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## Weak Mixing

[The Weak Mixing Matrix](#)

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**Abstract:** In 1963 Cabbibo [Cabb] proposed that flavor changing of quarks in the weak interaction, follows the rule

$$u \rightarrow d \cos \theta_C + s \sin \theta_C \equiv d'$$

$\theta_C$ , the Cabbibo angle, was found in experiment to be approximately 0.23 radians. Thus, the interaction produces mostly the  $d$  quark.

The GIM team, later predicted the  $c$  quark, and proposed a similar rule for its flavor changing

$$c \rightarrow -d \sin \theta_C + s \cos \theta_C \equiv s'.$$

The two rules are combined in the matrix equation

$$\begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} d' \\ s' \end{pmatrix}$$

Namely, the vector  $\begin{pmatrix} d \\ s \end{pmatrix}$  is rotated into the vector  $\begin{pmatrix} d' \\ s' \end{pmatrix}$  by the

transition rates matrix

$$\begin{aligned}
\begin{pmatrix} U_{ud} & U_{us} \\ U_{cd} & U_{cs} \end{pmatrix} &= \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \\
&\approx \begin{pmatrix} 1 - \frac{1}{2}\theta_C^2 & \theta_C \\ -\theta_C & 1 - \frac{1}{2}\theta_C^2 \end{pmatrix} \\
&\approx \begin{pmatrix} 1 - \frac{1}{2}(0.23)^2 & 0.23 \\ -0.23 & 1 - \frac{1}{2}(0.23)^2 \end{pmatrix}
\end{aligned}$$

In 1973, Kobayashi and Maskawa [K-M] predicted two more quarks  $b$ , and  $t$ . Then, the rules for flavor changing are

$$u \rightarrow U_{ud}d + U_{us}s + U_{ub}b \equiv d',$$

$$c \rightarrow U_{cd}d + U_{cs}s + U_{cb}b \equiv s',$$

$$t \rightarrow U_{td}d + U_{ts}s + U_{tb}b \equiv b'.$$

The three rules are combined in the matrix equation

$$\begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

Namely, the vector  $\begin{pmatrix} d \\ s \\ b \end{pmatrix}$  is rotated into the vector  $\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$  by the Weak

Mixing Matrix

$$\begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}.$$

We show that If the Weak Mixing Matrix,  $\begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}$ , is equal

to the product of rotations, we cannot tell in which order the non-commuting rotations are written. In particular, the Weak Mixing Matrix may not be equal to either the KM, or the PDG Matrices.

At most we may find an approximating matrix for the Weak Mixing Matrix that is based on the rotation matrices.

We show that one such approximating matrix for the Real part of the Weak Mixing Matrix is

$$\begin{pmatrix} \cos \theta_C \cos \theta_C^3 & \sin \theta_C \cos \theta_C^3 & \sin^3 \theta_C \cos \theta_C^2 \\ -\sin \theta_C \cos \theta_C^2 & \cos \theta_C \cos \theta_C^2 & \sin^2 \theta_C \\ -\cos \theta_C \sin^3 \theta_C & -\cos \theta_C \cos \theta_C^3 \sin^2 \theta_C & \cos \theta_C^2 \cos \theta_C^3 \end{pmatrix},$$

where  $\theta_C$  is the Cabbibo angle.

The approximating matrix depends on  $\theta_C$  alone, and predicts the Real part of the Weak Mixing Matrix to a high degree of accuracy.