

The Fine Structure Constant and the Paradoxial Natural Units

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Abstract In the Meter-Kilogram-Second-Ampere unit system,
the speed of light is

$$c = 3 \times 10^8 \text{ meter/second.}$$

And Planck Constant divided by 2π is

$$\hbar = 1.054\ 572\ 66 \times 10^{-34} \text{ Joule} \times \text{second.}$$

Feynman proposed setting both constants to 1, in what became known as the **Natural Units** System.

Most writers on Gauge Theories, Symmetries in Particle Physics, and Weak Interaction adapted this system without ever checking its meaning, and paradoxial consequences.

To prevent those consequences from being far more uglier than the MKSA values, the Natural Units System requires also that the Fine Structure constant

$$\alpha \text{ will keep its MKSA value, } \frac{1}{137}.$$

In turn, this requirement replaces the Ampere with an even uglier Natural Current Unit. The non-physical meanings of the Natural Units render them erroneous, confusing, and useless.

1.

The Natural Length Unit

Setting

$$c = 1 \text{ Natural Speed Unit,}$$

it follows that

$$3 \times 10^8 \text{ meter/sec} = 1 \text{ (Natural Length Unit)/sec}$$

Hence,

$$\boxed{\text{Natural Length Unit} = 3 \times 10^8 \text{ meter}}.$$

The classical electron radius is of the order of 10^{-15} .

And one fails to see how a unit which is 10^{23} larger will be applicable to the electron size.

Perhaps, the Natural Length is not supposed to be ever used in computations involving electrons.

2.**The Natural Work Unit**

Setting

$$\hbar = 1 \text{ (Natural Work Unit)} \times \text{Second},$$

it follows that

$$1.05457266 \times 10^{-34} \text{ Joule} \times \text{sec} = 1 \text{ (Natural Work Unit)} \times \text{sec}$$

Hence,

$$\boxed{\text{Natural Work Unit} = 1.05457266 \times 10^{-34} \text{ Joule}}.$$

The Electron Volt unit is of the order of 10^{-19} Joule .

The Mega Electron Volt is of the order of 10^{-13} Joule .

Here, one fails to see how a unit which is 10^{21} smaller than one MeV will be applicable to particle interactions in a collider.

Perhaps, the Natural Work Unit is not supposed to be ever used in computations involving colliding particles.

3.

The Natural Force Unit

The Natural Force Unit is

$$\begin{aligned} \frac{\text{Natural Work Unit}}{\text{Natural Length Unit}} &= \frac{1.05457266 \times 10^{-34} \text{ J}}{3 \times 10^8 \text{ meter}} \\ &= 3.5152422 \times 10^{-43} \text{ Newton} \end{aligned}$$

That is,

$$\boxed{\text{The Natural Force Unit} = 3.5152422 \times 10^{-43} \text{ Newton}}$$

The Bohr Radius of the Hydrogen Atom is

$$5.29177249 \times 10^{-11} \text{ meter.}$$

The electrical force between the Hydrogen's electron and proton is

$$\begin{aligned} \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} &= \frac{c^2}{10^7} \frac{e^2}{r^2} \\ &= \frac{9 \cdot 10^{16} (1.60217733)^2 \cdot 10^{-38}}{10^7 (5.2977249)^2 10^{-22}} \\ &= (8.23161157) \times 10^{-8} \text{ Newton.} \end{aligned}$$

Here, one fails to see how a unit which is 10^{36} smaller than the Hydrogen's proton-electron force will be applicable to particle interactions in a collider.

Perhaps, the Natural Force Unit is not supposed to be ever used in computations involving colliding particles.

4.

The Natural Mass Unit

The Natural Mass Unit is

$$\begin{aligned} \frac{\text{Natural Force Unit}}{\text{Natural Acceleration Unit}} &= \frac{\text{Natural Force Unit}}{(\text{Natural Length Unit})/\text{sec}^2} \\ &= \frac{3.5152422 \times 10^{-43} \text{Newton}}{(3 \times 10^8 \text{meter})/\text{sec}^2} \\ &= (1.717474)10^{-51} \text{Kg} \end{aligned}$$

That is,

$$\boxed{\text{The Natural Mass Unit} = (1.717474)10^{-51} \text{Kg}}$$

The electron mass is

$$9.1093897 \times 10^{-31} \text{Kg}.$$

Then, one fails to see how a unit mass 10^{20} times smaller than the electron's mass will be applicable to particle's mass.

Moreover, an orange colored photon with frequency

$$\nu = 5 \times 10^{14} \text{cycles/sec}$$

has mass

$$\begin{aligned} \frac{h\nu}{c^2} &= \frac{(6.6260755)10^{-34} \cdot 5 \cdot 10^{14}}{9 \cdot 10^{16}} \\ &= (3.681153056)10^{-36} \end{aligned}$$

which is 10^{15} times larger than the Natural Mass unit.

Perhaps, the Natural Mass Unit is not supposed to be ever used in any physical computations.

5.

The Fine Structure Constant in Natural Units

The Fine Structure Constant is

$$\alpha = \frac{e^2}{4\pi\epsilon_0 c\hbar}.$$

In [Dan1], we showed that the speed of the Hydrogen electron in its ground orbit is

$$v = \alpha c.$$

Indeed, from the force balance on the electron,

$$m \frac{v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2},$$

$$v = \frac{1}{4\pi\epsilon_0} \frac{e^2}{cmvr} c.$$

Applying the angular momentum quantization,

$$mvr = \hbar,$$

we conclude

$$v = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\underbrace{c\hbar}_{\alpha}} c = \alpha c.$$

Clearly,

$$\alpha = \frac{v}{c}$$

is a unit-free number.

In the MKSA units,

$$\begin{aligned}\alpha &= \frac{e^2}{4\pi\epsilon_0 c \hbar} \approx \frac{1}{137}, \\ &= (7.29735308)10^{-3}.\end{aligned}$$

Namely, the Hydrogen electron speed in its ground orbit is

$$\frac{3 \cdot 10^8}{137} = 2,189,781 \text{ meter/sec},$$

in the MKSA System.

In the Natural Units System,

$$\begin{aligned}\alpha &= \frac{e^2}{4\pi\epsilon_0} \\ &= \frac{c^2}{10^7} e^2 \\ &= \frac{9 \cdot 10^{16}}{10^7} (1.60217733)^2 10^{-38} \\ &= (2.310274977)10^{-28}\end{aligned}$$

That is,

$$\boxed{\hbar = c = 1 \Rightarrow \alpha = (2.310274977)10^{-28}}$$

This number is not particularly natural, and is not related to any physics.

It was more natural to keep α at its MKSA value of $1/137$, even though $1/137$ means nothing in the Natural Units System.

As a bonus, this messed up the units of electric Current, and electric Voltage

6.**The Natural Units System defined****by $\hbar = c = 1$ and $\alpha = 1 / 137$**

The Fine Structure constant α , was set arbitrarily in the Natural Units system to its 1/137 value in the MKSA system.

There are infinitely many options of setting it:

at the Natural Units favorite 1,
 or to 2 if you like the only even prime,
 or to 3 if you like the smallest odd prime,
 or to 4 if you are not keen about primes,

.....

While the conditions

$$\hbar = c = 1$$

define the Mechanical Units, any shifting of α from its value of $(2.310274977)10^{-28}$ redefines the Electric Units. Then, we obtain non-physical units for which no voltmeter exists, and no one can measure, or tell what they mean.

6.1 The Natural Current Unit

Setting α to 1/137,

$$\frac{1}{137} = \frac{c^2}{10^7} e^2,$$

$$\begin{aligned} \frac{10^3}{c} \sqrt{\frac{10}{137}} (\text{Natural Current Unit}) \times \text{second} &= e, \\ &= (1.60217733)10^{-19} (\text{Ampere}) \times \text{second}. \end{aligned}$$

$$\begin{aligned} \text{Natural Current Unit} &= \frac{c}{10^3} \sqrt{\frac{137}{10}} (1.60217733)10^{-19} \text{ Ampere} \\ &= \frac{3 \cdot 10^8}{10^3} \sqrt{\frac{137}{10}} (1.60217733)10^{-19} \text{ Ampere} \\ &= (1.779066249)10^{-13} \text{ Ampere} \end{aligned}$$

That is,

Natural Current Unit = (1.779066249)10 ⁻¹³ Ampere
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6.2 The Natural Voltage Unit

The Natural Voltage Unit equals

$$\begin{aligned} (\text{Natural Current Unit}) \times (\text{One Ohm Resistance}) &= \\ &= (1.779066249)10^{-13} \underbrace{\text{Ampere} \times \text{Ohm}}_{\text{Volt}} \end{aligned}$$

That is,

Natural Voltage Unit = (1.779066249)10 ⁻¹³ Volt
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6.3 The Natural electron-Volt Unit

The Natural electron-Volt Unit equals

$$\begin{aligned} (\text{electron-Charge}) \times (\text{Natural Voltage Unit}) &= \\ &= (1.779066249)10^{-13} \text{ electron-Volt} \end{aligned}$$

Hence,

$$\boxed{\text{Natural electron-Volt} = (1.779066249)10^{-13} \text{electron-Volt}}$$

The users of the Natural Units System prefer the MKSA electron-Volts to the Natural electron-Volt Units

This results in confusion about formulas and numbers. Books that use the Natural Units pack errors, and no credibility.

References

[Aitchison] Aitchison & Hey “*Gauge Theories in Particle Physics*” Second Edition, IOP, 1989

[Guidry] Mike Guidry, “*Gauge Field Theories, An Introduction with Applications*” Wiley, 1991

[Benson] Benson & Harris & Stocker & Lutz, “*Handbook of Physics*” , Springer, 2002.

[Dan1], Vic Dannon, [Zero Point Energy, and the Charge-Radiation Equation in Bohr's Atom](#) Gauge Institute Journal Volume 8, No.4, November 2012

[Dan2], Vic Dannon, [Non-Physical Unit Systems in General Relativity, and in Quantum Field Theory](#), Posted to www.gauge-institute.org

[Woan], Graham Woan, “*The Cambridge Handbook of Physics Formulas*”. Cambridge University Press, 2000.