Mass Conversion into Neutrino Energy

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Abstract Neutrinos are packets of Neutrino energy propagating in the vacuum at speeds c_ν which can be greater than light speed. Neutrino speeds cannot be assumed to be the same for each neutrino, because the momentum and energy carried by a neutrino may be different in each beta interaction. Here, we assume that c_ν is the average speed of a neutrino beam.

Neutrino energy, is not electromagnetic. Thus, Neutrinos are non-photonic signals, of their own energy.

Supernova Models establish that about 1% of the gravitational binding is released as nuclear binding energy, supernova shell kinetic energy, and electromagnetic radiation. The rest, 99% of the Gravitational Binding Energy of a collapsing star, is emitted in the form of Neutrinos' Radiation, and is carried away by the radiated Neutrinos.

Thus, Gravitational Radiation is made of Neutrinos.

Since neutrinos are made of no sub-particles, the Neutrino is the Quantum of Gravitational Radiation. Alternatively, The Neutrino is the Signal of Gravitational Radiation.

Consequently, we may follow a particle by detecting its neutrino radiation. This leads to a special relativity theory, where the signals are neutrinos: Lorentz transformations for a lab moving slower than neutrinos, and Lorentz-like transformations for a lab moving faster than neutrinos

Then, Mass conversion into Neutrino Energy depends on neutrino speed c_ν. For a body moving at speed $u < c_\nu$, the mass $m(u) = \dfrac{1}{\sqrt{1 - \dfrac{u^2}{c_\nu^2}}} m_0$ converts into the Neutrino Energy

$$E_\nu(u) = \frac{1}{\sqrt{1 - \dfrac{u^2}{c_\nu^2}}} m_0 c_\nu^2 .$$

Hence, $m(u_2) - m(u_1)$ converts into $\left[m(u_2) - m(u_1)\right] c_\nu^2$.

For $u \ll c_\nu$, $$E(u) \approx m_0 c_\nu^2 + \underbrace{\frac{1}{2} m_0 u^2}_{\text{T=Kinetic Energy}}$$

For a body moving at speed $u > c_\nu$, the mass, $m(u) = \frac{1}{\sqrt{\frac{u^2}{c_\nu^2} - 1}} m_0$, converts into the Neutrino Energy

$$E_\nu(u) = \frac{1}{\sqrt{\frac{u^2}{c_\nu^2} - 1}} m_0 c_\nu^2 .$$

Hence, $m(u_2) - m(u_1)$ converts into $\left[m(u_2) - m(u_1)\right] c_\nu^2$.

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Contents

The Neutrino is the Signal of Gravitational Radiation

0.1 Neutrinos as Signals of Non-Electromagnetic Energy

Photons are packets of Electromagnetic energy propagating in the vacuum as light speed c. Those energy packets are the signals of Electromagnetic propagation. They are characterized by their frequency. There are numerous such frequencies. Photons are generated by electrons accelerating at these frequencies: Radio waves frequencies are generated by electrons oscillating along an antenna. Optical frequencies are generated by spectral electrons moving from one orbit to another. X rays frequencies are generated in the nucleus.

Neutrinos are packets of Neutrino energy propagating in the vacuum at speeds c_ν which can be greater than light speed. Neutrino speeds cannot be assumed to be the same for each neutrino, because the momentum and energy carried by a neutrino may be different in each beta interaction. To account for these missing energy, and momentum in the balance of Radioactive Interaction, Pauli proposed the emission of electrically neutral, hence, unseen particle. Fermi coined the name Neutrino, ν.

It is believed that there are three varieties of Neutrinos, termed "oscillations". it is not known how these varieties are generated, or what causes one variety to oscillate into another. A Neutrino beam mixes the three types of Neutrinos, ν_e, ν_μ, and ν_τ. The Neutrino is one of the longest living particle, along with electrons, protons, and photons.

Neutrinos are generated for instance

- in the Atmosphere, by $\pi \to \mu + \nu_\mu \to e + 2\nu_e$
- in the Sun Core, by the fusion $4\text{H} \to \text{He} + \nu_e + \gamma$
- in the fusion of protons,
- in the fusion of Boron ^8B,
- in the fusion of Beryllium $^7\text{B}_\text{e}$
- in Meson decay,
- in Supernova

And they are believed to fill the Universe. But their chance to collide with a particle is very low.

Their Cross Section, the effective target area that they present to other particles, is as small as $\frac{1}{10^{43}}\text{cm}^2$.

Thus, millions of them are believed to pass through the earth in a fraction of a second, with out interaction with any particles, as if the earth was transparent to them.

In terms of particles, in a beta decay process in which anti-neutrinos are generated, a neutron, which may be explained as a condensed Hydrogen Atom, [Dan4], decomposes into its proton, and electron. The anti-neutrino balances the energy, and the momentum in the interaction,

$$\mathrm{n} \to \mathrm{p} + \mathrm{e} + \overline{\nu}_e .$$

In terms of energy levels, beta decay may be interpreted as a transformation of a nucleon from its d-quark energy level into its u-quark energy level, releasing the electron, and the anti-neutrino in the interaction,

$$\mathrm{d} \to \mathrm{u} + \mathrm{e} + \overline{\nu}_e .$$

Either way, Neutrinos are Signals carrying Neutrino energy, which is not considered electromagnetic. Therefore, Neutrinos are non-photonic signals, of their own energy.

0.2 Faster than Light Neutrinos

Faster Than Light Neutrinos are detected by the Cherenkov Radiation that they emit in water.

The interaction of Faster Than Light neutrinos with nucleus of atoms in water molecules generates bursts of Cherenkov

Radiation, that are counted by photo-multiplier tubes that surround the water tank.

Photon's speed decreases by third in water, and particles that can move in water faster than photons, can move in the air faster than photons.

Faster than Light Neutrinos in the Air are invisible to us, but in water they move slower than photons in the air, and become visible.

electrically charged particles leave behind them an optical wake of Cherenkov Radiation: A blue glow of radiation on the verge of becoming invisible.

That optical wake is similar to the sonic shock wave created by a jet plane moving at supersonic speed.

0.3 Neutrino is the Signal of Gravitational Radiation

Supernova Models establish that 99% of the Gravitational Binding energy of a collapsing star is emitted in the form of Neutrinos' Radiation.

This fact seems to be well-known. By [Close, p.139],

> *"…most of the energy produced in a supernova is radiated away in the form of an immense burst of neutrinos…"*

By [Bahcall, p.428],

"Most of the binding energy that is released when a neutron star is formed is believed to be emitted in the form of neutrinos"

Since we could not find any believable substantiation to this claim, we worked it out in the following:

By [Swihart, p.120], to build a Star of constant density

$$\rho,$$

with mass

$$M,$$

and radius

$$R,$$

we add to the mass

$$m(r) = \tfrac{4\pi}{3} r^3 \rho,$$

that fills a ball of radius

$$r,$$

the infinitesimal mass of a shell

$$dm(r) = 4\pi r^2 \rho dr,$$

which will add the Gravitational energy

$$-G\frac{m(r)dm(r)}{r} = -G\tfrac{16\pi^2}{3}\rho^2 r^4 dr.$$

Thus, the total Gravitational energy of the Star is the Sum

$$-G\int_{m=0}^{m=M}\frac{m(r)}{r}dm(r) = -G\tfrac{16\pi^2}{3}\rho^2\int_{r=0}^{r=R} r^4 dr = -\tfrac{3}{5}G\frac{M^2}{R}.$$

[Landau-Lifshitz, pp.327-331], supplies the analysis for the

Supernova creation of a neutron star, and concludes (p.330-331) with the following:

> *"The conversion of the whole mass M from the electron-nucleus state to the neutron state requires an expenditure of energy…to counterbalance the binding energy of the nuclei.*
>
> *In the process, energy is released because of the contraction of the body…This gain of energy is*
>
> $$\tfrac{3}{7}GM^2\left(\frac{1}{R_{\text{Neutron Star}}}-\frac{1}{R_{\text{Atom Star}}}\right)$$

The second term in the formula is negligible compared with the first, and

the Gravitational Binding released is

$$\tfrac{3}{7}GM^2\frac{1}{R_{\text{Neutron Star}}}.$$

Computing with the Chandrasekhar Limit,

$$M = 1.4M_{\text{Sun}} = 1.4\times 2\times 10^{30}\text{kg},$$

and with

$$R_{\text{Neutron Star}} = 10\text{km}=10^4\text{m},$$

the **Gravitational Binding** is

$$\tfrac{3}{7}6.7\times 10^{-11}(2.8)^2 10^{60}\frac{1}{10^4} = 2.25\times 10^{46}\text{Joul}$$

$$= 2.25\times 10^{55}\text{erg}$$

The **Nuclear Binding energy** is

$$\begin{pmatrix} 3.2 \underbrace{\text{MeV}}_{1.6\times10^{-11}\text{Joul}} \\ \text{per nucleus} \end{pmatrix} \times \begin{pmatrix} 6\times10^{23} \\ \#\text{ of nuclei/kg} \end{pmatrix} \times \underbrace{M}_{1.4\times2\times10^{30}\text{kg}} = 8.6\times10^{43}\text{Joul},$$

$$= 8.6\times10^{52}\text{erg}.$$

This is less than 0.5% of the gravitational binding.

By [Kundt, p.40],

> *“Supernova shells tend to have masses…of order $3M_{\text{Sun}}$-inferred from the times at which their spectra changes from optically thick (photospheric) to optically thin (nebular), usually between 6 and 18 weeks after launch- and radial velocities ranging from several 10^5 m/sec up to several 10^7 m/sec, with a quadratic mean near $10^{6.8}$ m/sec.”*

Their **kinetic Energy** is of order

$$\tfrac{1}{2}3\underbrace{M_{\text{Sun}}}_{2\times10^{30}}10^{13.6} = 1.2\times10^{44}\text{Joul}$$

$$= 1.2\times10^{53}\text{erg}$$

This is a little over 0.5% of the gravitational binding

By [Kundt],

Radio waves, optical, and X-ray radiation

average

$$3 \times 10^{40}\text{Joul} = 3 \times 10^{49}\text{erg}$$

This is negligible compared with the gravitational binding.

In conclusion, about 1% of the gravitational binding is released as nuclear binding energy, supernova shell kinetic energy, and electromagnetic radiation.

Consequently, 99% of the Gravitational Binding Energy is carried away by the radiated Neutrinos.

Thus,

Gravitational Radiation is made of Neutrinos.

Since neutrinos are made of no sub-particles,

Neutrino is the Quantum of Gravitational Radiation.

Alternatively,

The Neutrino is the Signal of Gravitational Radiation.

Consequently, we may follow a particle by detecting its neutrino radiation. This leads to a special relativity theory, where the signals are neutrinos.

I.

SPECIAL RELATIVITY of the NEUTRINO

1.

Lorentz Transformation for Slower than Neutrino Speed

A particle moves with speed $v < c_\nu$ in the x direction, passes through the Lab's origin, and emits a neutrino that moves with speed c_ν, in the x direction.

That neutrino signal, that may be invisible to us, will be detected in a neutrino detector.

We will assume that the speed of the neutrino is the same in all uniformly moving reference frames.

In the Lab frame, the neutrino speed is

$$c_\nu = \frac{dx}{dt}$$

$$= \frac{\frac{dp}{dt}dx}{dp}$$

$$= \frac{Fdx}{dp}$$

$$= \frac{dE}{dp}.$$

Therefore,

$$c_\nu dp = dE$$

In the particle frame,

$$c_\nu = \frac{d\xi}{d\tau}$$

$$= \frac{\frac{dp_\xi}{d\tau}d\xi}{dp_\xi}$$

$$= \frac{F_\xi d\xi}{dp_\xi}$$

$$= \frac{dE_\tau}{dp_\xi}$$

Hence,

$$c_\nu dp_\xi = dE_\tau$$

Therefore,

$$(dE_\tau)^2 - (c_\nu dp_\xi)^2 = 0 = (dE)^2 - (c_\nu dp)^2.$$

Thus, the infinitesimal momentum

$$\left(\frac{i}{c_\nu}dE\right)^2 + (dp)^2 \text{ is Lorentz invariant.}$$

Since

$$\frac{dE}{c_\nu dp} = \frac{dx}{c_\nu dt},$$

distance transforms similarly to momentum, and the infinitesimal distance

$$-c_\nu^2(dt)^2 + (dx)^2 \quad \text{is Lorentz invariant}$$

In the Lab, the event coordinates are

$$\left(ic_\nu t, x\right),$$

and the momentum coordinates are

$$\left(\frac{i}{c_\nu}E, p\right).$$

In the particle's frame, the event coordinates are

$$\left(ic_\nu \tau, \xi\right),$$

and the momentum coordinates are

$$\left(\frac{i}{c_\nu}E_\tau, p_\xi\right).$$

Assuming that the infinitesimal energies transform linearly

$$\begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \begin{bmatrix} \gamma_1 & \beta_1 \\ \beta_2 & \gamma_2 \end{bmatrix} \begin{bmatrix} dE \\ c_\nu dp \end{bmatrix},$$

we have

$$dE_\tau = \gamma_1 dE + \beta_1 c_\nu dp,$$

$$c_\nu dp_\xi = \beta_2 dE + \gamma_2 c_\nu dp,$$

$$(dE)^2 - (c_\nu dp)^2 = (dE_\tau)^2 - (c_\nu dp_\xi)^2$$

$$= (\gamma_1 dE + \beta_1 c_\nu dp)^2 - (\beta_2 dE + \gamma_2 c_\nu dp)^2$$

$$= (\gamma_1 dE)^2 + 2\gamma_1\beta_1 c_\nu dEdp + (\beta_1 c_\nu dp)^2$$

$$-(\beta_2 dE)^2 - 2\beta_2\gamma_2 c_\nu dEdp - (\gamma_2 c_\nu dp)^2$$

Equating Coefficients,

$$\gamma_1^2 - \beta_2^2 = 1,$$

$$\beta_1^2 - \gamma_2^2 = -1 \Rightarrow \gamma_2^2 - \beta_1^2 = 1,$$

$$\gamma_1\beta_1 - \beta_2\gamma_2 = 0 \quad \Rightarrow \quad \gamma_1\beta_1 = \beta_2\gamma_2.$$

We obtain

$$\gamma_1 = \gamma_2 \equiv \gamma_\nu,$$

$$\beta_1 = \beta_2 \equiv b,$$

or

$$\gamma_1 = -\gamma_2 \equiv \gamma_\nu,$$

$$\beta_1 = -\beta_2 \equiv b,$$

so that in either case

$$\gamma_\nu^2 - b^2 = 1.$$

Now, for $\delta E_\tau = 0$,

$$0 = \gamma_\nu \delta E + b c_\nu \delta p,$$

$$\frac{bc_\nu}{\gamma_\nu} = -\frac{\delta E}{\delta p} = -\frac{\delta x}{\delta t} = -\upsilon\,,$$

$$b = -\gamma_\nu \frac{\upsilon}{c_\nu}$$

Substituting in $\gamma_\nu^2 - b^2 = 1$,

$$\gamma_\nu^2 - \gamma_\nu^2 \frac{\upsilon^2}{{c_\nu}^2} = 1\,,$$

$$\gamma_\nu^2 (1 - \frac{\upsilon^2}{{c_\nu}^2}) = 1\,,$$

$$\gamma_\nu = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{{c_\nu}^2}}}.$$

$\underline{\upsilon < c_\nu}$

1.1
$$\begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{{c_\nu}^2}}} \begin{bmatrix} 1 & -\frac{\upsilon}{c_\nu} \\ -\frac{\upsilon}{c_\nu} & 1 \end{bmatrix} \begin{bmatrix} dE \\ c_\nu dp \end{bmatrix}$$

In the inverse transformation, $-\upsilon$ replaces υ

1.2
$$\begin{bmatrix} dE \\ c_\nu dp \end{bmatrix} = \frac{1}{\sqrt{1-\frac{\upsilon^2}{{c_\nu}^2}}} \begin{bmatrix} 1 & \frac{\upsilon}{c_\nu} \\ \frac{\upsilon}{c_\nu} & 1 \end{bmatrix} \begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix}$$

The distance transforms by

1.3
$$\begin{bmatrix} d\xi \\ c_\nu d\tau \end{bmatrix} = \frac{1}{\sqrt{1-\frac{\upsilon^2}{{c_\nu}^2}}} \begin{bmatrix} 1 & -\frac{\upsilon}{c_\nu} \\ -\frac{\upsilon}{c_\nu} & 1 \end{bmatrix} \begin{bmatrix} dx \\ c_\nu dt \end{bmatrix}$$

In the inverse transformation, $-\upsilon$ replaces υ,

1.4
$$\begin{bmatrix} dx \\ c_\nu dt \end{bmatrix} = \frac{1}{\sqrt{1-\frac{\upsilon^2}{{c_\nu}^2}}} \begin{bmatrix} 1 & \frac{\upsilon}{c_\nu} \\ \frac{\upsilon}{c_\nu} & 1 \end{bmatrix} \begin{bmatrix} d\xi \\ c_\nu d\tau \end{bmatrix}$$

1.5 The infinitesimal distance is Lorentz invariant

Proof:

$$(dx)^2 - c_\nu^2 (dt)^2 = \gamma_\nu^2 \left\{ \left[d\xi + \upsilon d\tau \right]^2 - \left[\frac{\upsilon}{c_\nu} d\xi + c_\nu d\tau \right]^2 \right\}$$

$$= \gamma_\nu^2 \left\{ (d\xi)^2 \left(1 - \frac{\upsilon^2}{c_\nu^2} \right) - c_\nu^2 (d\tau)^2 \left(1 - \frac{\upsilon^2}{c_\nu^2} \right) \right\}$$

$$= \underbrace{\gamma_\nu^2 \left(1 - \frac{\upsilon^2}{c_\nu^2}\right)}_{1} \left\{(d\xi)^2 - c_\nu^2 (d\tau)^2\right\} = (d\xi)^2 - c_\nu^2 (d\tau)^2 .$$

2.

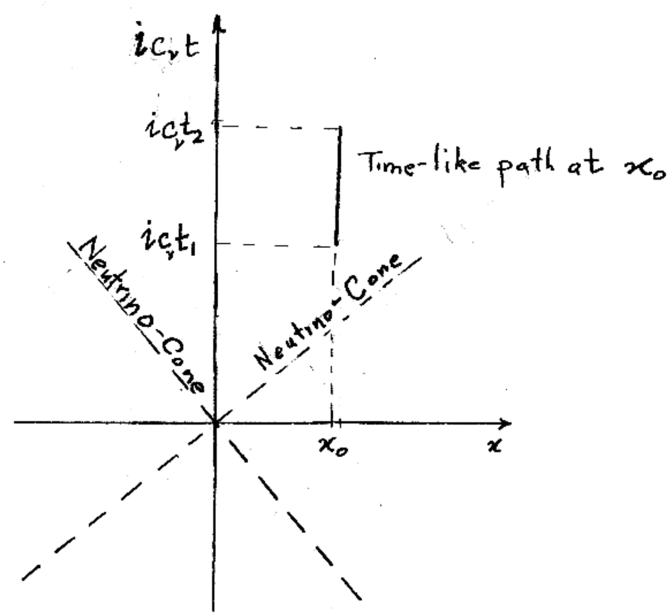
$\upsilon < c_\nu$, and Time-Like Path

If $\underline{\upsilon < c_\nu}$, then

$$-c_\nu^2(dt)^2 + (dx)^2 = \left[-c_\nu^2 + \upsilon^2\right](dt)^2 < 0$$

That is, the infinitesimal distance is negative, and the path is inside the neutrino cone.

Then, there is a Lorentz transformation of the Lab into an inertial frame where both events occur at the same point, at two different times causally ordered.



Time ordering of events on T**ime-like path** is causal:

$v < c_\nu \Rightarrow$ the infinitesimal momentum is positive

$$-\left(\frac{1}{c_\nu} dE\right)^2 + (dp)^2 > 0\,.$$

Since $dp_\mu dp^\mu$ is Lorentz invariant, it remains positive, and there is a Lorentz transformation of the Lab into an inertial frame where only momentum changes along the path.

The arc-lengths $-c_\nu{}^2 t^2 + x^2 + y^2$, and $-c_\nu{}^2 \tau^2 + \xi^2 + \eta^2$ lie on the same two sheet hyperboloid generated by revolving $-c_\nu{}^2 t^2 + x^2 + y^2$ about the ict axis in the $\left(ic_\nu t, x, y\right)$ space.

The arc-lengths $-c_\nu{}^2 t^2 + x^2 + y^2 + z^2$, and $-c_\nu{}^2 \tau^2 + \xi^2 + \eta^2 + \zeta^2$ lie on the same two sheet hyper-surface generated by revolving $-c_\nu{}^2 t^2 + x^2 + y^2 + z^2$ about the $ic_\nu t$ axis in the $\left(ic_\nu t, x, y, z\right)$ space.

The time ordering of two events is the same under any Lorentz transformation to another frame. That is,

2.1 Time-like Path is Causal

3.

Orientation Preserving Lorentz Transformation for $\upsilon < c_\nu$

For

$$\gamma_1 = \gamma_2 \equiv \gamma_\nu,$$

$$\beta_1 = \beta_2 \equiv b = -\gamma_\nu \frac{\upsilon}{c_\nu},$$

we obtained

$$\begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \gamma_\nu \begin{bmatrix} 1 & -\frac{\upsilon}{c_\nu} \\ -\frac{\upsilon}{c_\nu} & 1 \end{bmatrix} \begin{bmatrix} dE \\ c_\nu dp \end{bmatrix}.$$

In the coordinates $(iE_\tau, c_\nu p_\xi)$, we rewrite

$$\begin{bmatrix} idE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \underbrace{\gamma_\nu \begin{bmatrix} 1 & -i\frac{\upsilon}{c_\nu} \\ i\frac{\upsilon}{c_\nu} & 1 \end{bmatrix}}_{L_\nu} \begin{bmatrix} idE \\ c_\nu dp \end{bmatrix}.$$

L_ν is

- Hermitian, because $L_\nu = \overline{L}_\nu^{\,T} = L_\nu^{\dagger}$
- Orthogonal, because its columns are perpendicular
- Rotation, because it is orthogonal

- Proper Rotation, preserving the orientation of the frame, since $\det L_\nu = \gamma_\nu^2 \left(1 - \frac{\upsilon^2}{c_\nu^2}\right) = 1$.

The inverse transformation is

$$L_\nu{}^{-1} = \gamma_\nu \begin{bmatrix} 1 & i\frac{\upsilon}{c_\nu} \\ -i\frac{\upsilon}{c_\nu} & 1 \end{bmatrix} = L_\nu{}^T = L_\nu .$$

Hence L_ν is

- Unitary since $L_\nu L_\nu{}^T = I$
- Normal since $L_\nu L_\nu{}^T = L_\nu{}^T L_\nu$

4.

Orientation Reversing Mapping for $\upsilon < c_\nu$

For

$$\gamma_1 = -\gamma_2 \equiv \gamma_\nu,$$

$$\beta_1 = -\beta_2 \equiv b = -\gamma_\nu \frac{\upsilon}{c_\nu},$$

we obtained

$$\begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \gamma_\nu \begin{bmatrix} 1 & -\frac{\upsilon}{c_\nu} \\ \frac{\upsilon}{c_\nu} & -1 \end{bmatrix} \begin{bmatrix} dE \\ c_\nu dp \end{bmatrix}.$$

In the coordinates $(iE_\tau, c_\nu p_\xi)$, we rewrite

$$\begin{bmatrix} idE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \gamma_\nu \underbrace{\begin{bmatrix} 1 & -i\frac{\upsilon}{c_\nu} \\ -i\frac{\upsilon}{c_\nu} & -1 \end{bmatrix}}_{B} \begin{bmatrix} idE \\ c_\nu dp \end{bmatrix}$$

B is

- Hermitian, because $B = \overline{B}^T = B^\dagger$
- Orthogonal, because its columns are perpendicular
- Rotation, because it is orthogonal
- Improper Rotation, reversing the orientation of the

frame since

$$\det B = \begin{vmatrix} \gamma_\nu & -i\gamma_\nu \dfrac{\upsilon}{c_\nu} \\ -i\gamma_\nu \dfrac{\upsilon}{c_\nu} & -\gamma_\nu \end{vmatrix} = -\gamma_\nu^2 + \gamma_\nu^2 \frac{\upsilon^2}{c_\nu^2} = -\gamma_\nu^2 \left(1 - \frac{\upsilon^2}{c_\nu^2}\right) = -1$$

The inverse transformation is

$$B^{-1} = \begin{bmatrix} \gamma_\nu & -i\gamma_\nu \dfrac{\upsilon}{c} \\ -i\gamma_\nu \dfrac{\upsilon}{c} & -\gamma_\nu \end{bmatrix} = B^T = B .$$

Hence, B is

- Unitary as $BB^T = I$
- Normal as $BB^T = B^T B$

Thus, the analysis of B is similar to that of L.

5.

Lorentz Transformation for $\upsilon < c_\nu$, is Inadequate for $\upsilon > c_\nu$

For $\upsilon > c_\nu$,

$$\frac{1}{\sqrt{1-\frac{\upsilon^2}{c_\nu^2}}} = \frac{-i}{\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}},$$

$$\begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \frac{-i}{\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}} \begin{bmatrix} 1 & -\frac{\upsilon}{c_\nu} \\ -\frac{\upsilon}{c_\nu} & 1 \end{bmatrix} \begin{bmatrix} dE \\ c_\nu dp \end{bmatrix}$$

In the inverse transformation, $-\upsilon$ replaces υ

$$\begin{bmatrix} dE \\ c_\nu dp \end{bmatrix} = \frac{-i}{\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}} \begin{bmatrix} 1 & \frac{\upsilon}{c_\nu} \\ \frac{\upsilon}{c_\nu} & 1 \end{bmatrix} \begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix}.$$

5.1 *The energy transformations*

$$dE = \frac{-i}{\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}}\left(dE_\tau + \upsilon dp_\xi\right)$$

and

$$c_\nu dp = \frac{-i}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}\left(\frac{\upsilon}{c_\nu} dE_\tau + c_\nu dp_\xi\right)$$

are not physical.

For the distance,

$$\begin{bmatrix} d\xi \\ c_\nu d\tau \end{bmatrix} = \frac{-i}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}\begin{bmatrix} 1 & -\frac{\upsilon}{c_\nu} \\ -\frac{\upsilon}{c_\nu} & 1 \end{bmatrix}\begin{bmatrix} dx \\ c_\nu dt \end{bmatrix}.$$

In the inverse transformation, $-\upsilon$ replaces υ,

$$\begin{bmatrix} dx \\ c_\nu dt \end{bmatrix} = \frac{-i}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}\begin{bmatrix} 1 & \frac{\upsilon}{c_\nu} \\ \frac{\upsilon}{c_\nu} & 1 \end{bmatrix}\begin{bmatrix} d\xi \\ c_\nu d\tau \end{bmatrix}.$$

5.2 *The distance transformations*

$$dx = \frac{-i}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}\left(d\xi + \upsilon d\tau\right)$$

and

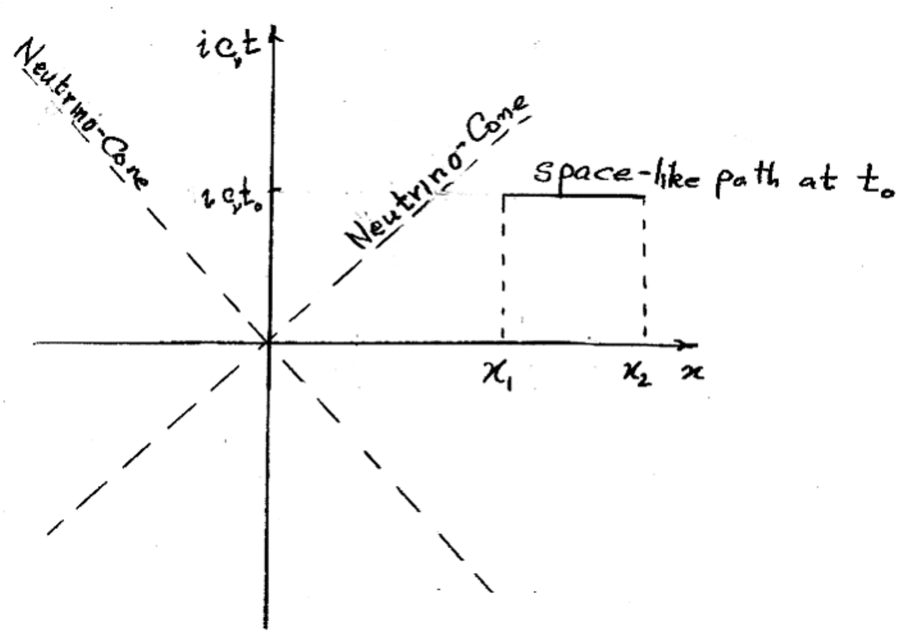
$$c_{\nu} dt = \frac{-i}{\sqrt{\frac{\upsilon^2}{c_{\nu}^2} - 1}} \left(\frac{\upsilon}{c_{\nu}} d\xi + c_{\nu} d\tau \right)$$

are not physical.

6.

Lorentz Transformation and Time Paradox at $v > c_\nu$

If $v > c_\nu$, then $dx_\mu dx^\mu > 0$,
and the path is out of the neutrino cone.
Then, there is a Lorentz Transformation to an inertial frame where the two events occur at the same time, at two different points. Only distance changes along the Path.



Time-ordering of events on **space-like path** depends on the frames, and has $dp_\mu dp^\mu < 0$.

Since $dp_\mu dp^\mu$ is Lorentz invariant, it stays negative, and there is a Lorentz Transformation to an inertial frame where only energy changes along the path.

6.1 *For $v > c_\nu$, transformations 5.1, and 5.2 can change the time ordering of two events, and violate causality.*

In other words, there is a Lorentz Transformation for motion at $v < c_\nu$, given by 5.1, and 5.2, that can transfer an inertial system moving at $v > c_\nu$, where a chicken lays an egg, into an inertial system in which the egg is laid before the chicken is born. Thus, space-like-path may lead to a time paradox.

We seek here a Lorentz-like transformation that when applied to an inertial system moving at $v > c_\nu$, will avoid a time paradox.

7.

Lorentz-like Transformations for $\upsilon > c_\nu$

For motions faster than light, we need Lorentz-like transformations that have physical meaning, and avoid time paradoxes,

We obtain such formula by exchanging υ, and c_ν in the Lorentz transformation formulas. That is, we will use the factor

$$\frac{1}{\sqrt{1-\frac{c_\nu^2}{\upsilon^2}}} = \frac{1}{\frac{c_\nu}{\upsilon}\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}}$$

and the transformation matrix

$$\begin{bmatrix} 1 & -\frac{c_\nu}{\upsilon} \\ -\frac{c_\nu}{\upsilon} & 1 \end{bmatrix}$$

Then, the energy transforms by

7.1
$$\begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \frac{1}{\frac{c_\nu}{\upsilon}\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}} \begin{bmatrix} 1 & -\frac{c_\nu}{\upsilon} \\ -\frac{c_\nu}{\upsilon} & 1 \end{bmatrix} \begin{bmatrix} dE \\ c_\nu dp \end{bmatrix}$$

$$= \frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c_\nu} & -1 \\ -1 & \frac{\upsilon}{c_\nu} \end{bmatrix} \begin{bmatrix} dE \\ c_\nu dp \end{bmatrix}$$

The distance transforms similarly by

7.2
$$\begin{bmatrix} c_\nu d\tau \\ d\xi \end{bmatrix} = \frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c_\nu} & -1 \\ -1 & \frac{\upsilon}{c_\nu} \end{bmatrix} \begin{bmatrix} c_\nu dt \\ dx \end{bmatrix}$$

Replacing υ with $-\upsilon$

7.3
$$\begin{bmatrix} c_\nu dt \\ dx \end{bmatrix} = \frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} \begin{bmatrix} -\frac{\upsilon}{c_\nu} & -1 \\ -1 & -\frac{\upsilon}{c_\nu} \end{bmatrix} \begin{bmatrix} c_\nu d\tau \\ d\xi \end{bmatrix}$$

This Lorentz-like transformation is valid because

7.4 The infinitesimal Distance is Lorentz-like Invariant

Proof:

$$-c_\nu^2(dt)^2 + (dx)^2 = \frac{1}{\frac{\upsilon^2}{c_\nu^2} - 1}\left\{-\left[-d\xi - \frac{\upsilon}{c_\nu}c_\nu d\tau\right]^2 + \left[-\frac{\upsilon}{c_\nu}d\xi - \upsilon d\tau\right]^2\right\}$$

$$= \frac{1}{\frac{\upsilon^2}{c_\nu^2} - 1}\left\{-c_\nu^2(d\tau)^2\left(\frac{\upsilon^2}{c_\nu^2} - 1\right) + (d\xi)^2\left(\frac{\upsilon^2}{c_\nu^2} - 1\right)\right\}$$

$$= \underbrace{\frac{1}{\frac{\upsilon^2}{c_\nu^2} - 1}\left(\frac{\upsilon^2}{c_\nu^2} - 1\right)}_{1}\left\{-c_\nu^2(d\tau)^2 + (d\xi)^2\right\}$$

$$= -c_\nu^2(d\tau)^2 + (d\xi)^2. \square$$

8.

Orientation Preserving for Motion at $\upsilon > c_\nu$

we obtained

$$\begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c_\nu} & -1 \\ -1 & \frac{\upsilon}{c_\nu} \end{bmatrix} \begin{bmatrix} dE \\ c_\nu dp \end{bmatrix}.$$

In the coordinates $(iE_\tau, c_\nu p_\xi)$, we rewrite

$$\begin{bmatrix} idE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \underbrace{\frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c_\nu} & -i \\ i & \frac{\upsilon}{c_\nu} \end{bmatrix}}_{L_{\upsilon > c_\nu}} \begin{bmatrix} idE \\ c_\nu dp \end{bmatrix}.$$

The Lorentz-like Transformation $L_{\upsilon > c_\nu}$ is

- Hermitian, because $L_{\upsilon > c_\nu} = \overline{L}_{\upsilon > c_\nu}{}^T = L_{\upsilon > c_\nu}{}^\dagger$
- Orthogonal, because its columns are perpendicular
- Rotation, because it is orthogonal
- Proper Rotation, preserving the orientation of the

frame, since $\det L_{\upsilon > c_\nu} = \frac{1}{\frac{\upsilon^2}{c_\nu^2} - 1}\left(\frac{\upsilon^2}{c_\nu^2} - 1\right) = 1$.

The inverse transformation is

$$L_{\upsilon > c_\nu}^{-1} = \frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c_\nu} & i \\ -i & \frac{\upsilon}{c_\nu} \end{bmatrix} = L_{\upsilon > c_\nu}^{T}.$$

Hence $L_{\upsilon > c_\nu}$ is

- Unitary since $L_{\upsilon > c_\nu} L_{\upsilon > c_\nu}^{T} = I$
- Normal since $L_{\upsilon > c} L_{\upsilon > c}^{T} = L_{\upsilon > c}^{T} L_{\upsilon > c}$

9.

Motion at Signal Speed $v \approx c_\nu$

For $v \approx c_\nu$,

$$\frac{1}{\sqrt{1 - \frac{v^2}{c_\nu^2}}} \approx \frac{1}{0}.$$

Thus, the transformation does not exist. The particle is the emitted neutrino, and there is no transformation from the particle frame to the Lab frame.

10.

$v = c_{\nu}$, and Neutrino-Like Path

If $v = c_{\nu}$, then

$$dx_{\mu} dx^{\mu} = 0 .$$

and the path is on the neutrino cone.

The neutrino path is causal, and has

$$dp_{\mu} dp^{\mu} = 0 .$$

11.

Energy Transformation detected by neutrino signal

11.1 *For* $\upsilon < c_\nu$,

$$\Delta E_\tau = \frac{1 - \frac{\upsilon}{c_\nu}}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}} \Delta E .$$

For $\upsilon > c_\nu$,

$$\Delta E_\tau = \frac{\frac{\upsilon}{c_\nu} - 1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} \Delta E ,$$

Proof:

For $\upsilon < c_\nu$,

$$\begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}} \begin{bmatrix} 1 & -\frac{\upsilon}{c_\nu} \\ -\frac{\upsilon}{c_\nu} & 1 \end{bmatrix} \begin{bmatrix} dE \\ c_\nu dp \end{bmatrix}$$

$$dE_\tau = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}} \left(dE - \frac{\upsilon}{c_\nu} c_\nu dp \right)$$

$$= \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}\left(dE - \frac{\upsilon}{c_\nu} dE\right)$$

$$= \frac{1 - \frac{\upsilon}{c_\nu}}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}} dE\,.$$

$$\Delta E_\tau = \frac{1 - \frac{\upsilon}{c_\nu}}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}} \Delta E\,.\square$$

For $\upsilon > c_\nu$,

$$\begin{bmatrix} dE_\tau \\ c_\nu dp_\xi \end{bmatrix} = \frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c_\nu} & -1 \\ -1 & \frac{\upsilon}{c_\nu} \end{bmatrix} \begin{bmatrix} dE \\ c_\nu dp \end{bmatrix},$$

$$dE_\tau = \frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}\left(\frac{\upsilon}{c_\nu} dE - c_\nu dp\right)$$

$$= \frac{\frac{\upsilon}{c_\nu} - 1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} dE\,.\square$$

12.

Contraction of a Moving Ruler detected by Neutrino Signal

A ruler moves with velocity υ in the x direction, and is detected by its emission of Neutrinos

The ruler has the Lab coordinates

$$(ic_\nu t, x),$$

and its own frame coordinates

$$(ic_\nu \tau, \xi).$$

12.1 *If* $\upsilon < c_\nu$,

$$\Delta x\Big|_{\Delta t=0} = \Delta\xi\sqrt{1-\frac{\upsilon^2}{c_\nu^2}}.$$

the ruler length appears shorter in the Lab frame by

$$\sqrt{1-\frac{\upsilon^2}{c_\nu^2}}.$$

If $\upsilon > c_\nu$,

$$\Delta x\Big|_{\Delta t=0} = \Delta\xi\sqrt{1-\frac{c_\nu^2}{\upsilon^2}},$$

the ruler length appears shorter in the Lab frame by

$$\sqrt{1-\frac{c_\nu^2}{\upsilon^2}}\,.$$

<u>*Proof*</u>:

<u>$\upsilon < c_\nu$,</u>

The origins of both frames coincide at time $t = 0$. Then,

$$\Delta\xi = \frac{1}{\sqrt{1-\frac{\upsilon^2}{c_\nu^2}}}\left(\Delta x - \upsilon\Delta t\right).$$

In the Lab, at any one time, $\Delta t = 0$, and

$$\Delta x\big|_{\Delta t=0} = \Delta\xi\sqrt{1-\frac{\upsilon^2}{c_\nu^2}}\,.\square$$

<u>$\upsilon > c_\nu$</u>

The origins of both frames coincide at time $t = 0$. Then,

$$\Delta\xi = \frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}}\left(\frac{\upsilon}{c_\nu}\Delta x - c_\nu\Delta t\right).$$

In the Lab, at any one time, $\Delta t = 0$, and

$$\Delta\xi = \frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}}\frac{\upsilon}{c_\nu}\Delta x\big|_{\Delta t=o}$$

$$\Delta x\big|_{\Delta t=0} = \frac{c_\nu}{\upsilon}\Delta\xi\sqrt{\frac{\upsilon^2}{c_\nu^2}-1} = \Delta\xi\sqrt{1-\frac{c_\nu^2}{\upsilon^2}}\,.\square$$

13.

Retardation of a Moving Clock Detected by Neutrino Signals

A clock moves with velocity υ in the x direction. Its time is measured by detected neutrinos.

The clock has the Lab coordinates

$$(ic_\nu t, x),$$

and its own frame coordinates

$$(ic_\nu \tau, \xi).$$

13.1 *For* $\upsilon < c_\nu$, $\Delta t\Big|_{\Delta\xi=0} = \dfrac{\Delta\tau}{\sqrt{1-\dfrac{\upsilon^2}{c_\nu^2}}}$.

clock will be retarded in the Lab by $\dfrac{1}{\sqrt{1-\dfrac{\upsilon^2}{c_\nu^2}}}$.

For $\upsilon > c_\nu$, $\Delta t\Big|_{\Delta\xi=0} = -\dfrac{\Delta\tau}{\sqrt{1-\dfrac{c_\nu^2}{\upsilon^2}}}$.

clock will be retarded in the Lab by

$$\left|\frac{\Delta t\big|_{\Delta\xi=0}}{\Delta\tau}\right| = \frac{1}{\sqrt{1-\dfrac{c_\nu^2}{\upsilon^2}}}.$$

<u>*Proof*</u>: <u>$\upsilon < c_\nu$</u>

The origins of both frames coincide at time $t = 0$. Then,

$$\Delta t = \frac{1}{\sqrt{1-\dfrac{\upsilon^2}{c_\nu^2}}}\left(\Delta\tau + \frac{\upsilon}{c_\nu^2}\Delta\xi\right).$$

In the clock frame, at any one location $\Delta\xi = 0$. Thus, in the Lab,

$$\Delta t\big|_{\Delta\xi=0} = \frac{\Delta\tau}{\sqrt{1-\dfrac{\upsilon^2}{c_\nu^2}}}.\square$$

<u>$\upsilon > c_\nu$</u>

The origins of both frames coincide at time $t = 0$. Then,

$$c_\nu\Delta t = \frac{1}{\sqrt{\dfrac{\upsilon^2}{c_\nu^2}-1}}\left(-\frac{\upsilon}{c_\nu}c_\nu\Delta t - \Delta\xi\right).$$

In the clock frame, at any one location $\Delta\xi = 0$. Thus, in the Lab,

$$\Delta t\Big|_{\Delta\xi=0} = -\frac{\upsilon}{c_\nu}\frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}}\Delta\tau$$

$$= \frac{-1}{\sqrt{1-\frac{c_\nu^2}{\upsilon^2}}}\Delta\tau\,.\square$$

14.

Velocities at $\upsilon < c_\nu$

A particle moves with velocity u_ξ in the frame $(ic_\nu \tau, \xi)$.

The frame moves with velocity $\upsilon < c_\nu$, in the x direction, in the Lab frame $(ic_\nu t, x)$.

The particle emits neutrinos, and is observed by detecting these neutrino signals.

14.1 *The particle velocity in the moving frame is*

$$u_\xi = \frac{u_x - \upsilon}{1 - \frac{u\upsilon}{c_\nu^2}}$$

<u>*Proof*</u>: $u_\xi = \frac{d\xi}{d\tau}$,

$$= \frac{\frac{1}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}\left(dx - \upsilon dt\right)}{\frac{1}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}\left(dt - \frac{\upsilon}{c_\nu^2} dx\right)},$$

$$= \frac{\frac{dx}{dt} - \upsilon}{1 - \frac{\upsilon}{c_\nu^2}\frac{dx}{dt}} = \frac{u_x - \upsilon}{1 - \frac{u\upsilon}{c_\nu^2}}.\square$$

14.2 $\upsilon < c_\nu$ *and* $u_x < c_\nu \Rightarrow u_\xi < c_\nu$

<u>*Proof*</u>:

$$0 < \left(1 - \frac{u_x}{c_\nu}\right)\left(1 + \frac{\upsilon}{c_\nu}\right) = 1 - \frac{u_x \upsilon}{c_\nu^2} - \frac{u_x}{c_\nu} + \frac{\upsilon}{c_\nu}$$

$$\frac{\frac{u_x}{c_\nu} - \frac{\upsilon}{c_\nu}}{1 - \frac{u_x \upsilon}{c_\nu^2}} < 1$$

$$u_\xi = \frac{u_x - \upsilon}{1 - \frac{u_x \upsilon}{c_\nu^2}}$$

$$= \frac{\frac{u_x}{c_\nu} - \frac{\upsilon}{c_\nu}}{1 - \frac{u_x \upsilon}{c_\nu^2}} c_\nu < c_\nu.\square$$

14.3 $\upsilon < c_\nu$ *and* $u_x = c_\nu \Rightarrow u_\xi = c_\nu$.

Proof: $u_\xi = \dfrac{u_x - v}{1 - \dfrac{u_x v}{c_\nu^2}}$

$$= \frac{c_\nu - v}{1 - \dfrac{c_\nu v}{c_\nu^2}}$$

$$= \frac{c_\nu - v}{c_\nu - v} c_\nu = c_\nu . \square$$

14.4 $v < c_\nu$ *and* $u_x > c_\nu \Rightarrow u_\xi > c_\nu$

Proof:

$$0 > \left(1 - \frac{u}{c_\nu}\right)\left(1 + \frac{v}{c_\nu}\right) = 1 - \frac{uv}{c_\nu^2} - \frac{u}{c_\nu} + \frac{v}{c_\nu},$$

$$\frac{\dfrac{u_x}{c_\nu} - \dfrac{v}{c_\nu}}{1 - \dfrac{u_x v}{c_\nu^2}} > 1 .$$

Therefore,

$$u_\xi = \frac{u_x - v}{1 - \dfrac{u_x v}{c_\nu^2}}$$

$$= \frac{\dfrac{u_x}{c_\nu} - \dfrac{\upsilon}{c_\nu}}{1 - \dfrac{u_x \upsilon}{c_\nu^2}} c_\nu > c_\nu . \square$$

14.5 *The particle velocity in the Lab is*

$$u_x = \frac{u_\xi + \upsilon}{1 + \dfrac{u_\xi \upsilon}{c_\nu^2}}$$

<u>*Proof*</u>: Replacing υ with $-\upsilon$

$$u_x = \frac{dx}{dt}$$

$$= \frac{\dfrac{1}{\sqrt{1 - \dfrac{\upsilon^2}{c_\nu^2}}}\left(d\xi + \upsilon d\tau\right)}{\dfrac{1}{\sqrt{1 - \dfrac{\upsilon^2}{c_\nu^2}}}\left(d\tau + \dfrac{\upsilon}{c_\nu^2} d\xi\right)}$$

$$= \frac{\dfrac{d\xi}{d\tau} + \upsilon}{1 + \dfrac{\upsilon}{c_\nu^2}\dfrac{d\xi}{d\tau}}$$

$$= \frac{u_\xi + v}{1 + \frac{u_\xi v}{c_\nu^2}}.\square$$

14.6 $$v < c_\nu \quad and \quad u_\xi < c_\nu \Rightarrow u_x < c_\nu$$

Proof:

$$0 < \left(1 - \frac{u_\xi}{c_\nu}\right)\left(1 - \frac{v}{c_\nu}\right) = 1 + \frac{u_\xi v}{c_\nu^2} - \frac{u_\xi}{c_\nu} - \frac{v}{c_\nu}$$

$$\frac{\frac{u_\xi}{c_\nu} + \frac{v}{c_\nu}}{1 + \frac{u_\xi v}{c_\nu^2}} < 1$$

$$u_x = \frac{u_\xi + v}{1 + \frac{u_\xi v}{c_\nu^2}},$$

$$= \frac{\frac{u_\xi}{c_\nu} + \frac{v}{c_\nu}}{1 + \frac{u_\xi v}{c_\nu^2}} c_\nu < c_\nu.\square$$

14.7 $$v < c_\nu \quad and \quad u_\xi = c_\nu \Rightarrow u_x = c_\nu$$

Proof: $u_x = \dfrac{c_\nu + \upsilon}{1 + \dfrac{c_\nu \upsilon}{c_\nu^2}}$

$$= \frac{c_\nu + \upsilon}{c_\nu + \upsilon} c_\nu = c_\nu$$

14.8 $\upsilon < c_\nu$ *and* $u_\xi > c_\nu \Rightarrow u_x > c_\nu$

Proof:

$$0 > \left(1 - \frac{u_\xi}{c_\nu}\right)\left(1 - \frac{\upsilon}{c_\nu}\right) = 1 + \frac{u_\xi \upsilon}{c_\nu^2} - \frac{u_\xi}{c_\nu} - \frac{\upsilon}{c_\nu},$$

$$\frac{\dfrac{u_\xi}{c_\nu} + \dfrac{\upsilon}{c_\nu}}{1 + \dfrac{u_\xi \upsilon}{c_\nu^2}} > 1.$$

Therefore

$$u_x = \frac{u_\xi + \upsilon}{1 + \dfrac{u_\xi \upsilon}{c_\nu^2}},$$

$$= \frac{\dfrac{u_\xi}{c_\nu} + \dfrac{\upsilon}{c_\nu}}{1 + \dfrac{u_\xi \upsilon}{c_\nu^2}} c_\nu > c_\nu . \square$$

14.9 *For* $\upsilon < c_\nu$

$$u_\xi < c_\nu \Leftrightarrow u_x < c_\nu\,,$$

$$u_x > c_\nu \Leftrightarrow u_\xi > c_\nu\,,$$

$$u_\xi = c_\nu \Leftrightarrow u_x = c_\nu$$

15.

Velocities at $\upsilon > c_\nu$

A particle moves with velocity u_ξ in the frame $(ic_\nu \tau, \xi)$.

The frame moves with velocity $\upsilon > c$, in the x direction, in the Lab frame (ict, x). The particle emits neutrinos, and is observed by detecting these neutrino signals.

15.1 *The particle velocity in the moving frame is*

$$u_\xi = \frac{\upsilon - u_x}{\frac{\upsilon u_x}{c_\nu^2} - 1}$$

<u>*Proof*</u>: $u_\xi = \dfrac{d\xi}{d\tau}$,

$$= \frac{\frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}\left(-dx + \upsilon dt\right)}{\frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}\left(\frac{\upsilon}{c_\nu^2} dx - dt\right)},$$

$$= \frac{-\frac{dx}{dt} + \upsilon}{\frac{\upsilon}{c_\nu^2}\frac{dx}{dt} - 1}$$

$$= \frac{-\frac{dx}{dt} + \upsilon}{\frac{\upsilon}{c_\nu^2}\frac{dx}{dt} - 1}$$

$$= \frac{-u_x + \upsilon}{\frac{\upsilon}{c_\nu^2}u_x - 1}. \square$$

15.2 $\upsilon > c_\nu \quad and \quad u_x < c_\nu \Rightarrow u_\xi > c_\nu$

<u>*Proof*</u>:

$$0 > \left(\frac{u_x}{c_\nu} - 1\right)\left(1 + \frac{\upsilon}{c_\nu}\right) = -1 + \frac{u_x \upsilon}{c_\nu^2} + \frac{u_x}{c_\nu} - \frac{\upsilon}{c_\nu}$$

$$\frac{\frac{\upsilon}{c_\nu} - \frac{u_x}{c_\nu}}{\frac{u_x \upsilon}{c_\nu^2} - 1} > 1$$

$$u_\xi = c_\nu \frac{\frac{\upsilon}{c_\nu} - \frac{u_x}{c_\nu}}{\frac{u_x \upsilon}{c_\nu^2} - 1} > c_\nu . \square$$

15.3 $\upsilon > c_\nu \quad and \quad u_x = c_\nu \Rightarrow u_\xi = c_\nu$.

<u>*Proof*</u>: $u_\xi = c_\nu \dfrac{\dfrac{\upsilon u_x}{c_\nu^2} - 1}{\dfrac{\upsilon}{c_\nu} - \dfrac{u_x}{c_\nu}}$

$$= c_\nu \frac{\dfrac{\upsilon c}{c_\nu^2} - 1}{\dfrac{\upsilon}{c_\nu} - \dfrac{c_\nu}{c_\nu}} = c_\nu . \Box$$

15.4 $\upsilon > c_\nu \quad and \quad u_x > c_\nu \Rightarrow u_\xi < c_\nu$

<u>*Proof*</u>:

$$0 < \left(\frac{u_x}{c_\nu} - 1\right)\left(1 + \frac{\upsilon}{c_\nu}\right) = -1 + \frac{u_x \upsilon}{c_\nu^2} + \frac{u_x}{c_\nu} - \frac{\upsilon}{c_\nu}$$

$$\frac{\dfrac{\upsilon}{c_\nu} - \dfrac{u_x}{c_\nu}}{\dfrac{\upsilon u_x}{c_\nu^2} - 1} < 1$$

$$u_\xi = c_\nu \frac{\dfrac{\upsilon}{c_\nu} - \dfrac{u_x}{c_\nu}}{\dfrac{\upsilon u_x}{c_\nu^2} - 1} < c_\nu . \Box$$

15.5 *The particle velocity in the Lab is* $u_x = c_\nu \dfrac{\dfrac{\upsilon}{c_\nu} + \dfrac{u_\xi}{c_\nu}}{\dfrac{u_\xi \upsilon}{c_\nu^2} + 1}$

<u>*Proof*</u>: Replacing υ with $-\upsilon$

$$u_x = \frac{dx}{dt}$$

$$= \frac{\dfrac{1}{\sqrt{\dfrac{\upsilon^2}{c_\nu^2} - 1}}\left(-d\xi - \upsilon d\tau\right)}{\dfrac{1}{\sqrt{\dfrac{\upsilon^2}{c_\nu^2} - 1}}\left(-\dfrac{\upsilon}{c_\nu^2} d\xi - d\tau\right)}$$

$$= \frac{\dfrac{d\xi}{d\tau} + \upsilon}{\dfrac{\upsilon}{c_\nu^2}\dfrac{d\xi}{d\tau} + 1}$$

$$= c_\nu \frac{\dfrac{u_\xi}{c_\nu} + \dfrac{\upsilon}{c_\nu}}{\dfrac{u_\xi \upsilon}{c_\nu^2} + 1}. \square$$

15.6 $\upsilon > c_\nu$ *and* $u_\xi > c_\nu \Rightarrow u_x < c_\nu$

<u>*Proof*</u>:
$$0 < \left(1 - \frac{u_\xi}{c_\nu}\right)\left(1 - \frac{\upsilon}{c_\nu}\right) = 1 + \frac{u_\xi \upsilon}{c_\nu^2} - \frac{u_\xi}{c_\nu} - \frac{\upsilon}{c_\nu}$$

$$\frac{\dfrac{u_\xi}{c_\nu} + \dfrac{\upsilon}{c_\nu}}{\dfrac{u_\xi \upsilon}{c_\nu^2} + 1} < 1$$

$$u = c_\nu \frac{\dfrac{u_\xi \upsilon}{c^2} + 1}{\dfrac{u_\xi}{c_\nu} + \dfrac{\upsilon}{c_\nu}} < c_\nu . \square$$

15.7 $\upsilon > c_\nu$ *and* $u_\xi = c_\nu \Rightarrow u_x = c_\nu$

<u>*Proof*</u>: $$u_x = c_\nu \frac{\dfrac{c\upsilon}{c_\nu^2} + 1}{\dfrac{c_\nu}{c_\nu} + \dfrac{\upsilon}{c_\nu}} = c_\nu . \square$$

15.8 $\upsilon > c_\nu$ *and* $u_\xi < c_\nu \Rightarrow u_x > c_\nu$

<u>*Proof*</u>:
$$0 > \left(1 - \frac{u_\xi}{c_\nu}\right)\left(1 - \frac{\upsilon}{c_\nu}\right) = 1 + \frac{u_\xi \upsilon}{c_\nu^2} - \frac{u_\xi}{c_\nu} - \frac{\upsilon}{c_\nu},$$

$$\frac{\dfrac{u_\xi}{c_\nu} + \dfrac{\upsilon}{c_\nu}}{\dfrac{u_\xi \upsilon}{c_\nu^2} + 1} > 1$$

$$u_x = c_\nu \frac{\dfrac{u_\xi}{c_\nu} + \dfrac{\upsilon}{c_\nu}}{\dfrac{u_\xi \upsilon}{c_\nu^2} + 1} > c_\nu . \square$$

15.9 *For* $\upsilon > c_\nu$

$$u_\xi < c_\nu \Leftrightarrow u_x > c_\nu ,$$

$$u_x < c_\nu \Leftrightarrow u_\xi > c_\nu ,$$

$$u_\xi = c_\nu \Leftrightarrow u_x = c_\nu$$

16.

Plane-Velocity at $\upsilon < c$ Detected by Neutrino Signal

A particle has velocity (u_ξ, u_η) in the frame $(ic_\nu \tau, \xi, \eta)$.

That frame moves with velocity υ, in the x direction, in the Lab frame $(ic_\nu t, x, y)$.

16.1 *The particle velocities in the moving frame are*

$$u_\xi = \frac{u_x - \upsilon}{1 - \frac{\upsilon u_x}{c_\nu^2}}$$

$$u_\eta = \frac{u_y \sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}{1 - \frac{\upsilon u_x}{c_\nu^2}}$$

<u>*Proof*</u>: $u_\xi = \dfrac{d\xi}{d\tau}$,

$$= \frac{\frac{1}{\sqrt{1-\frac{\upsilon^2}{c_\nu^2}}}\left(dx - \upsilon dt\right)}{\frac{1}{\sqrt{1-\frac{\upsilon^2}{c_\nu^2}}}\left(dt - \frac{\upsilon}{c_\nu^2}dx\right)}$$

$$= \frac{\frac{dx}{dt} - \upsilon}{1 - \frac{\upsilon}{c_\nu^2}\frac{dx}{dt}},$$

$$= \frac{u_x - \upsilon}{1 - \frac{\upsilon u_x}{c_\nu^2}}. \square$$

$$u_\eta = \frac{d\eta}{d\tau}$$

$$= \frac{dy}{\frac{1}{\sqrt{1-\frac{\upsilon^2}{c_\nu^2}}}(dt - \frac{\upsilon}{c_\nu^2}dx)}$$

$$= \frac{\frac{dy}{dt}\sqrt{1-\frac{\upsilon^2}{c_\nu^2}}}{1 - \frac{\upsilon}{c_\nu^2}\frac{dx}{dt}}$$

$$= \frac{u_y \sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}{1 - \frac{\upsilon u_x}{c_\nu^2}}. \square$$

Replacing υ with $-\upsilon$,

16.2 *The particle velocities in the Lab are*

$$u_x = \frac{u_\xi + \upsilon}{1 + \frac{\upsilon u_\xi}{c_\nu^2}}$$

$$u_y = \frac{u_\eta \sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}{1 + \frac{\upsilon u_\xi}{c_\nu^2}}$$

17.

Plane-Velocities at $\upsilon > c_\nu$ Detected by Neutrino Signal

A particle has velocity (u_ξ, u_η) in the frame $(ic_\nu\tau, \xi, \eta)$.

That frame moves with velocity $\upsilon > c_\nu$, in the x direction, in the Lab frame $(ic_\nu t, x, y)$.

17.1 *The particle velocities in the moving frame are*

$$u_\xi = \frac{\upsilon - u_x}{\frac{u_x\upsilon}{c_\nu^2} - 1}$$

$$u_\eta = \frac{u_y}{\left(\frac{u_x\upsilon}{c_\nu^2} - 1\right)}\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}$$

Proof: $u_\xi = \frac{d\xi}{d\tau}$,

$$= \frac{\frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}}\left(-dx+\upsilon dt\right)}{\frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}}\left(\frac{\upsilon}{c_\nu^2}dx-dt\right)}$$

$$= \frac{-\frac{dx}{dt}+\upsilon}{\frac{\upsilon}{c_\nu^2}\frac{dx}{dt}-1}$$

$$= c_\nu \frac{\frac{\upsilon}{c_\nu}-\frac{u_x}{c_\nu}}{\frac{u_x\upsilon}{c_\nu^2}-1}.\square$$

$$u_\eta = \frac{d\eta}{d\tau}$$

$$= \frac{dy}{\frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}}\left(\frac{\upsilon}{c_\nu^2}dx-dt\right)}$$

$$= \frac{\frac{dy}{dt}}{\left(\frac{\upsilon}{c_\nu^2}\frac{dx}{dt}-1\right)}\sqrt{\frac{\upsilon^2}{c_\nu^2}-1}$$

$$= \frac{u_y}{\left(\frac{u_x \upsilon}{c_\nu^2} - 1\right)} \sqrt{\frac{\upsilon^2}{c_\nu^2} - 1} \,. \square$$

Replacing υ with $-\upsilon$,

17.2 *The particle velocities in the Lab are*

$$u_x = \frac{\upsilon + u_\xi}{\frac{u_\xi \upsilon}{c_\nu^2} + 1}$$

$$u_y = -\frac{u_\eta}{\frac{u_\xi \upsilon}{c_\nu^2} + 1} \sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}$$

18.

Plane-Force at $\upsilon < c_\nu$ as Detected by Neutrino Signal

A particle has velocity (u_ξ, u_η), and momentum $\left(i\frac{1}{c_\nu}E_\tau, p_\xi, p_\eta\right)$ in the frame $(ic_\nu\tau, \xi, \eta)$.

That frame moves with velocity υ, in the x direction, in the Lab frame $(ic_\nu t, x, y)$.

The Forces on the particle in the moving frame are

$$F_\xi = \frac{dp_\xi}{d\tau}$$

$$= \frac{\dfrac{1}{\sqrt{1-\dfrac{\upsilon^2}{c_\nu^2}}}\left(-\dfrac{\upsilon}{c_\nu^2}dE + dp_x\right)}{\dfrac{1}{\sqrt{1-\dfrac{\upsilon^2}{c_\nu^2}}}\left(dt - \dfrac{\upsilon}{c_\nu^2}dx\right)}$$

$$= \frac{-\dfrac{\upsilon}{c_\nu^2}\dfrac{dE}{dt} + \dfrac{dp_x}{dt}}{1 - \dfrac{\upsilon}{c_\nu^2}\dfrac{dx}{dt}}$$

$$= \frac{-\frac{\upsilon}{c_\nu^2} F_x u_x + F_x}{1 - \frac{\upsilon}{c_\nu^2} u_x} = F_x$$

$$F_\eta = \frac{dp_\eta}{d\tau}$$

$$= \frac{dp_y}{\frac{1}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}(dt - \frac{\upsilon}{c_\nu^2} dx)}$$

$$= \frac{\frac{dp_y}{dt}\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}{1 - \frac{\upsilon}{c_\nu^2}\frac{dx}{dt}}$$

$$= \frac{F_y\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}{1 - \frac{\upsilon u_x}{c_\nu^2}}$$

If the particle moves only in the y direction, its x-velocity in the Lab is $u_x = v$, and

$$F_{\eta} = \frac{F_y\sqrt{1-\dfrac{\upsilon^2}{c_\nu^2}}}{1-\dfrac{\upsilon v}{c_\nu^2}}$$

$$= \frac{F_y}{\sqrt{1-\dfrac{\upsilon^2}{c_\nu^2}}}.$$

19.

Plane-Force at $\upsilon > c_\nu$ as Detected by Neutrino Signal

A particle has velocity (u_ξ, u_η), and momentum $\left(i\frac{1}{c_\nu}E_\tau, p_\xi, p_\eta\right)$ in the frame $(ic\tau, \xi, \eta)$.

That frame moves with velocity $\upsilon > c$, in the x direction, in the Lab frame (ict, x, y).

19.1 *The Forces on the particle in the moving frame are*

$$F_\xi = \frac{\frac{\upsilon}{c_\nu} - \frac{u_x}{c_\nu}}{\frac{\upsilon u_x}{c_\nu^2} - 1} F_x,$$

$$F_\eta = \sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}\,\frac{1}{\frac{\upsilon u_x}{c_\nu^2} - 1} F_y$$

Proof:

$$F_\xi = \frac{dp_\xi}{d\tau}$$

$$= \frac{\frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}\left(-\frac{1}{c_\nu}dE + \frac{\upsilon}{c_\nu}dp_x\right)}{\frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}\left(\frac{\upsilon}{c_\nu^2}dx - dt\right)}$$

$$= \frac{-\frac{1}{c_\nu}\frac{dE}{dt} + \frac{\upsilon}{c_\nu}\frac{dp_x}{dt}}{\frac{\upsilon}{c_\nu^2}\frac{dx}{dt} - 1}$$

$$= \frac{-\frac{1}{c_\nu}\frac{F_x dx}{dt} + \frac{\upsilon}{c_\nu}F_x}{\frac{u_x \upsilon}{c_\nu^2} - 1}$$

$$= \frac{\frac{\upsilon}{c_\nu} - \frac{u_x}{c_\nu}}{\frac{\upsilon u_x}{c_\nu^2} - 1}F_x . \square$$

$$F_\eta = \frac{dp_\eta}{d\tau}$$

$$= \frac{dp_y}{\frac{1}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}\left(\frac{\upsilon}{c_\nu^2}dx - dt\right)}$$

$$= \frac{\frac{dp_y}{dt}}{\frac{\upsilon}{c_\nu^2}\frac{dx}{dt} - 1}\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}$$

$$= \sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}\,\frac{1}{\frac{\upsilon u_x}{c_\nu^2} - 1} F_y . \square$$

If the particle moves only in the y direction, its x-velocity in the Lab is $u_x = v$, and

$$F_\eta = \sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}\,\frac{1}{\frac{\upsilon^2}{c_\nu^2} - 1} F_y .$$

$$= \frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} F_y$$

II.

MASS CONVERSION into NEUTRINOS' ENERGY

20.

Mass Conversion into Neutrinos' Energy at $v \ll c_{\nu}$

20.1 *At* $v \ll c$, *the mass* m *converts into Photons' energy*

$$E = mc^2$$

<u>*Proof*</u>:

Following [Einstein1], and [Einstein2],

A body at rest in the (x, y, z) system, has the energy

$$E_0.$$

In the (ξ, η, ζ) system, which is moving along the x axis with velocity $v \ll c_{\nu}$, the body has the energy

$$H_0.$$

Hence, the kinetic energy of the body is

$$T_0 = H_0 - E_0$$

Let the body radiate light energy $\frac{1}{2}L$ in the direction of x, and $\frac{1}{2}L$ in the opposite direction. Then, it has energy E_1, in the (x, y, z) system, and energy H_1 in the (ξ, η, ζ) system.

Hence, the kinetic energy of the body after radiation is

$$T_1 = H_1 - E_1 .$$

By energy conservation in the(x, y, z) system,

$$E_0 = E_1 + \tfrac{1}{2}L + \tfrac{1}{2}L ,$$

$$= E_1 + L$$

By energy conservation in the (ξ, η, ζ) system,

$$H_0 = H_1 + \tfrac{1}{2}L \frac{1 - \frac{\upsilon}{c}}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} + \tfrac{1}{2}L \frac{1 + \frac{\upsilon}{c}}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}$$

$$= H_1 + L \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}$$

The change in the kinetic energy is

$$T_0 - T_1 = (H_0 - E_0) - (H_1 - E_1)$$

$$= (H_0 - H_1) - (E_0 - E_1)$$

$$= L\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - L$$

Assuming $v / c \ll 1$,

$$= L\left[1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{1}{2}\frac{3}{2}\left(\frac{v^2}{c^2}\right)^2 + \frac{1}{2}\frac{3}{2}\frac{5}{2}\left(\frac{v^2}{c^2}\right)^3 + ...\right] - L$$

$$= \frac{1}{2}\frac{L}{c^2}v^2 + \frac{1}{2}\frac{3}{2}L\left(\frac{v}{c}\right)^4 + \frac{1}{2}\frac{3}{2}\frac{5}{2}L\left(\frac{v}{c}\right)^6 + ...$$

Assuming $v / c \ll 1$, and keeping terms up to order 2,

$$\approx \frac{1}{2}\frac{L}{c^2}v^2$$

The change in kinetic energy is due to the loss of mass that is radiated as light. That is,

$$T_0 - T_1 = \tfrac{1}{2}(\Delta m)v^2.$$

Therefore,

$$\frac{1}{2}(\Delta m)v^2 \approx \frac{1}{2}\frac{L}{c^2}v^2,$$

$$(\Delta m)c^2 \approx L.$$

20.2 Each Radiation has its Mass-Energy Equivalence

Einstein concludes with

“If the body releases radiation energy, its mass

decreases by $\frac{L}{c^2}$ *"*

This is true only for electromagnetic radiation that propagates at light speed c. For neutrino radiation, its speed replaces the light speed.

Einstein proceeds with

> *"the fact that the energy withdrawn from the body becomes energy of radiation rather than some other kind of energy makes no difference"*

In fact, each energy has its own equivalence formula:

For photon radiation that propagates at speed c, the radiated mass Δm is equivalent to photon energy $(\Delta m)c^2$.

For neutrino radiation that propagates at speed c_ν, we show next that the mass Δm converts into neutrino energy $(\Delta m)c_\nu^2$.

20.3 Mass Conversion into Neutrinos' Energy

If the body above radiates neutrino energy $\frac{1}{2}L$ in the direction of x, and $\frac{1}{2}L$ in the opposite direction. Then,

by energy conservation in the (ξ, η, ζ) system,

$$H_0 = H_1 + \tfrac{1}{2}L\frac{1-\dfrac{\upsilon}{c_\nu}}{\sqrt{1-\dfrac{\upsilon^2}{c_\nu^2}}} + \tfrac{1}{2}L\frac{1+\dfrac{\upsilon}{c_\nu}}{\sqrt{1-\dfrac{\upsilon^2}{c_\nu^2}}}$$

$$= H_1 + L\frac{1}{\sqrt{1-\dfrac{\upsilon^2}{c_\nu}}}$$

The change in the kinetic energy is

$$T_0 - T_1 = L\left(1-\frac{\upsilon^2}{c_\nu^2}\right)^{-\frac{1}{2}} - L$$

Assuming $\upsilon \ll c_\nu$,

$$= L\left[1+\frac{1}{2}\frac{\upsilon^2}{c_\nu^2}+\frac{1}{2}\frac{3}{2}\left(\frac{\upsilon^2}{c_\nu^2}\right)^2+\frac{1}{2}\frac{3}{2}\frac{5}{2}\left(\frac{\upsilon^2}{c_\nu^2}\right)^3+...\right]-L$$

Assuming $\upsilon \ll c_\nu$, and keeping terms up to order 2,

$$\approx \frac{1}{2}\frac{L}{c_\nu^2}\upsilon^2$$

The change in kinetic energy is due to the loss of mass that is radiated as neutrinos. That is,

$$T_0 - T_1 = \tfrac{1}{2}(\Delta m)\upsilon^2.$$

Therefore,

$$\frac{1}{2}(\Delta m)v^2 \approx \frac{1}{2}\frac{L}{c_\nu^2}v^2,$$

$$(\Delta m)c_\nu^2 \approx L.$$

21.

Mass Moving at $\upsilon < c_\nu$

The frame $(ic_\nu \tau, \xi, \eta)$ is aligned with the Lab frame $(ic_\nu t, x, y)$, and moves at speed $\upsilon < c_\nu$ along the x axis.

21.1 *The mass measured in the Lab, by the detected neutrinos is*

$$m = \frac{m_\xi}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}$$

Proof: A mass m_ξ, moving in the y direction with velocity

$$u_\eta$$

collides with a mass m_ξ moving in the $-y$ direction with

$$-u_\eta .$$

In the Lab, the second mass has mass m, and velocities

$$\underbrace{u_x}_{0} = \frac{\upsilon + u_\xi}{\frac{u_\xi \upsilon}{c_\nu^2} + 1} \quad \Rightarrow u_\xi = -\upsilon$$

$$u_y = \frac{-u_\eta}{1 + \frac{\upsilon u_\xi}{c_\nu^2}} \sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}\,,$$

$$= \frac{-u_\eta}{1 + \frac{\upsilon(-\upsilon)}{c_\nu^2}} \sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}\,,$$

$$= \frac{-u_\eta}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}$$

After the collision,

$$u_\eta \to -U_\eta.$$

Hence,

$$\frac{-u_\eta}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}} \to \frac{U_\eta}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}.$$

By momentum conservation in the y direction,

$$m_\xi u_\eta + m\frac{-u_\eta}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}} = m_\xi(-U_\eta) + m\frac{U_\eta}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}.$$

Therefore, the mass measured in the Lab is

$$m = \frac{m_\xi}{\sqrt{1 - \frac{v^2}{c_\nu^2}}}. \square$$

21.2 *If a particle moves with velocity* $u < c_\nu$ *in the Lab, and if* m_0 *is the rest mass of the particle in its frame.*

Then, the particle's mass in the Lab is

$$m(u) = \frac{m_0}{\sqrt{1 - \frac{u^2}{c_\nu^2}}}.$$

The particle will be heavier in the Lab by

$$\frac{1}{\sqrt{1 - \frac{u^2}{c_\nu^2}}}.$$

22.

Mass Moving at $\upsilon > c_\nu$

The frame $(ic_\nu\tau, \xi, \eta)$ is aligned with the Lab frame $(ic_\nu t, x, y)$, and moves at speed $\upsilon < c_\nu$ along the x axis.

22.1 *A mass* m_ξ *in* $(ic_\nu\tau, \xi, \eta)$, *measures in the Lab as*

$$m = \frac{m_\xi}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}$$

<u>*Proof*</u>: A mass m_ξ, moving in the y direction with velocity

$$u_\eta$$

collides with a mass m_ξ moving in the $-y$ direction with

$$-u_\eta .$$

In the Lab, the second mass has mass m_x, and velocities

$$\underbrace{u_x}_{0} = \frac{\upsilon + u_\xi}{\frac{u_\xi \upsilon}{c_\nu^2} + 1} \Rightarrow u_\xi = -\upsilon$$

$$u_y = -\frac{-u_\eta}{\frac{u_\xi \upsilon}{c_\nu^2} + 1}\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}$$

$$= \frac{u_\eta}{\frac{(-\upsilon)\upsilon}{c_\nu^2} + 1}\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1},$$

$$= \frac{-u_\eta}{\frac{\upsilon^2}{c_\nu^2} - 1}\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1},$$

$$= \frac{-u_\eta}{\sqrt{1 - \frac{\upsilon^2}{c_\nu^2}}}$$

After the collision,

$$u_\eta \to -U_\eta.$$

Hence,

$$\frac{-u_\eta}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} \to \frac{U_\eta}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}.$$

By momentum conservation in the y direction,

$$m_\xi u_\eta + m\frac{-u_\eta}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}} = m_\xi(-U_\eta) + m\frac{U_\eta}{\sqrt{\frac{\upsilon^2}{c_\nu^2} - 1}}.$$

Therefore, the mass measured in the Lab is

$$m = \frac{m_\xi}{\sqrt{\frac{v^2}{c_\nu^2} - 1}}.\square$$

22.2 *If a particle moves with velocity* $u > c_\nu$ *in the Lab, and if* m_0 *is the rest mass of the particle in its frame.*

Then, the particle's mass in the Lab is

$$m(u) = \frac{m_0}{\sqrt{\frac{u^2}{c_\nu^2} - 1}}.$$

The particle will be heavier in the Lab by

$$\frac{1}{\sqrt{\frac{u^2}{c_\nu^2} - 1}}.$$

23.

Neutrino's Energy

23.1 The Neutrino's Mass is the same in all frames

Proof: For $u = c_\nu$, the particles are neutrinos.

If they bounce at the collision and remain neutrinos, their momentum conservation is

$$m_\xi c_\nu + m(-c_\nu) = m_\xi(-c_\nu) + m(c_\nu),$$

$$m_{\text{neutrino}} = (m_\xi)_{\text{neutrino}}.$$

Thus, the neutrino's mass is the same in all frames.

23.2 $$\boxed{E_{\text{neutrino}} = m_{\text{neutrino}} c_\nu^2}$$

Proof:

$$m_{\text{neutrino}} = \frac{p_{\text{neutrino}}}{c_\nu}$$

$$= \frac{\frac{1}{c_\nu} E_{\text{neutrino}}}{c_\nu}$$

$$= \frac{E_{\text{neutrino}}}{c_\nu^2}.$$

Thus,

$$E_{\text{neutrino}} = m_{\text{neutrino}} c_\nu^2. \square$$

23.3 *An aggregate of Neutrinos has the energy*

$$\boxed{E_{\text{neutrinos}} = \sum m_{\text{neutrino}} c_\nu^2}$$

24.

Mass Conversion into Neutrinos' Energy for $u < c_\nu$

24.1 *At speed* $u < c_\nu$, *the mass* $m(u) = \dfrac{1}{\sqrt{1 - \dfrac{u^2}{c_\nu^2}}} m_0$

converts into the Neutrino Energy $E_\nu(u) = \dfrac{1}{\sqrt{1 - \dfrac{u^2}{c_\nu^2}}} m_0 c_\nu^2$.

Hence, $m(u_2) - m(u_1)$ *converts into* $\left[m(u_2) - m(u_1)\right] c_\nu^2$.

<u>*Proof*</u>:

$$dm(u) = m_0 d\left(1 - \frac{u^2}{c_\nu^2}\right)^{-\frac{1}{2}}$$

$$= m_0(-\tfrac{1}{2})\left(1 - \frac{u^2}{c_\nu^2}\right)^{-\frac{3}{2}} \left(-\frac{1}{c_\nu^2}\right) d(u^2)$$

$$= \tfrac{1}{2} \frac{1}{c_\nu^2 - u^2} \underbrace{m_0 \left(1 - \frac{u^2}{c_\nu^2}\right)^{-\frac{1}{2}}}_{m(u)} d(u^2).$$

Hence,

$$(c_\nu^2 - u^2)dm(u) = \tfrac{1}{2} m(u) d(u^2).$$

Then,

$$\begin{aligned} dE(u) &= \frac{d\vec{p}(u)}{dt} \cdot d\vec{x} \\ &= d\vec{p}(u) \cdot \vec{u} \\ &= d\left(m(u)\vec{u} \right) \cdot \vec{u} \\ &= dm(u) \underbrace{(\vec{u} \cdot \vec{u})}_{u^2} + \underbrace{m(u)\vec{u} \cdot d\vec{u}}_{\frac{1}{2} m(u) d(\vec{u} \cdot \vec{u})} \\ &= u^2 dm(u) + \underbrace{\tfrac{1}{2} m(u) d(u^2)}_{(c_\nu^2 - u^2) dm(u)} \\ &= u^2 dm(u) + (c_\nu^2 - u^2) dm(u) \\ &= c_\nu^2 dm(u). \end{aligned}$$

Assuming that c_ν is the average of neutrinos speeds, we may integrate

$$\underbrace{E(u) - E(0)}_{\text{kinetic Energy T}} = c_\nu^2 \int_{\mu=0}^{\mu=u} dm(\mu) = [\underbrace{m(u)}_{m_0 \frac{1}{\sqrt{1-\frac{u^2}{c_\nu^2}}}} - m_0] c_\nu^2,$$

$$E(u) = \frac{1}{\sqrt{1 - \dfrac{u^2}{c_\nu^2}}} m_0 c_\nu^2. \square$$

24.2 *For* $u \ll c_\nu$,

$$E(u) = m_0 c_\nu^2 + \underbrace{\frac{1}{2} m_0 u^2 + \frac{1}{2}\frac{3}{2}\frac{m_0}{c_\nu^2} u^4 + \frac{1}{2}\frac{3}{2}\frac{5}{2}\frac{m_0}{c_\nu^4} u^6 + ...}_{\text{T=Kinetic Energy}}$$

$$\approx m_0 c_\nu^2 + \underbrace{\frac{1}{2} m_0 u^2}_{\text{T=Kinetic Energy}}$$

<u>*Proof*</u>:

$$E(u) = m_0 c_\nu^2 \left(1 - \frac{u^2}{c_\nu^2}\right)^{-\frac{1}{2}}$$

For $u \ll c_\nu$,

$$= m_0 c_\nu^2 \left(1 + \frac{1}{2}\frac{u^2}{c_\nu^2} + \frac{1}{2}\frac{3}{2}\left(\frac{u^2}{c_\nu^2}\right)^2 + \frac{1}{2}\frac{3}{2}\frac{5}{2}\left(\frac{u^2}{c_\nu^2}\right)^3 + ...\right)$$

$$= m_0 c_\nu^2 + \underbrace{\frac{1}{2} m_0 u^2 + \frac{1}{2}\frac{3}{2}\frac{m_0}{c_\nu^2} u^4 + \frac{1}{2}\frac{3}{2}\frac{5}{2}\frac{m_0}{c_\nu^4} u^6 + ...}_{\text{T=Kinetic Energy}}$$

$$\approx m_0 c_\nu^2 + \underbrace{\frac{1}{2} m_0 u^2}_{\text{T=Kinetic Energy}} \quad .\square$$

25.

Mass Conversion into Neutrinos' Energy for $u > c_\nu$

25.1 *At speed* $u > c_\nu$, *the mass* $m(u) = \dfrac{1}{\sqrt{\dfrac{u^2}{c_\nu^2} - 1}} m_0$

converts into the Neutrino Energy $E_\nu(u) = \dfrac{1}{\sqrt{\dfrac{u^2}{c_\nu^2} - 1}} m_0 c_\nu^2$.

Hence, $m(u_2) - m(u_1)$ *converts into* $\left[m(u_2) - m(u_1)\right] c_\nu^2$.

Proof:

$$dm(u) = m_0 d\left(\frac{u^2}{c_\nu^2} - 1\right)^{-\frac{1}{2}}$$

$$= m_0(-\tfrac{1}{2})\left(\frac{u^2}{c_\nu^2} - 1\right)^{-\frac{3}{2}}\left(\frac{1}{c_\nu^2}\right) d(u^2)$$

$$= \frac{1}{2}\frac{1}{c_\nu^2 - u^2}\underbrace{m_0\left(\frac{u^2}{c_\nu^2} - 1\right)^{-\frac{1}{2}}}_{m(u)} d(u^2).$$

Hence,

$$(c_\nu^2 - u^2)dm(u) = \tfrac{1}{2}m(u)d(u^2).$$

Then,

$$\begin{aligned}
dE(u) &= \frac{d\vec{p}(u)}{dt}\cdot d\vec{x} \\
&= d\vec{p}(u)\cdot\vec{u} \\
&= d\Big(m(u)\vec{u}\Big)\cdot\vec{u} \\
&= dm(u)\underbrace{(\vec{u}\cdot\vec{u})}_{u^2} + \underbrace{m(u)\vec{u}\cdot d\vec{u}}_{\frac{1}{2}m(u)d(\vec{u}\cdot\vec{u})} \\
&= u^2dm(u) + \underbrace{\tfrac{1}{2}m(u)d(u^2)}_{(c_\nu^2-u^2)dm(u)} \\
&= u^2dm(u) + (c_\nu^2 - u^2)dm(u) \\
&= c_\nu^2dm(u).
\end{aligned}$$

Integrating,

$$\begin{aligned}
\underbrace{E(u)-E(0)}_{\text{kinetic Energy T}} &= c_\nu^2\int_{\mu=0}^{\mu=u} dm(\mu), \\
&= [m(u)-m_0]c_\nu^2 \\
&= \left(\frac{m_0}{\sqrt{\frac{u^2}{c_\nu^2}-1}} - m_0\right)c_\nu^2
\end{aligned}$$

$$E(u) = \frac{m_0}{\sqrt{\frac{u^2}{c_\nu^2} - 1}} c_\nu^2 . \square$$

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