# The Weak Mixing Matrix

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**Abstract:** We show that the Weak Mixing Matrix, $\begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}$,

is not equal to the product of rotations, and in particular, it is not equal to the KM, or the PDG Matrices.

At most we may find an approximating matrix for the Weak Mixing Matrix that is based on the rotation matrices.

We show that one such approximating matrix for the Real part of the Weak Mixing Matrix is

$$\begin{pmatrix} \cos\theta_C \cos\theta_C^3 & \sin\theta_C \cos\theta_C^3 & \sin^3\theta_C \cos\theta_C^2 \\ -\sin\theta_C \cos\theta_C^2 & \cos\theta_C \cos\theta_C^2 & \sin^2\theta_C \\ -\cos\theta_C \sin^3\theta_C & -\cos\theta_C \cos\theta_C^3 \sin^2\theta_C & \cos\theta_C^2 \cos\theta_C^3 \end{pmatrix},$$

where $\theta_C$ is the Cabbibo angle.

The approximating matrix depends on $\theta_C$ alone, and predicts the Real part of the Weak Mixing Matrix to a high degree of accuracy.

We establish, with a Chi-Squared Goodness-of-Fitness-Test, that our approximating matrix can be used with extremely high level of statistical confidence.

**Keywords**: CKM Matrix. Wolfenstein approximation for the CKM matrix. Weak Mixing Matrix.

**PACS**:12.15Hh;

## Introduction.

In 1963 Cabbibo [Cabb] proposed that flavor changing of quarks in the weak interaction, follows the rule

$$u \to d\cos\theta_C + s\sin\theta_C \equiv d'$$

$\theta_C$, the Cabbibo angle, was found in experiment to be approximately *0.23* radians. Thus, the interaction produces mostly the $d$ quark.

The GIM team, later predicted the $c$ quark, and proposed a similar rule for its flavor changing

$$c \to -d\sin\theta_C + s\cos\theta_C \equiv s'.$$

The two rules are combined in the matrix equation

$$\begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} d' \\ s' \end{pmatrix}$$

Namely, the vector $\begin{pmatrix} d \\ s \end{pmatrix}$ is rotated into the vector $\begin{pmatrix} d' \\ s' \end{pmatrix}$ by the transition rates matrix

$$\begin{pmatrix} U_{ud} & U_{us} \\ U_{cd} & U_{cs} \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

$$\approx \begin{pmatrix} 1-\frac{1}{2}\theta_C^2 & \theta_C \\ -\theta_C & 1-\frac{1}{2}\theta_C^2 \end{pmatrix}$$

$$\approx \begin{pmatrix} 1-\frac{1}{2}(0.23)^2 & 0.23 \\ -0.23 & 1-\frac{1}{2}(0.23)^2 \end{pmatrix}$$

In 1973, Kobayashi and Maskawa [K-M] predicted two more quarks $b$, and $t$. Then, the rules for flavor changing are

$$u \to U_{ud}d + U_{us}s + U_{ub}b \equiv d',$$

$$c \to U_{cd}d + U_{cs}s + U_{cb}b \equiv s',$$

$$t \to U_{td}d + U_{ts}s + U_{tb}b \equiv b'.$$

The three rules are combined in the matrix equation

$$\begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

Namely, the vector $\begin{pmatrix} d \\ s \\ b \end{pmatrix}$ is rotated into the vector $\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$ by the Weak Mixing Matrix

$$\begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}.$$

## 1. Weak Mixing Matrix decomposition into rotations

Kobayashi and Maskawa proposed that the Weak Mixing Matrix is the product of four rotation matrices, based on four mixing angles, $\theta_1, \theta_2, \theta_3,$ and $\delta$ [K-M]:

$$\begin{pmatrix} c_1 & s_1 & \\ -s_1 & c_1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & c_2 & s_2 \\ & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} 1 & & \\ & c_3 & s_3 \\ & -s_3 & c_3 \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{pmatrix},$$

where $c_i = \cos\theta_i$, and $s_i = \sin\theta_i$.

This matrix became known as the CKM Matrix, but Cabbibo had no part in it, and we shall refer to it as the KM Matrix

Multiplying the four matrices we have,

$$\begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 \cos\delta & c_1 c_2 s_3 + s_2 c_3 \cos\delta \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 \cos\delta & -c_1 s_2 s_3 + c_2 c_3 \cos\delta \end{pmatrix} + i\sin\delta \begin{pmatrix} 0 & 0 & 0 \\ 0 & -s_2 s_3 & s_2 c_3 \\ 0 & -s_3 c_2 & c_2 c_3 \end{pmatrix}$$

As first simplification, we take $\delta \approx 0$, and the KM matrix is

$$\begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 & c_1 c_2 s_3 + s_2 c_3 \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 & -c_1 s_2 s_3 + c_2 c_3 \end{pmatrix}$$

The KM Matrix has a flaw that lasted long enough to appear in many textbooks, (for instance, [Fay, p. 473], [Dono, p. 64], and [Grein, p. 216]) but eventually was observed [e.g. Seid]

Note that in the product

$$\begin{pmatrix} c_1 & s_1 & \\ -s_1 & c_1 & \\ & & 1 \end{pmatrix}\begin{pmatrix} 1 & & \\ & c_2 & s_2 \\ & -s_2 & c_2 \end{pmatrix}\begin{pmatrix} 1 & & \\ & c_3 & s_3 \\ & -s_3 & c_3 \end{pmatrix}\begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{pmatrix},$$

the first matrix rotates the vector $\begin{pmatrix} d \\ s \\ b \end{pmatrix}$ about the vector $\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix}$,

the second matrix rotates the vector $\begin{pmatrix} d \\ s \\ b \end{pmatrix}$ about the vector $\begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix}$,

and the third matrix, erroneously, repeats the action of the second matrix.

Instead, the third matrix should rotate the vector $\begin{pmatrix} d \\ s \\ b \end{pmatrix}$ about the vector $\begin{pmatrix} 0 \\ s \\ 0 \end{pmatrix}$. To that end, the third matrix ought to be

$$\begin{pmatrix} c_3 & & s_3 \\ & 1 & \\ -s_3 & & c_3 \end{pmatrix}.$$

Using this rotation, the Particle Data Group [PDG, p.169] claims that the KM Matrix is given by

$$\begin{pmatrix} c_1c_3 & s_1c_3 & s_3 \\ -s_1c_2 - c_1s_2s_3 & c_1c_2 - s_1s_2s_3 & s_2c_3 \\ s_1s_2 - c_1c_2s_3 & -c_1s_2 - s_1c_2s_3 & c_2c_3 \end{pmatrix}$$

Obviously, the PDG Matrix, and the KM Matrix are two different matrices, because two matrices are equal if and only if all of their entries are equal.

In addition, the PDG changed the order of the product, and plugged the corrected rotation between the first, and the second rotations. That is, the PDG Matrix is the product of the rotations

$$\begin{pmatrix} 1 & & \\ & c_2 & s_2 \\ & -s_2 & c_2 \end{pmatrix}\begin{pmatrix} c_3 & & s_3 \\ & 1 & \\ -s_3 & & c_3 \end{pmatrix}\begin{pmatrix} c_1 & s_1 & \\ -s_1 & c_1 & \\ & & 1 \end{pmatrix}$$

The PDG might have expected the rotations to commute, but in fact, they do not commute.

For instance, the product

$$\begin{pmatrix} c_3 & & s_3 \\ & 1 & \\ -s_3 & & c_3 \end{pmatrix}\begin{pmatrix} 1 & & \\ & c_2 & s_2 \\ & -s_2 & c_2 \end{pmatrix}\begin{pmatrix} c_1 & s_1 & \\ -s_1 & c_1 & \\ & & 1 \end{pmatrix}$$

is the matrix

$$\begin{pmatrix} c_1c_3 + s_1s_2s_3 & s_1c_3 - c_1s_2s_3 & s_3c_2 \\ -s_1c_2 & c_1c_2 & s_2 \\ -c_1s_3 + s_1s_2s_3 & -c_1c_3s_2 - s_1s_3 & c_2c_3 \end{pmatrix}$$

while the product

$$\begin{pmatrix} c_1 & s_1 & \\ -s_1 & c_1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & c_2 & s_2 \\ & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_3 & & s_3 \\ & 1 & \\ -s_3 & & c_3 \end{pmatrix}$$

is the matrix

$$\begin{pmatrix} c_1c_3 - s_1s_2s_3 & s_1c_2 & s_3c_1 + s_1s_2c_3 \\ -s_1c_3 - c_1s_2s_3 & c_1c_2 & -s_1s_3 + c_1s_2c_3 \\ -c_2s_3 & -s_2 & c_2c_3 \end{pmatrix}.$$

We cannot tell which of these different matrices is the Weak Mixing Matrix $\begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}$.

Consequently, *the decomposition of the Weak Mixing Matrix into simple rotations cannot be realized credibly, even after we replace Kobayashi and Maskawa's erroneous rotation.*

Clearly, *The Weak Mixing Matrix is not equal to the product of rotations, and in particular, it is not equal to the KM or the PDG Matrices.*

At most, we may find an *approximation* for the Weak Mixing Matrix that is based on the rotation matrices.

We intend our approximation to supply experimenters with prediction of the values of the Weak Mixing Matrix .

## 2. Approximating the Weak Mixing Matrix

It is well-known [PDG, p.169] that

$$\sin\theta_3 = s_3 \ll \sin\theta_2 = s_2 \ll \sin\theta_C = s_1 .$$

Therefore, for the Real part of the KM Matrix

$$\begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3 & c_1c_2s_3 + s_2c_3 \\ s_1s_2 & -c_1s_2c_3 - c_2s_3 & -c_1s_2s_3 + c_2c_3 \end{pmatrix} \approx \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 & s_2c_3 \\ s_1s_2 & -c_1c_3s_2 & c_2c_3 \end{pmatrix}.$$

For the Real part of the PDG Matrix

$$\begin{pmatrix} c_1c_3 & s_1c_3 & s_3 \\ -s_1c_2 - c_1s_2s_3 & c_1c_2 - s_1s_2s_3 & s_2c_3 \\ s_1s_2 - c_1c_2s_3 & -c_1s_2 - s_1c_2s_3 & c_2c_3 \end{pmatrix} \approx \begin{pmatrix} c_1c_3 & s_1c_3 & s_3 \\ -s_1c_2 & c_1c_2 & s_2c_3 \\ s_1s_2 - c_1c_2s_3 & -c_1s_2 & c_2c_3 \end{pmatrix}$$

For the Real part of $\begin{pmatrix} c_3 & & s_3 \\ & 1 & \\ -s_3 & & c_3 \end{pmatrix}\begin{pmatrix} 1 & & \\ & c_2 & s_2 \\ & -s_2 & c_2 \end{pmatrix}\begin{pmatrix} c_1 & s_1 & \\ -s_1 & c_1 & \\ & & 1 \end{pmatrix}$,

$$\begin{pmatrix} c_1c_3 + s_1s_2s_3 & s_1c_3 - c_1s_2s_3 & s_3c_2 \\ -s_1c_2 & c_1c_2 & s_2 \\ -c_1s_3 + s_1s_2s_3 & -c_1c_3s_2 - s_1s_3 & c_2c_3 \end{pmatrix} \approx \begin{pmatrix} c_1c_3 & s_1c_3 & s_3c_2 \\ -s_1c_2 & c_1c_2 & s_2 \\ -c_1s_3 & -c_1c_3s_2 & c_2c_3 \end{pmatrix}$$

We see that with few exceptions, the three matrices approximating the rotations products have identical components.

Consequently, we may choose any one of the approximating matrices to predict the numerical values of the Weak Mixing Matrix.

In the proceedings, we will use the third approximating matrix,

$$\begin{pmatrix} c_1c_3 & s_1c_3 & s_3c_2 \\ -s_1c_2 & c_1c_2 & s_2 \\ -c_1s_3 & -c_1c_3s_2 & c_2c_3 \end{pmatrix} = \begin{pmatrix} \cos\theta_C \cos\theta_3 & \sin\theta_C \cos\theta_3 & \sin\theta_3 \cos\theta_2 \\ -\sin\theta_C \cos\theta_2 & \cos\theta_C \cos\theta_2 & \sin\theta_2 \\ -\cos\theta_C \sin\theta_3 & -\cos\theta_C \cos\theta_3 \sin\theta_2 & \cos\theta_2 \cos\theta_3 \end{pmatrix}.$$

It has two parameters $\theta_2$, and $\theta_3$, that we intend to approximate in such a way that the resulting matrix will depend only on the Cabbibo angle $\theta_C$.

To motivate our discussion we turn to the Wolfenstein approximation of the KM Matrix

## 3. Wolfenstein Approximation of the KM Matrix

In 1983, Wolfenstein [Wolf] obtained an approximating formula by expanding the KM matrix in powers of $\lambda = \sin\theta_C$. The Wolfenstein approximation [Wolf, equation 10] for the KM matrix to order of $\lambda^5$ is

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3\rho \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho) & -A\lambda^2 & 1 \end{pmatrix} + i\eta A\lambda^3 \begin{pmatrix} 0 & 0 & -(1+\frac{1}{2}\lambda^2) \\ 0 & -A\lambda & \lambda \\ -1 & 0 & 0 \end{pmatrix},$$

where $\lambda = \sin\theta_C$, $A \approx 1.25,$ , $\rho < 1$, and $|\eta| < 0.1$.

Experiments determine $A \approx 0.8$.

The powers of $\lambda = \sin\theta_C$ in the Matrix suggest that perhaps

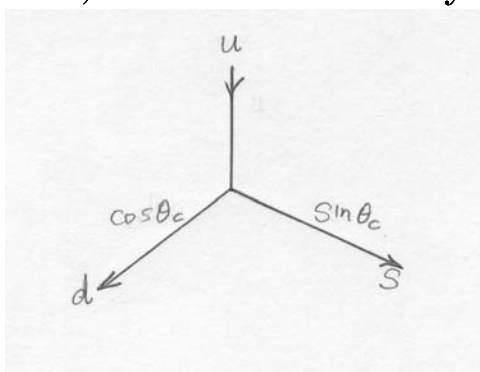
$$\sin\theta_2 \approx \sin^2\theta_C$$

$$\sin\theta_3 \approx \sin^3\theta_C .$$

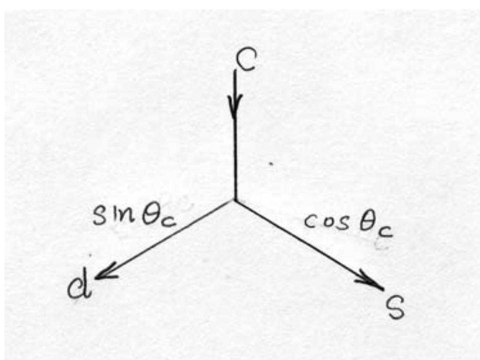
We will assume the first relation, and prove the second.

**4.** $\sin\theta_2 \approx \sin^2\theta_C \Rightarrow \sin\theta_3 \approx \sin^3\theta_C$

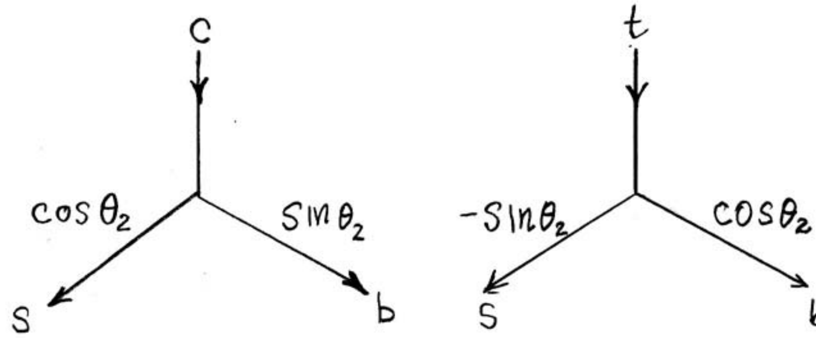
The $u$ transition into $d$, and $s$ is described by



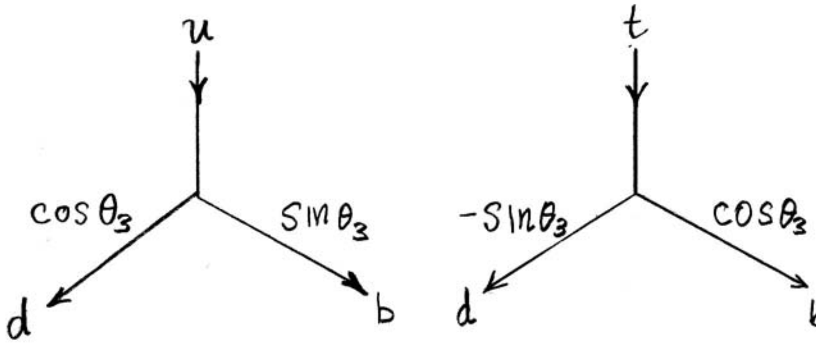

The $c$ transition into $d$, and $s$ is described by




The 2nd rotation matrix, $\begin{pmatrix} 1 & & \\ & \cos\theta_2 & \sin\theta_2 \\ & -\sin\theta_2 & \cos\theta_2 \end{pmatrix}$, adds the transitions




The $3^{\text{rd}}$ rotation matrix, $\begin{pmatrix} \cos\theta_3 & & \sin\theta_3 \\ & 1 & \\ -\sin\theta_3 & & \cos\theta_3 \end{pmatrix}$, adds the transitions

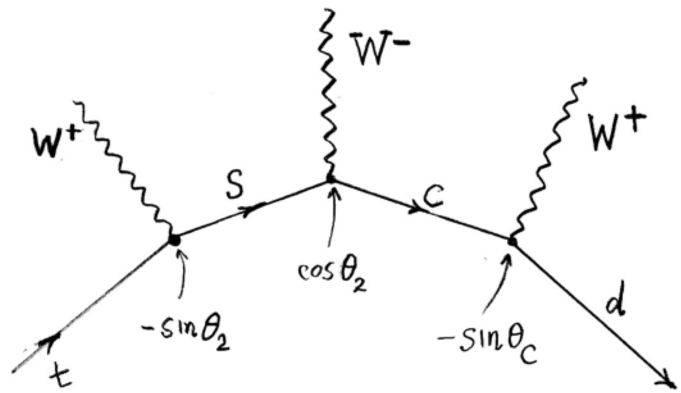



We assume that

$$\begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C\cos\theta_3 & \sin\theta_C\cos\theta_3 & \sin\theta_3\cos\theta_2 \\ -\sin\theta_C\cos\theta_2 & \cos\theta_C\cos\theta_2 & \sin\theta_2 \\ -\cos\theta_C\sin\theta_3 & -\cos\theta_C\cos\theta_3\sin\theta_2 & \cos\theta_2\cos\theta_3 \end{pmatrix} \quad (1)$$

To prove that $\sin\theta_3 \approx \sin^3\theta_C$, we construct a Feynman diagram for the relation

$$|U_{td}| \approx \cos\theta_C \sin\theta_3 .$$




From the diagram

$$|U_{td}| = \sin\theta_2 \cos\theta_2 \sin\theta_C$$

Therefore,

$$\cos\theta_C \sin\theta_3 \approx \sin\theta_2 \cos\theta_2 \sin\theta_C .$$

Since $\cos\theta_C \approx 1$, and $\cos\theta_2 \approx 1$, we have

$$\sin\theta_3 \approx \sin\theta_2 \sin\theta_C .$$

Since we assume $\sin\theta_2 \approx \sin^2\theta_C$, we conclude that

$$\sin\theta_3 \approx \sin^3\theta_C .$$

A similar proof for our assumption that $\sin\theta_2 \approx \sin^2\theta_C$ may exist, but its pursuit is not crucial to our purpose here. The validity of the approximation to the Weak Mixing Matrix that we are seeking, will be established by a Chi-Squared Goodness-of-Fitness Statistical test.

Substituting the relations above in (1), we will obtain an approximation for the Weak Mixing Matrix that depends only on the Cabbibo angle $\theta_C$.

## 5. Cabbibo angle approximation for the Real part of the Weak Mixing Matrix.

Assuming that

$$\sin\theta_2 \approx \sin^2\theta_C,$$

we have

$$\sin\theta_3 \approx \sin^3\theta_C,$$

$$\cos\theta_2 \approx \cos(\sin\theta_2) \approx \cos((\sin\theta_C)^2) \approx \cos\theta_C^2,$$

$$\cos\theta_3 \approx \cos(\sin\theta_3) \approx \cos((\sin\theta_C)^3) \approx \cos\theta_C^3.$$

Substituting into (1), we obtain

$$\begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C \cos\theta_C^3 & \sin\theta_C \cos\theta_C^3 & \sin^3\theta_C \cos\theta_C^2 \\ -\sin\theta_C \cos\theta_C^2 & \cos\theta_C \cos\theta_C^2 & \sin^2\theta_C \\ -\cos\theta_C \sin^3\theta_C & -\cos\theta_C \cos\theta_C^3 \sin^2\theta_C & \cos\theta_C^2 \cos\theta_C^3 \end{pmatrix} \quad \textbf{(2)}$$

The approximation Matrix is parametrized by the Cabbibo angle alone, with no other parameters.

In comparison, the Wolfenstein Matrix added the parameters $A$, $\rho$, and $\eta$.

The Weak Mixing Angle given by [PDG, p.166] is

$$\theta_C = 0.22308 \pm 0.00030.$$

We list the transition rates

| | *predicted* | *observed* PDG 2008 |
|---|---|---|
| $\lvert U_{ud}\rvert$ | $\cos\theta_C \cos\theta_C^3 = 0.97516058$ | $0.97418 \pm 0.00027$ |
| $\lvert U_{us}\rvert$ | $\sin\theta_C \cos\theta_C^3 = 0.221220714$ | $0.2255 \pm 0.0019$ |
| $\lvert U_{cd}\rvert$ | $\sin\theta_C \cos\theta_C^2 = 0.220960457$ | $0.230 \pm 0.011$ |
| $\lvert U_{cs}\rvert$ | $\cos\theta_C \cos\theta_C^2 = 0.974013344$ | $0.99 \pm 0.01$ |
| $\lvert U_{cb}\rvert$ | $\sin^2\theta_C = 0.048944636$ | $0.0412 \pm 0.0011$ |
| $\lvert U_{ub}\rvert$ | $\sin^3\theta_C \cos\theta_C^2 = 0.010814829$ | $0.00393 \pm 0.00036$ |
| $\lvert U_{td}\rvert$ | $\cos\theta_C \sin^3\theta_C = 0.010559918$ | $0.0081 \pm 0.0006$ |
| $\lvert U_{ts}\rvert$ | $\cos\theta_C \cos\theta_C^3 \sin^2\theta_C = 0.04772888$ | $0.0387 \pm 0.0023$ |
| $\lvert U_{tb}\rvert$ | $\cos\theta_C^2 \cos\theta_C^3 = 0.998700448$ | $0.99875 \pm 0.00005$ |

For $\lvert U_{cs}\rvert$ we chose PDG 2008 value $0.99 \pm 0.01$ that is not over $1$. PDG proceeds to average it with $1.07 \pm 0.08$, which is a flawed measurement that should not be considered.

For $\lvert U_{tb}\rvert$, we stated the PDG 1998 value, that is compatible with dozen prior years values.[see references 1-7] The PDG 2008 statement that $\lvert U_{tb}\rvert > 0.74$ is unsatisfying.

We test the validity of our approximation with a Chi-Squared Goodness-of-Fitness Statistical Test

# 6. Chi-Squared Goodness-of-Fit-Test of our Approximation to the Weak Mixing Matrix

## 6.1 The Null Hypothesis:

The approximation matrix is valid approximation to the Weak Mixing matrix $U(i,j) = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}$.

We aim to show that the values expected for the Weak Mixing Matrix fit well the observed values of the Weak Mixing Matrix

## 6.2 The Chi-Square Statistic for the Weak Mixing Matrix

$$\chi^2_{\text{computed}}(\alpha, \nu) = \sum_{i,j} \frac{\left[U_{\text{observed}}(i,j) - U_{\text{expected}}(i,j)\right]^2}{U_{\text{expected}}(i,j)},$$

where $\alpha$ is the test level of confidence, and $\nu$ is the number of degrees of freedom.

## 6.3 The Number of Degrees of Freedom

Here, the number of degrees of freedom is $\nu = 9 - 1 = 8$.

At least 5 degrees of freedom are recommended for Chi-Squared Goodness-of-Fit testing. Greater confidence of the test, requires greater $\nu$. As $\nu$ increases, the skewed Chi-Squared distribution gets close to the symmetric normal distribution.

## 6.4 The Chi-Square Test

| | *predicted* | *observed* | $\frac{\lvert error \rvert^2}{predicted}$ |
|---|---|---|---|
| $\lvert U_{ud} \rvert$ | 0.97516058 | 0.97418 | $9.860295382 * 10^{-7} < 0.000001$ |
| $\lvert U_{us} \rvert$ | 0.221220714 | 0.2255 | $8.277836347 * 10^{-5} < 0.000083$ |
| $\lvert U_{cd} \rvert$ | 0.220960457 | 0.230 | $3.698097784 * 10^{-4} < 0.000370$ |
| $\lvert U_{cs} \rvert$ | 0.974013344 | 0.99 | $2.623918570 * 10^{-4} < 0.000263$ |
| $\lvert U_{cb} \rvert$ | 0.048944636 | 0.0412 | $1.225453730 * 10^{-3} < 0.001226$ |
| $\lvert U_{ub} \rvert$ | 0.010814829 | 0.00393 | $4.382951442 * 10^{-3} < 0.004383$ |
| $\lvert U_{td} \rvert$ | 0.010559918 | 0.0081 | $5.730344276 * 10^{-4} < 0.000574$ |
| $\lvert U_{ts} \rvert$ | 0.04772888 | 0.0387 | $1.707994700 * 10^{-3} < 0.001708$ |
| $\lvert U_{tb} \rvert$ | 0.998700448 | 0.99875 | $2.458595777 * 10^{-9} < 0.000001$ |

Therefore,

$$\chi^2_{\text{computed}}(8) < 0.008609$$
$$< 0.0087 = \chi^2(\alpha, 8)$$

We need to find the confidence level $\alpha$ at which $\chi^2(\alpha, 8)$ is $0.0087$.

## 6.5 The Confidence Level

By [Abram], the Chi-Squared distribution is the Cumulative Probability Function

$$Q(\chi^2;\nu) = \frac{1}{2^{\nu/2}\Gamma(\nu / 2)} \int_{t=\chi^2}^{\infty} e^{-t/2} t^{\nu/2-1} dt\,.$$

To use the MATHEMATICA Gamma function

$$\text{Gamma}[a,z] = \int_{u=z}^{u=\infty} e^{-u} u^{a-1} du\,,$$

we identify $a = \frac{\nu}{2}$, $u = \frac{t}{2}$, $z = \frac{\chi^2}{2}$. Then,

$$Q(\chi^2;\nu) = \frac{1}{\Gamma(\nu / 2)} \int_{u=\chi^2/2}^{\infty} e^{-u} u^{\nu/2-1} du$$

$$= \frac{1}{\Gamma(\nu / 2)} \text{Gamma}[\nu/2, \chi^2 / 2]$$

For $\nu = 8$, and $\chi^2 = 0.0087$, we have $\Gamma(8 / 2) = (4-1)!$, and with MATHEMATICA we obtain

$$Q(0.0087;8) = N[\frac{1}{3!}\text{Gamma}[4,0.00435]] = 0.9999999999851326`$$

This means that our approximation to the Weak Mixing Matrix gives the correct transition rates with confidence of

$$99.99999999851326\%$$

The uncertainty in the transition rates is

$$P(\chi^2, 8) = P(0.00435, 8) = 1.48674 * 10^{-11} < \frac{2}{10^{11}}$$

In Conclusion, we have shown that the approximation

$$\begin{pmatrix} \cos\theta_C \cos\theta_C^3 & \sin\theta_C \cos\theta_C^3 & \sin^3\theta_C \cos\theta_C^2 \\ -\sin\theta_C \cos\theta_C^2 & \cos\theta_C \cos\theta_C^2 & \sin^2\theta_C \\ -\cos\theta_C \sin^3\theta_C & -\cos\theta_C \cos\theta_C^3 \sin^2\theta_C & \cos\theta_C^2 \cos\theta_C^3 \end{pmatrix}$$

for the real part of the Weak Mixing Matrix, can be used with extremely high level of confidence.

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