Zero Point Energy, and the Charge-Radiation Equation in Bohr's Atom

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Abstract Bohr's postulate of Quantized Angular Momentum is equivalent to the assumption of Zero Point Energy for the Hydrogen Atom.

The confirmation of Bohr's Atom in spectroscopy, validates the existence of Zero Point Energy for the Hydrogen Atom

For an electron that orbits a proton at the first Bohr orbit r_B, at frequency ν_B, the following are equivalent

1) The Angular Momentum is quantized, $m_e v_B r_B = \hbar$.

2) The Fine-Structure constant Formula, $\alpha = \dfrac{e^2}{4\pi\varepsilon_0 c\hbar}$.

3) The Charge-Radiation Equation, $\dfrac{e^2}{4\pi\varepsilon_0 r_B} = h\nu_B$.

4) Ground State Energy is Zero Point Energy $\dfrac{-e^2}{8\pi\varepsilon_0 r_B} = -\frac{1}{2}h\nu_B$,

$$\nu_B = 6.576928348 \times 10^{15}\,\text{cycles/second}$$

The Electron's Magnetic Energy in the Bohr Orbit is

$$U_{magnetic} = \tfrac{1}{4}\mu_0 r_B e^2 \nu_B^2 = \tfrac{1}{(4\pi)^2}\mu_0 v_B^2 e^2 \tfrac{1}{r_B}$$

It is negligible compared to its Electric Energy.

For an electron in the n[th] Bohr orbit r_n, at frequency ν_n,

the following are equivalent

1) The <u>Angular Momentum is quantized</u>, $m_e v_n r_n = n\hbar$.

2) The <u>Charge-Radiation Equation</u>, $\dfrac{e^2}{4\pi\varepsilon_0 r_n} = nh\nu_n$.

3) <u>Orbit Energy is of n photons</u>, $h\nu_n$ $\dfrac{-e^2}{8\pi\varepsilon_0 r_n} = -\tfrac{1}{2}\underbrace{nh\nu_n}_{\frac{1}{n^2}h\nu_B}$.

Keywords: Zero Point Energy, Bohr's Atom, Quantized Angular Momentum, Fine Structure Constant, Electron, Photon, Proton, Binding Electric Energy, Radiation Energy, Charge-Radiation Equation

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Introduction

In 1901, Planck showed that the radiation-energy density per unit volume at frequencies between ν, and $\nu + \delta\nu$ of an ideal radiator (black body) is

$$u(\nu, T) = \frac{8\pi}{c^3}\nu^2 \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}.$$

The assumption of discrete radiation energy, conflicted with Planck's belief in radiation of continuous waves, and he kept searching for a more believable law

To reconcile his quantum hypothesis with his conception of wave radiation, he avoided the conclusion that radiation energy must be made of particles, and postulated that radiation is a transition between the energy levels of an oscillator. Furthermore, ignoring the symmetry between emission and absorption, he maintained that the absorption of radiation energy is continuous.

Under these assumptions, Planck derived in 1912 his second radiation law in which zero point energy in the amount of $\frac{1}{2}h\nu$ is added to $\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$, of his 1901 radiation law.

In [Dan], we showed that Planck's derivation of his 1912 radiation law only recovers the Zero Point Energy that he unknowingly assumed in his model for that derivation.

In particular, we showed that Planck's ZPE radiation law is equivalent to the combined three assumptions of

1) Zero Point Energy Hypothesis,
2) the Quantum Law, and
3) the approximated Boson Statistics distribution law.

Planck's 1901 radiation law resolves the Black body radiation problem, and confirms the boson statistics distribution law.

The Quantum law holds independently in the photoelectric effect, in Compton's scattering, and in spectroscopy.

But Planck's Zero Point Energy remains a hypothesis. Thus, the validity of Planck's 1912 radiation law, and the existence of Planck's Zero Point Energy are doubtful.

However, we will show here that Bohr's postulate of Quantized Angular Momentum is equivalent to the assumption of Zero Point Energy for the Hydrogen Atom.

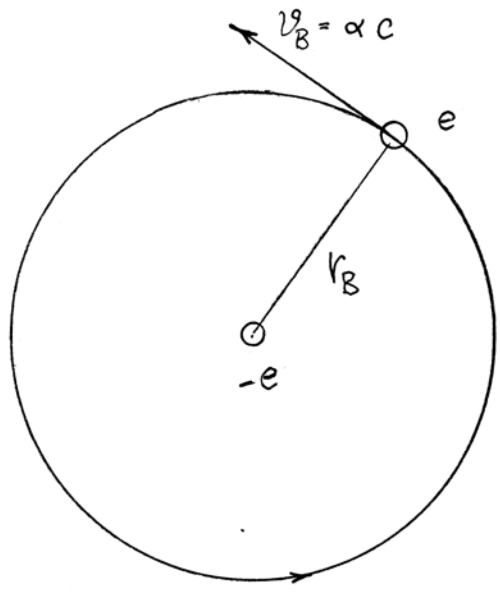
The confirmation of Bohr's Atom in spectroscopy, validates the existence of Zero Point Energy for the Hydrogen Atom.

0.1 Bohr's Quantized Angular Momentum

Consider an electron with mass m_e, and charge e, in the first Bohr orbit of radius $r_1 = r_B$, at frequency $\nu_1 = \nu_B$, and speed

$$v_1 = v_B = \omega_B r_B = 2\pi\nu_B r_B,$$

encircling a proton with charge $-e$,



An electron with charge e, orbiting at frequency ν_1, and speed $v_1 = \alpha c = \omega_1 r_1 = 2\pi\nu_1 r_1$, a proton with charge $-e$ at distance r_1.

Bohr postulated that the angular momentum is quantized:

For the electron's first, Bohr's orbit, $n = 1$, and

$$m_e v_1 r_1 = \hbar = \tfrac{h}{2\pi}.$$

For the electron's 2^{nd} orbit, $n = 2$, and

$$m_e v_2 r_2 = 2\hbar = 2\tfrac{h}{2\pi},$$

For the 3^{rd} electron orbit, $n = 3$, and

$$m_e v_3 r_3 = 3\hbar = 3\tfrac{h}{2\pi},$$

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Angular Momentum is the vector product

$$m\vec{v} \times \vec{r} = m(\vec{\omega} \times \vec{r}) \times \vec{r}\,,$$

and the quantization of that quantity cannot be visualized like the quantization of a scalar quantity, such as energy.

We aim to show that Quantized Angular Momentum is equivalent to the assumption of Zero Point Energy for Bohr's Atom.

We'll first show that Quantized Angular Momentum is equivalent to the Fine-Structure constant.

0.2 The Fine-Structure Constant

Sommerfeld suggested that

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 c\hbar}\,,$$

determines the spacing between the fine structure of spectral lines.

α, known as the Fine Structure Constant, is a pure number that is approximately $\frac{1}{137}$ in any unit system.

The fine-structure constant depends on fundamental constants,

- e, the electron charge, which was thought to be the smallest electric charge,
- c, light speed in the vacuum, which is thought to be the greatest speed,

- $h = 2\pi\hbar$, Planck Constant of radiation energy,
- $\varepsilon_0 = \frac{1}{\mu_0 c^2}$, the permittivity of the vacuum.

Consequently, it was suspected to hide some truth of nature. Eddington, Pauli, and their contemporaries [Miller] searched for a meaning for the appearance of these fundamental constant in one formula.

We show that the Formula for the Fine Structure Constant can be written in a form that equates the electron-proton binding electric energy to the radiation energy of a photon.

We call that equivalent form the Charge-Radiation equation.

0.3 The Charge-Radiation Equation

The binding electric energy $\dfrac{e^2}{4\pi\varepsilon_0 r_B}$ *of the electron in the first Bohr orbit* r_B *with frequency* ν_B, *equals the radiation energy* $h\nu_B$ *of a photon of frequency* ν_B.

The binding electric energy of the Hydrogen Atom in the ground state is precisely the radiation energy of a photon with frequency that equals the frequency of the electron motion.

The Charge-Radiation equation is equivalent to the equality of the electron's energy in the ground state to $\frac{1}{2}h\nu_B$ of Zero Point radiation Energy.

0.4 The Zero Point Energy of Bohr's Atom

The Zero Point Energy of Bohr's Atom is the Ground state energy. Its value, $\frac{1}{2}h\nu_B$ is the lowest energy of the Bohr Atom.

The loss of that Zero Point Energy, would mean the destruction of the Atom.

Therefore, at absolute zero temperature, when all thermal motions cease, the atom still retains that amount of energy.

The existence of Zero Point Energy has been always difficult to confirm in experiments. But the confirmation of Bohr's quantization of Angular Momentum in spectroscopy, validates Zero Point Energy in the amount of $\frac{1}{2}h\nu_B$, for the Bohr Atom.

1.

Quantized Angular Momentum and the Fine Structure Constant

1.1 $$mv_B r_B = \hbar \quad \Leftrightarrow \quad \alpha = \frac{e^2}{4\pi\varepsilon_0 c\hbar}$$

Proof:

Ignoring the magnetic Field of the proton on the electron's charge, the centripetal repulsion is balanced by the electric attraction

$$m_e \frac{v_B^2}{r_B} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_B^2}.$$

$$\underbrace{m_e v_B r_B}_{\hbar} v_B = \frac{1}{4\pi\varepsilon_0} e^2,$$

$$v_B = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar},$$

$$\underbrace{\frac{v_B}{c}}_{\alpha} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}.$$

2.

The Fine-Structure Constant and the Charge-Radiation Equation

2.1 $\alpha = \frac{e^2}{4\pi\varepsilon_0 c\hbar} \quad \Leftrightarrow \quad \frac{e^2}{4\pi\varepsilon_0 r_B} = h\nu_B$

<u>*Proof*</u>:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 c\hbar},$$

Hence,

$$\hbar = \frac{e^2}{4\pi\varepsilon_0 \alpha c}$$

$$= \frac{e^2}{4\pi\varepsilon_0 v_B}$$

$$= \frac{e^2}{4\pi\varepsilon_0 \omega_B r_B}.$$

Therefore,

$$\hbar\omega_B = \frac{e^2}{4\pi\varepsilon_0 r_B}.$$

3.

The Charge-Radiation Equation and Zero Point Energy

3.1

$$\frac{e^2}{4\pi\varepsilon_0 r_B} = h\nu_B \quad \Leftrightarrow \quad U_{1st\ \text{Orbit}} = \underbrace{\underbrace{\tfrac{1}{2}m\omega_B^2 r_B^2}_{U_{\text{rotation}}} + \underbrace{\frac{-e^2}{4\pi\varepsilon_0 r_B}}_{U_{\text{electric}}}}_{\frac{-e^2}{8\pi\varepsilon_0 r_B}} = -\tfrac{1}{2}h\nu_B$$

<u>Proof</u>:

Ignoring the magnetic forces,

$$m_{\text{e}}\frac{v_B^2}{r_B} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_B^2},$$

$$\tfrac{1}{2}m_{\text{e}}\underbrace{v_B^2}_{\omega_B^2 r_B^2} = \tfrac{1}{2}\underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_B}}_{h\nu_B},$$

The electron's energy at the ground orbit is the sum of the electron's kinetic energy of rotation,

$$U_{\text{rotation}} = \tfrac{1}{2}m\omega_B^2 r_B^2 = \tfrac{1}{2}mv_B^2,$$

and the electron's binding electric energy

$$U_{\text{electric}} = \frac{-e^2}{4\pi\varepsilon_0 r_B},$$

$$\tfrac{1}{2}\underbrace{m\omega_B^2 r_B^2}_{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_B}} + \frac{-e^2}{4\pi\varepsilon_0 r_B} = \frac{-e^2}{8\pi\varepsilon_0 r_B} = -\tfrac{1}{2}h\nu_B.$$

In the Bohr orbit, the electron's energy is of photon with energy

$$-\tfrac{1}{2}h\nu_B.$$

That is, the Ground state Energy of the Bohr Atom is Zero Point Energy of $\tfrac{1}{2}h\nu_B$.

3.2 Bohr's Atom Zero Point Energy

$$\boxed{E_B = -\tfrac{1}{2}h\nu_B = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_B} = -13.6\text{eV}}$$

3.3 $$\boxed{\nu_B = 2\frac{13.6\text{eV}}{h} = 6.576928348 \times 10^{15}\text{cycles/second}}$$

Computation: $h = 4.1356692 \times 10^{-15}\text{eV}$, [Woan].

4.

The Electron's magnetic Energy

4.1 Electron's Total Quantized Energy

Bohr's quantization of the Angular Momentum ignores the electron's Orbital Magnetic Energy, and the electron's Spin Magnetic Energy.

Including orbital magnetic forces, in the force balance, the electron's total quantized energy in the Bohr orbit is

$$-\frac{e^2}{8\pi\varepsilon_0 r_B} + U_{\text{magnetic}} = h\nu_B.$$

4.2 The Electron's Magnetic Energy

$$U_{magnetic} = \tfrac{1}{4}\mu_0 r_B e^2 \nu_B^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 v_B^2 e^2 \frac{1}{r_B}$$

Proof: The current due to the electron's charge e that turns ν_B cycles/second is

$$I = e\nu_B$$

The Magnetic Energy of this current is [Benson, p.486]

$$\tfrac{1}{2}LI^2.$$

By [Fischer, p.97]

$$L = \mu_0 \frac{\pi {r_B}^2}{2\pi r_B} = \tfrac{1}{2}\mu_0 r_B.$$

Thus, the magnetic energy due to the electron charge is

$$\begin{aligned} \tfrac{1}{2}\tfrac{1}{2}\mu_0 r_B (e\nu_B)^2 &= \tfrac{1}{4}\mu_0 r_B e^2 {\nu_B}^2 \\ &= \tfrac{1}{4}\mu_0 e^2 r_B \underbrace{\nu_B^2}_{\frac{1}{4\pi^2}\omega_B^2} \\ &= \frac{1}{(4\pi)^2}\mu_0 v_B^2 e^2 \frac{1}{r_B}. \end{aligned}$$

4.3 The Electron's Magnetic Energy in the Bohr's Orbit is negligible compared to its Electric Energy

Proof:

$$\frac{U_{\text{magnetic}}}{U_{\text{electric}}} = \frac{\frac{1}{(4\pi)^2}\mu_0 v_B^2 e^2 \frac{1}{r_B}}{\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_B}} = \frac{1}{2\pi}\frac{v_B^2}{c^2} = \frac{1}{2\pi}\alpha^2 \approx \tfrac{1}{2\pi(137)^2} \approx 8.5\times 10^{-6}$$

5.

The n^th Orbit Charge-Radiation

In the n^th orbit,

the Angular momentum is $m_e v_n r_n = n\hbar$,

The orbit energy is $E_n = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_n}$

5.1 The Electron's Energy

$$\tfrac{1}{2}m_e\omega_n^2 r_n^2 + \frac{-e^2}{4\pi\varepsilon_0 r_n} = \frac{-e^2}{8\pi\varepsilon_0 r_n} = -\tfrac{1}{2}m_e\omega_n^2 r_n^2 = -\tfrac{1}{2}\frac{\hbar\alpha c}{r_n}$$

Proof: $\tfrac{1}{2}\underbrace{m_e\omega_n^2 r_n^2}_{\frac{e^2}{4\pi\varepsilon_0 r_n}} + \frac{-e^2}{4\pi\varepsilon_0 r_n} = \frac{-e^2}{8\pi\varepsilon_0 r_n} = \frac{-e^2}{\underbrace{8\pi\varepsilon_0 r_B}_{-\frac{1}{2}\hbar\omega_B}}\frac{r_B}{r_n} = -\tfrac{1}{2}\hbar\frac{v_B}{r_n} = -\tfrac{1}{2}\frac{\hbar\alpha c}{r_n}$

5.2

$$v_n = \frac{v_B}{n} = \frac{\alpha}{n}c$$

Proof: $v_n = \frac{(m_e\omega_n^2 r_n^2)r_n}{m_e v_n r_n} = \frac{\hbar\alpha c}{n\hbar} = \frac{\alpha}{n}c$

5.3

$$r_n = n^2 r_B$$

Proof: $r_n = \dfrac{n\hbar}{m_e v_n} = \dfrac{n\hbar}{m_e \frac{1}{n} v_B} = n^2 \dfrac{\hbar}{m_e v_B} = n^2 r_B$

5.4 $$\nu_n = \tfrac{1}{n^3}\nu_B$$

Proof: $\nu_n = \tfrac{1}{2\pi}\omega_n = \tfrac{1}{2\pi}\dfrac{v_n}{r_n} = \tfrac{1}{2\pi}\dfrac{\frac{1}{n}v_B}{n^2 r_B} = \dfrac{1}{n^3}\tfrac{1}{2\pi}\omega_B = \dfrac{1}{n^3}\nu_B$

5.5 The Orbit Energy

$$E_n = \frac{1}{n^2}E_B = -\frac{1}{n^2}(\tfrac{1}{2}h\nu_B)$$

Proof: $E_n = -\dfrac{1}{8\pi\varepsilon_0}\dfrac{e^2}{r_n} = -\underbrace{\dfrac{1}{8\pi\varepsilon_0}\dfrac{e^2}{r_B}}_{\frac{1}{2}h\nu_B}\underbrace{\dfrac{r_B}{r_n}}_{\frac{1}{n^2}}$

5.6 The Charge-Radiation Equation

The binding electric energy $\dfrac{e^2}{4\pi\varepsilon_0 r_n}$ *of the electron-proton Atom, where the electron is in the* n^{th} *Bohr orbit* r_n *with frequency* ν_n*, equals the radiation energy* $nh\nu_n$ *of* n *photons of frequency* ν_n.

$$\boxed{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = nh\nu_n}$$

Proof: $$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = \frac{\hbar v_B}{r_n} = \hbar \underbrace{\omega_B}_{n^3\omega_n}\frac{r_B}{n^2 r_B} = nh\nu_n$$

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