{v}dv$$

$$= \frac{1}{T}du + \frac{1}{v}dv$$

That is,

$$\frac{\partial s}{\partial u} = \frac{1}{T},$$

$$\frac{\partial s}{\partial v} = \frac{1}{v}.$$

And

$$s(T, v) = \int \frac{1}{T}du + \underbrace{\int \frac{1}{v}dv}_{\log v}$$

We proceed to derive a Radiation Law for the electromagnetic entropy s equivalent to Planck's Radiation Laws for the electromagnetic internal energy u.

We first observe that the quantum of electromagnetic energy implies quantum electromagnetic entropy

2.

Entropy Quantum

The quantum of radiation energy implies quantum entropy.

The relation

$$\frac{\partial s}{\partial u} = \frac{1}{T}$$

means that at fixed concentration v, for an infinitesimal change in the electromagnetic radiation energy δu, and the related infinitesimal change in the entropy δs,

$$\frac{\delta s}{\delta u} = \frac{1}{T},$$

$$\delta s = \frac{\delta u}{T}$$

Since the quantum of electromagnetic energy is

$$\varepsilon_{\omega} = \hbar\omega,$$

the quantum of electromagnetic entropy is

$$\sigma_{\omega}(T) = \frac{\varepsilon_{\omega}}{T} = \frac{\hbar\omega}{T}$$

At temperatures above 1° Kelvin, the entropy quantum is smaller than the energy quantum. But at temperatures under 1° Kelvin, and close to 0° Kelvin, the entropy quantum is greater than the photon.

As $T \downarrow 0$, the entropy quantum is indefinitely large.

3.

Electromagnetic Entropy without Zero Point Energy

In 1901, Planck proved that the average internal radiation energy per mode in the frequency interval $[\omega, \omega + d\omega]$ is

$$u_{\omega}(T) = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1}.$$

There are $\frac{\omega^2}{\pi^2 c^3}$ modes in $[\omega, \omega + d\omega]$, and the average radiation energy in it is

$$\frac{\omega^2}{\pi^2 c^3} u_{\omega}(T)$$

For the photon gas, the entropy is

$$s(T, \upsilon) = \underbrace{\int \frac{1}{T} du}_{\text{depends on } T \text{ alone}} + \underbrace{\log \upsilon}_{\text{we'll keep } \upsilon \text{ fixed}}.$$

Since υ, and T are independent variables, we may fix υ, and seek only the integral that depends on T alone, hence, on u alone.

Thus, we aim to write $\frac{1}{T}$ in terms of u:

Denoting $\varepsilon = \hbar\omega$, and $u_{\omega}(T) = u$,

$$u = \frac{\varepsilon}{e^{\frac{\varepsilon}{kT}} - 1}$$

Solving for T,

$$\frac{u}{\varepsilon} = \frac{1}{e^{\frac{\varepsilon}{kT}} - 1},$$

$$\frac{u}{\varepsilon}(e^{\frac{\varepsilon}{kT}} - 1) = 1,$$

$$\frac{u}{\varepsilon}e^{\frac{\varepsilon}{kT}} = \frac{u}{\varepsilon} + 1,$$

$$e^{\frac{\varepsilon}{kT}} = \frac{\frac{u}{\varepsilon} + 1}{\frac{u}{\varepsilon}},$$

$$\frac{\varepsilon}{kT} = \log\frac{\frac{u}{\varepsilon} + 1}{\frac{u}{\varepsilon}},$$

$$\frac{1}{T} = \frac{k}{\varepsilon}\log\frac{\frac{u}{\varepsilon} + 1}{\frac{u}{\varepsilon}},$$

$$\frac{1}{T} = \frac{k}{\varepsilon}\left\{\log(\frac{u}{\varepsilon} + 1) - \log(\frac{u}{\varepsilon})\right\}$$

Since $\dfrac{\partial s}{\partial u} = \dfrac{1}{T}$,

$$\frac{\partial s}{\partial u} = \frac{k}{\varepsilon}\left\{\log(\frac{u}{\varepsilon} + 1) - \log(\frac{u}{\varepsilon})\right\}$$

Integrating with respect to u, the average entropy per mode is

$$s = k\left\{(\frac{u}{\varepsilon} + 1)\log(\frac{u}{\varepsilon} + 1) - (\frac{u}{\varepsilon})\log(\frac{u}{\varepsilon})\right\}$$

$$= k\left\{ (\frac{u}{\varepsilon})\log(\frac{u}{\varepsilon}+1) - (\frac{u}{\varepsilon})\log(\frac{u}{\varepsilon}) + \log(\frac{u}{\varepsilon}+1) \right\}$$

$$= k\left\{ (\frac{u}{\varepsilon})\log\frac{\frac{u}{\varepsilon}+1}{\frac{u}{\varepsilon}} + \log(\frac{u}{\varepsilon}+1) \right\}$$

Substituting from above

$$\log\frac{\frac{u}{\varepsilon}+1}{\frac{u}{\varepsilon}} = \frac{\varepsilon}{kT}.$$

Also, from $u = \frac{\varepsilon}{e^{\frac{\varepsilon}{kT}}-1}$

$$\frac{u}{\varepsilon} = \frac{1}{e^{\frac{\varepsilon}{kT}}-1}$$

$$\frac{u}{\varepsilon}+1 = \frac{e^{\frac{\varepsilon}{kT}}}{e^{\frac{\varepsilon}{kT}}-1},$$

Therefore,

$$s = k\left\{ \frac{1}{e^{\frac{\varepsilon}{kT}}-1}\frac{\varepsilon}{kT} + \log\frac{e^{\frac{\varepsilon}{kT}}}{e^{\frac{\varepsilon}{kT}}-1} \right\}$$

$$= k\left\{ \frac{1}{e^{\frac{\varepsilon}{kT}}-1}\frac{\varepsilon}{kT} + \frac{\varepsilon}{kT} + \log\frac{1}{e^{\frac{\varepsilon}{kT}}-1} \right\}$$

$$= \frac{\varepsilon}{T}\left(\frac{1}{e^{\frac{\varepsilon}{kT}}-1} + 1 \right) + k\log\frac{1}{e^{\frac{\varepsilon}{kT}}-1}$$

The electromagnetic entropy radiation law gives the average entropy per mode in $[\omega, \omega + d\omega]$

$$\boxed{s_\omega(T) = \frac{\varepsilon_\omega}{T}\left(\frac{1}{e^{\frac{\varepsilon_\omega}{kT}} - 1} + 1\right) + k\log\frac{1}{e^{\frac{\varepsilon_\omega}{kT}} - 1}}$$

$$= \sigma_\omega\left(\frac{1}{e^{\frac{\sigma_\omega}{k}} - 1} + 1\right) + k\log\frac{1}{e^{\frac{\sigma_\omega}{k}} - 1}$$

There are $\dfrac{\omega^2}{\pi^2 c^3}$ modes in the frequency interval $[\omega, \omega + d\omega]$, and the average electromagnetic entropy in the interval is

$$\frac{\omega^2}{\pi^2 c^3} s_\omega(T).$$

Since the direction of the derivation can be reversed, we have

3.1 The average electromagnetic radiation energy per mode in $[\omega, \omega + dw]$ is

$$u_\omega(T) = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1},$$

$\Leftrightarrow$ the average electromagnetic entropy at fixed υ, per mode in $[\omega, \omega + dw]$ is

$$s_\omega(T) = \frac{\varepsilon_\omega}{T}\left(\frac{1}{e^{\frac{\varepsilon_\omega}{kT}} - 1} + 1\right) + k\log\frac{1}{e^{\frac{\varepsilon_\omega}{kT}} - 1}$$

In [Dan5], we proved that

The average electromagnetic radiation energy per mode in $[\omega, \omega + dw]$ is

$$u_\omega(T) = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1},$$

$\Leftrightarrow$ per mode in $[\omega, \omega + dw]$,

- The radiation energy consists of quanta of $\varepsilon_\omega = \hbar\omega$
- The Bose Statistics applies: then P quanta can be distributed among N radiators in $\dfrac{(P + N - 1)!}{P!(N-1)!}$ ways.

4.

Electromagnetic Entropy with Zero Point Energy

In 1912, Planck assumed Zero Point Energy in his Radiation Law of 1901, and proved that the average internal radiation energy per mode at frequency ω is

$$u_{\omega}(T) = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1} + \tfrac{1}{2}\hbar\omega\,.$$

There are

$$\frac{\omega^2}{\pi^2 c^3}$$

modes in the frequency interval

$$[\omega, \omega + d\omega],$$

and the average radiation energy in the interval is

$$\frac{\omega^2}{\pi^2 c^3} u_{\omega}(T)$$

For the photon gas, the entropy is

$$s(T, v) = \underbrace{\int \frac{1}{T} du}_{\text{depends on } T \text{ alone}} + \underbrace{\log v}_{\text{we'll keep } v \text{ fixed}}\,.$$

Since v, and T are independent variables, we may fix v, and seek only the integral that depends only on T, hence, on u.

Thus, we aim to write $\frac{1}{T}$ in terms of u:

Denoting $\varepsilon = \hbar\omega$, and $u_\omega(T) = u$,

$$u = \frac{\varepsilon}{e^{\frac{\varepsilon}{kT}} - 1} + \tfrac{1}{2}\varepsilon$$

$$= \tfrac{1}{2}\varepsilon\left(\frac{2}{e^{\frac{\varepsilon}{kT}} - 1} + \frac{e^{\frac{\varepsilon}{kT}} - 1}{e^{\frac{\varepsilon}{kT}} - 1}\right)$$

$$= \tfrac{1}{2}\varepsilon\frac{e^{\frac{\varepsilon}{kT}} + 1}{e^{\frac{\varepsilon}{kT}} - 1}$$

Solving for T,

$$2\frac{u}{\varepsilon} = \frac{e^{\frac{\varepsilon}{kT}} + 1}{e^{\frac{\varepsilon}{kT}} - 1},$$

$$2\frac{u}{\varepsilon}(e^{\frac{\varepsilon}{kT}} - 1) = e^{\frac{\varepsilon}{kT}} + 1,$$

$$e^{\frac{\varepsilon}{kT}}(2\frac{u}{\varepsilon} - 1) = 2\frac{u}{\varepsilon} + 1,$$

$$e^{\frac{\varepsilon}{kT}} = \frac{2\frac{u}{\varepsilon} + 1}{2\frac{u}{\varepsilon} - 1},$$

$$e^{\frac{\varepsilon}{kT}} = \frac{\frac{u}{\varepsilon} + \frac{1}{2}}{\frac{u}{\varepsilon} - \frac{1}{2}},$$

$$\frac{\varepsilon}{kT} = \log \frac{\frac{u}{\varepsilon} + \frac{1}{2}}{\frac{u}{\varepsilon} - \frac{1}{2}},$$

$$\frac{1}{T} = \frac{k}{\varepsilon} \log \frac{\frac{u}{\varepsilon} + \frac{1}{2}}{\frac{u}{\varepsilon} - \frac{1}{2}},$$

$$\frac{1}{T} = \frac{k}{\varepsilon}\left\{ \log(\frac{u}{\varepsilon} + \frac{1}{2}) - \log(\frac{u}{\varepsilon} - \frac{1}{2}) \right\}$$

Since $\dfrac{\partial s}{\partial u} = \dfrac{1}{T}$,

$$\frac{\partial s}{\partial u} = \frac{k}{\varepsilon}\left\{ \log(\frac{u}{\varepsilon} + \frac{1}{2}) - \log(\frac{u}{\varepsilon} - \frac{1}{2}) \right\}$$

Integrating with respect to u, the average entropy per mode in $[\omega, \omega + d\omega]$ is

$$s = k\left\{ (\frac{u}{\varepsilon} + \frac{1}{2})\log(\frac{u}{\varepsilon} + \frac{1}{2}) - (\frac{u}{\varepsilon} - \frac{1}{2})\log(\frac{u}{\varepsilon} - \frac{1}{2}) \right\}$$

$$= k\left\{ (\frac{u}{\varepsilon} - \frac{1}{2})\log(\frac{u}{\varepsilon} + \frac{1}{2}) - (\frac{u}{\varepsilon} - \frac{1}{2})\log(\frac{u}{\varepsilon} - \frac{1}{2}) + \log(\frac{u}{\varepsilon} + \frac{1}{2}) \right\}$$

$$= k\left\{ (\frac{u}{\varepsilon} - \frac{1}{2})\log \frac{\frac{u}{\varepsilon} + \frac{1}{2}}{\frac{u}{\varepsilon} - \frac{1}{2}} + \log(\frac{u}{\varepsilon} + \frac{1}{2}) \right\}$$

Substituting from above

$$\log \frac{\frac{u}{\varepsilon} + \frac{1}{2}}{\frac{u}{\varepsilon} - \frac{1}{2}} = \frac{\varepsilon}{kT}.$$

Also, from $u = \dfrac{\varepsilon}{e^{\frac{\varepsilon}{kT}} - 1} + \frac{1}{2}\varepsilon$

$$\frac{u}{\varepsilon} + \frac{1}{2} = \frac{1}{e^{\frac{\varepsilon}{kT}} - 1} + 1,$$

$$= \frac{e^{\frac{\varepsilon}{kT}}}{e^{\frac{\varepsilon}{kT}} - 1},$$

and

$$\frac{u}{\varepsilon} - \frac{1}{2} = \frac{1}{e^{\frac{\varepsilon}{kT}} - 1}$$

Therefore,

$$s = k\left\{\frac{1}{e^{\frac{\varepsilon}{kT}} - 1}\frac{\varepsilon}{kT} + \log\frac{e^{\frac{\varepsilon}{kT}}}{e^{\frac{\varepsilon}{kT}} - 1}\right\}$$

$$= k\left\{\frac{1}{e^{\frac{\varepsilon}{kT}} - 1}\frac{\varepsilon}{kT} + \frac{\varepsilon}{kT} + \log\frac{1}{e^{\frac{\varepsilon}{kT}} - 1}\right\}$$

$$= \frac{\varepsilon}{T}\left(\frac{1}{e^{\frac{\varepsilon}{kT}} - 1} + 1\right) + k\log\frac{1}{e^{\frac{\varepsilon}{kT}} - 1}$$

The electromagnetic entropy radiation law gives the average entropy per mode in $[\omega, \omega + d\omega]$

$$\boxed{s_\omega(T) = \frac{\varepsilon_\omega}{T}\left(\frac{1}{e^{\frac{\varepsilon_\omega}{kT}} - 1} + 1\right) + k\log\frac{1}{e^{\frac{\varepsilon_\omega}{kT}} - 1}}$$

$$= \sigma_\omega \left(\frac{1}{e^{\frac{\sigma_\omega}{k}} - 1} + 1 \right) + k \log \frac{1}{e^{\frac{\sigma_\omega}{k}} - 1}$$

Thus, the assumption of Zero Point Energy does not affect the radiation entropy.

The entropy of radiation that includes Zero Point Energy is the same as the entropy of radiation that avoids Zero Point Energy.

There are $\frac{\omega^2}{\pi^2 c^3}$ modes in the frequency interval $[\omega, \omega + d\omega]$, and the average electromagnetic entropy in the interval is

$$\frac{\omega^2}{\pi^2 c^3} s_\omega(T).$$

Since the direction of the derivation can be reversed, we have

4.1 the average electromagnetic radiation energy per mode

in $[\omega, \omega + dw]$ is

$$u_\omega(T) = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1} + \tfrac{1}{2}\hbar\omega$$

$\Leftrightarrow$ the average electromagnetic entropy at fixed υ, per mode in $[\omega, \omega + dw]$ is

$$s_\omega(T) = \frac{\varepsilon_\omega}{T} \left(\frac{1}{e^{\frac{\varepsilon_\omega}{kT}} - 1} + 1 \right) + k \log \frac{1}{e^{\frac{\varepsilon_\omega}{kT}} - 1}$$

In [Dan5], we proved that

the average electromagnetic radiation energy per mode in $[\omega, \omega + dw]$ is

$$u_{\omega}(T) = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1} + \tfrac{1}{2}\hbar\omega$$

$\Leftrightarrow$ per mode in$[\omega, \omega + dw]$,

- Each oscillator has Zero Point Energy $\frac{1}{2}\hbar\omega$
- The radiation energy consists of quanta of $\varepsilon_{\omega} = \hbar\omega$
- The Bose Statistics applies. then P quanta can be distributed among N radiators in $\dfrac{(P + N - 1)!}{P!(N - 1)!}$ ways.

5.

Electromagnetic Radiation Zero Point Entropy is Zero, Satisfying the 3rd Law of Thermodynamics

5.1 The Self-Evident Third Law of Thermodynamics

The First Law of Thermodynamics expresses our experience that energy is always conserved. It is a most believable fact that need not be axiomatized.

The Second Law postulates that a thermodynamic system has entropy, a heat function that unlike heat energy does not depend on the process. The second law does not create entropy. Being able to check if entropy exists is far superior to postulating its existence.

The Third Law says that at zero absolute Temperature, entropy vanishes. It is self-evident that at zero absolute Temperature, heat vanishes. It is just as self-evident that if entropy exists, it vanishes whenever heat vanishes.

5.2 The Third Law for Electromagnetic Entropy

For $T \approx 0$,

$$e^{\frac{\varepsilon_\omega}{kT}} - 1 \approx e^{\frac{\varepsilon_\omega}{kT}}$$

$$s_\omega(T) = \frac{\varepsilon_\omega}{T}\left(\frac{1}{e^{\frac{\varepsilon_\omega}{kT}} - 1} + 1\right) + k\log\frac{1}{e^{\frac{\varepsilon_\omega}{kT}} - 1}$$

$$\approx \frac{\varepsilon_\omega}{T}\left(\frac{1}{e^{\frac{\varepsilon_\omega}{kT}}} + 1\right) + k\log\frac{1}{e^{\frac{\varepsilon_\omega}{kT}}}$$

$$= \frac{\frac{\varepsilon_\omega}{T}}{e^{\frac{\varepsilon_\omega}{kT}}} + \frac{\varepsilon_\omega}{T} - \underbrace{k\log e^{\frac{\varepsilon_\omega}{kT}}}_{\frac{\varepsilon_\omega}{T}}$$

$$= \frac{\frac{\varepsilon_\omega}{T}}{e^{\frac{\varepsilon_\omega}{kT}}} \approx \frac{\infty}{\infty}$$

Applying L'hospital

$$\frac{\frac{d}{dT}\left(\frac{\varepsilon_\omega}{T}\right)}{\frac{d}{dT}(e^{\frac{\varepsilon_\omega}{kT}})} = \frac{-\frac{\varepsilon_\omega}{T^2}}{e^{\frac{\varepsilon_\omega}{kT}}\left(-\frac{\varepsilon_\omega}{kT^2}\right)}$$

$$= \frac{k}{e^{\frac{\varepsilon_\omega}{kT}}}$$

$$\approx \frac{k}{\infty} = 0$$

Thus, Zero Point Entropy of Electromagnetic Radiation is zero, and the Third Law of Thermodynamics holds for the system of Photon Gas.

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