The Aharonov-Bohm Magnetic Field is Not Zero and the Electron Spirals towards the Center

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Abstract Aharonov and Bohm claimed that a Magnetic Potential may generate a vanishing Magnetic Induction, ignoring experimental evidence to the contrary.

We show that the suppressed experiments were correct, and compute the non-negligible Magnetic Induction.

Namely, if an electron encircles at radius $r = b$, a solenoid of radius a, that generates along its axis a uniform Magnetic Induction B, then, the Magnetic Induction at the electron is the non-negligible $\vec{B} = \frac{a^2 B}{b^2} \vec{1}_z$, that forces the electron to spiral towards the center. This spiraling motion

- cannot happen in Aharonov-Bohm vanishing field, and, being a global effect

- cannot be predicted from the Schrödinger Equation,

Eventually, the repetition of the Fallacy that the Induction is zero. became actual Physics.

Thus, the significance of the non-existing Aharonov-Bohm Effect is nil. In particular,

- The Magnetic Potential which is not observable, does not appear in the Lorentz Force, and is not unique, cannot replace the Magnetic Induction.
- The Electromagnetic field is characterized fully by its Electric Field $\vec{E}$, and Magnetic Induction $\vec{B}$. The Electric Potential φ, and the Magnetic Potential $\vec{A}$ do not offer a more complete description of the electromagnetic Field.

Keywords: Aharonov-Bohm Effect, Quantum Mechanics, Schrödinger Equation, Electron, Electrodynamics, Magnetic Potential, Magnetic Flux, Magnetic Induction.

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Introduction

Aharonov and Bohm claimed that a Magnetic Potential $\vec{A}$ may generate a vanishing Magnetic Induction $\vec{B}$.

They claimed that if an electron encircles at radius $r = b$, a solenoid of radius a, that generates along its axis a uniform Magnetic Induction B, then, the Magnetic Induction at the electron vanishes. And the electron is affected by the Non-zero Magnetic Potential that appears in Schrödinger's Equation.

The Aharonov-Bohm claims were avoided by most authors: The paradoxial Effect is not mentioned in Electrodynamics Texts, and is presented in few Quantum Mechanics Texts. Indeed,

- It does not seem credible that a Non-zero Magnetic Potential $\vec{A}$ will generate no Magnetic Induction $\vec{B}$

And

- Using the Schrödinger Equation, that accounts for quantum effects, to make up for Electrodynamics effects, is like using a microscope to find a tree in a forest.

In particular,

Experiments show a small non-zero Magnetic Induction, at the electron.

According to Feynman, [Feynman, part II, Chapter 13, p. 5],

> "***We observe experimentally that when a Solenoid is very long compared with its diameter, the field outside is very small compared with the field inside***"

Nevertheless, Feynman, believed that the Magnetic Induction vanishes, and gave the following "proof":

> "*Since the field stays inside*
>
> *(and has zero divergence),…*"

Then,

- "stays inside" insinuates that outside it vanishes, and
- "zero divergence", $\mathrm{Div}\,\vec{B}=0$, insinuates $\vec{B}=0$. while it is clear that $\vec{B}=\mathrm{Constant}$.

Thus, Feynman assumed that the Magnetic Induction is zero. In [Feynman, Part II, Chapter 15, p.11], he claimed

> "*You remember that for a long Solenoid carrying an electric current there is a* ***B****-field inside but none outside..*"

Assuming zero Induction, he described electron interference that proved that the Induction is zero…(pages 11-12 there).

Feynman concluded with

> "*Precisely this experiment has recently been done.* ***It is a very, very difficult experiment****. Because the wavelength of the electrons is so small, the apparatus must be on a tiny scale to observe the interference…*"

Why Feynman preferred an experiment that assumes its false result over experimental evidence that contradicts it, will never be known.

Clearly, the Aharonov-Bohm Effect is based on repetition of the Fallacy that the Induction is zero. Eventually, that repeated Fallacy became actual Physics.

Following our recent construction of Space-time Electrodynamics, we considered the Aharonov-Bohm Effect in Space-time.

We examined the Aharonov-Bohm Effect in space-time, expecting to find that the time component of the Magnetic Induction exists, and applies to the electron.

We found that Space-time Electrodynamics was not needed to resolve the Aharonov-Bohm Paradox.

The Aharonov-Bohm Effect does not exist already in 3-space.

Aharonov-Bohm vanishing of the Magnetic Induction is a Fallacy, due to ignorance of elementary Vector Analysis, and confusion about the relevance of Quantum Mechanics.

Its proponents were anxious to keep the Magnetic Potential in the Schrödinger Equation, instead of finding how the Magnetic Induction may replace the Potential in that Equation.

We show that the Aharonov-Bohm Effect is Pure Fiction.

As the suppressed experimental evidence shows, the Magnetic Induction at the electron does not vanish. It has a non-zero value.

That Magnetic Induction generates a Lorentz Force on the electron in the direction of the center, and results in a Magnetron-like effect. The electron spirals towards the solenoid.

1.

The Non-existent Aharonov-Bohm Effect

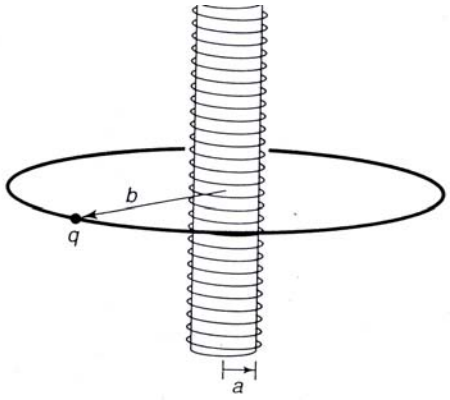
A Solenoid with cross-section radius a, generates a Uniform Magnetic Induction

$$\vec{B} = B\vec{1}_z,$$

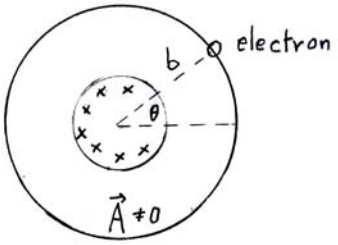
which is seemingly confined to the interior of the Solenoid.

And an electron encircles the Solenoid, at radius

$$b > a.$$



In the electron's plane,



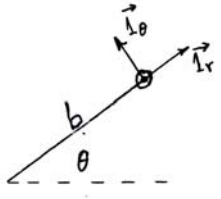
The Magnetic Flux of the Solenoid is its cross-section area πa^2, times the Magnetic Induction density B,

$$\pi a^2 B.$$

The Magnetic Potential at the Electron is

$$\vec{A} = \frac{\pi a^2 B}{2\pi b}\vec{1}_\theta = \frac{a^2 B}{2b}\vec{1}_\theta,$$

where $\vec{1}_\theta$ is the unit vector in the direction of increasing θ, in the cylindrical coordinate system (r, θ, z).

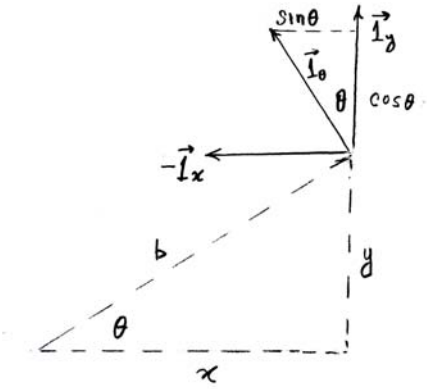


Aharonov-Bohm claimed that the Magnetic Induction out of the solenoid vanishes, and the effect of the Magnetic Potential $\vec{A}$ on the electron requires the Schrödinger Equation. In fact,

- the Magnetic Potential generates a non-zero Magnetic Induction that applies to the electron, and
- the Schrödinger Equation is not necessary for elementary Electrodynamics.

2.

The Non-Vanishing Magnetic Induction on the Electron



Clearly,

$$\vec{1}_\theta = -\vec{1}_x \sin\theta + \vec{1}_y \cos\theta$$

$$= -\vec{1}_x \frac{y}{b} + \vec{1}_y \frac{x}{b}$$

Hence,

$$\vec{A} = \frac{a^2 B}{2b} \vec{1}_\theta$$

$$= -\frac{a^2 B}{2b^2} y\vec{1}_x + \frac{a^2 B}{2b^2} x\vec{1}_y$$

$$= \frac{a^2B}{2b^2}\begin{bmatrix} -y \\ x \\ 0 \end{bmatrix}.$$

Therefore,

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$= \frac{a^2B}{2b^2}\vec{\nabla} \times \begin{bmatrix} -y \\ x \\ 0 \end{bmatrix}$$

$$= \frac{a^2B}{2b^2}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \partial_x & \partial_y & \partial_z \\ -y & x & 0 \end{vmatrix}$$

$$= \frac{a^2B}{2b^2}\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{a^2B}{b^2}\vec{1}_z .$$

That is, the Magnetic Induction at the electron, at $r = b$, is the non-zero

$$\vec{B} = \frac{a^2B}{b^2}\vec{1}_z .$$

This field is not negligible!

For instance, if the electron orbit has radius $b = 2a$, then the Magnetic Induction is $\vec{B} = \frac{B}{4}\vec{1}_z$, which is far from negligible.

3.

The Magnetic Induction Forces the Electron to Spiral towards the Center

The Lorentz Force on the electron is

$$q_e v_e \vec{1}_\theta \times \frac{a^2 B}{b^2} \vec{1}_z = q_e v_e \frac{a^2 B}{b} \vec{1}_r ,$$

where v_e is the electron speed, and q_e is the electron charge.

Since $v_e = \omega_e b$, where ω_e is the angular speed, the Lorentz Force on the electron is

$$q_e \omega_e \frac{a^2 B}{b} \vec{1}_r .$$

Since the electron charge q_e is negative, the force is

$$(-q_e)\omega_e \frac{a^2 B}{b}(-\vec{1}_r).$$

The force direction is opposite to $\vec{1}_r$. That is, the force pushes the electron towards the center, and increases as the electron gets closer to the solenoid

The force results in a path, similar to the one that appears in Magnetrons [Hinkel].

A Magnetron is a circular cavity between two concentric cylinders. The negatively charged internal cylinder serves as a Cathode, and the positively charged external cylinder serves as an Anode. The potential gradient will accelerate an electron emitted from the Cathode to the Anode. A magnetic field perpendicular to the electron path generates a Lorentz Force that curves each electron path between the cylinders into a cycloid arc, in the direction of the cathode. As the negative cathode will repel the electron, and the positive Anode will attract it, the electron will slow, and come to a halt. Then, the process will start again. The electron motion through consecutive cycloid arcs transfers electromagnetic energy to nearby cavities in the Anode block, and keeps them oscillating, at a characteristic frequency determined by their capacitance and inductivity.

In the Aharonov-Bohm setup, the potential gradient that drives the electron in the circle, must be along the loop, in the direction of $\vec{1}_\theta$.

Therefore, the electron will not come to a halt before it

reaches the solenoid. The electron will spiral towards the solenoid, and collide with it.

This spiraling motion

- cannot happen in the Aharonov-Bohm vanishing field, and, being a global effect,
- cannot be predicted from the Schrödinger Equation.

4.

The insignificance of the Aharonov-Bohm Effect

The significance of the non-existing Aharonov-Bohm Effect is nil. In particular,

- The Magnetic Potential which is not observable, does not appear in the Lorentz Force, and is not unique, cannot replace the Magnetic Induction.
- The Electromagnetic field is characterized fully by its Electric Field $\vec{E}$, and Magnetic Induction $\vec{B}$. The Electric Potential φ, and the Magnetic Potential $\vec{A}$ do not offer a more complete description of the electromagnetic Field.

Wikipedia lists many more consequences of the non-existent Aharonov Bohm effect.

Any consequence of the non-existent effect, may be an insignificant fallacy too.

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