A Hypothesis that Dark Matter is Regular Matter, Invisible, since it Moves Faster Than Light

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Abstract Cherenkov Radiation proves that matter can move Faster than Light. In particular, Faster Than Light neutrinos are detected by the Cherenkov radiation that they emit in water.

A review of Relativity suggests that it has no preference for speeds slower than light over speeds Faster Than Light.

Speeds Faster than Light are as probable as speeds Slower than Light, and lead to no singularity in the Lorentz Transformation, and to no contradiction in Newtonian or Lorentzian Mechanics.

Motion Faster Than Light enables our Hypothesis that Dark Matter is regular matter, invisible since it moves away from us Faster than Light.

To be Dark, matter need not have an inconceivable

structure. It only needs to move away from us faster than light.

Motion Faster Than Light allows Non-Causality. But the rejection of the Causality Hypothesis need not allow Time Paradoxes. The only Time paradoxes we know of are fictional.

The disappearance of Unidentified Objects as they fly away from the observers, may be the result of crossing the speed of light.

If dark matter is regular matter that travels faster the light, then, beyond the visible universe lies a huge, invisible universe, expanding at speeds faster than light into the void.

While such universe may be finite, Its expansion rules out the possibility that it is bounded.

In a Universe where light speed is the maximal speed, a traveler going in a straight line will end at his starting point.

But in a Universe where speeds Faster than light are allowed, there is a lot of travel space.

Then, going in a straight line, at speeds faster than light, will not get the traveler back home. Most likely, he will not be seen again till the universe will start its contraction towards a total collapse.

Keywords Special Relativity, Lorentz Transformation, Faster Than Light, Dark Matter, Cherenkov Radiation, Gravitation

Physics & Astronomy Classification Scheme: 03.30.+p; 95.35.+d;04;

Contents

Faster Than Light, and Dark Matter

0.1 Cherenkov Radiation

Cherenkov Radiation indicates that motion can be Faster Than Light. In particular, Faster Than Light Neutrinos are detected by the Cherenkov Radiation that they emit in water.

The interaction of Faster Than Light neutrinos with nucleus of atoms in water molecules generates bursts of Cherenkov Radiation that are counted by photo-multiplier tubes that surround the water tank.

Photon's speed decreases by third in water, and particles that can move in water faster than photons, can move in the air faster than photons.

Particles Faster than Light in the Air are invisible to us, but in Matter they may move slower than photons in the air, and become visible.

electrically charged particles leave behind them an optical wake of Cherenkov Radiation: A blue glow of radiation on the verge of becoming invisible.

That optical wake is similar to the sonic shock wave created by a jet plane faster than Sound.

0.2 The Lorentz Transformation at almost light speed

Light speed is perceived as a barrier because of the singularity that appears in the Lorentz Transformation at $v = c$. At speed v that approaches light speed c, the Lorentz factor

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

is very large. Then, the mass observed in the Lab is very large, and an observed force results in very small acceleration, and inability to accelerate, and reach light speed, what is more, cross it.

But in the particle's frame, the mass remains the same, and light speed is not a barrier.

0.3 The Lorentz Transformation at light speed

At $v = c$, the Lorentz factor is singular, and does not exist.

Then, the speed, and mass are undefined to the observer, but they are well defined in the particle's frame.

The particle is at the edge of our visible Universe, and we are at the edge of its visible Universe.

0.4 Lorentz Transformation for Invisible Matter

At speed faster than light speed, $v > c$, the Lorentz factor is

$$\frac{-i}{\sqrt{\frac{v^2}{c^2} - 1}}.$$

Since light speed is the largest that we can observe in the Lab, the particle becomes invisible to us in the Lab. But in the particle's frame, the speed of light is not a physical barrier, it is crossed, and the particle moves faster than light.

Then, $i = \sqrt{-1}$ in the Lorentz factor means that light from the particle does not reach us.

The particle's imaginary speed and mass that appear in the Lab mean physical invisibility.

At the same time, the signal from us does not reach the Faster Than Light particle, and we become invisible to it. To the particle, we become invisible matter with imaginary speed, and imaginary mass.

We hypothesize that the Unobserved Matter in the Universe, referred to as Dark Matter, is invisible to us not because it has a different structure, but because it travels Faster Than Light.

It is beyond the edge of our visible Universe, and we are beyond the edge of its visible Universe. Then, we are Dark Matter to our invisible Universe.

0.5 Time Paradox, and Faster Than Light

Causality is violated at speeds faster than light. Non-Causality may lead, in theory, to Time paradoxes, and speculations about Time Paradoxes are used to rule out motions Faster than Light.

For example, the speculation that you may travel to the past, and infect your father there with a flu that kills him before you were born.

However, what is the probability that you will find your father in the past?

That chance may be infinitesimal, and Time Paradox may be fictional.

It is likely that Time Paradoxes take place only in Sci-Fi movies.

Since Lorentz Transformation allows speeds Faster Than Light, and Cherenkov Radiation is the result of motion Faster Than Light, Causality should be allowed to be violated, without creating Time Paradoxes.

Causality holds at speeds smaller than Light.

At speeds Faster Than Light, evidenced by Cherenkov Radiation, Causality need not hold. Then, a belief in speculative Time Paradoxes is not enough to hold on to Causality. There is no Evidence that supports the Causality Postulate, at speeds Faster Than Light. And to date, Time Paradoxes seem to be unlikely, and not one Time Paradox has ever been reported

0.6 The Hypothesis that the Invisible Dark Matter Moves Faster Than Light

In the following, we establish that Relativity Theory has no preference for speeds slower than light over speeds Faster Than Light.

We show that speeds Faster than Light are as probable as speeds Slower than Light, and that it leads to no singularity in the Lorentz Transformation, and to no contradiction in Newtonian or Lorentzian Mechanics.

Motion Faster Than Light enables our Hypothesis that Dark Matter is regular matter, invisible since it moves away from us Faster than Light.

To be Dark, matter need not have inconceivable structure. It only needs to move away from us faster than light.

To that Faster Than Light matter, we are Faster Than

Light. Hence, invisible, Dark Matter.

Motion Faster Than Light allows Non-Causality. But the rejection of the Causality Hypothesis need not allow Time Paradoxes. The only Time paradoxes we know of, are fictional.

The disappearance of Unidentified Objects as they fly away from their observers, may be the result of their crossing the speed of light with respect to their observers.

0.7 Beyond the Boundary of the Visible Universe

If dark matter is regular matter that travels faster the light, then, beyond the visible universe lies a huge, invisible universe, expanding at speeds faster than light into the void. While such universe may be finite, Its expansion rules out the possibility that it is bounded.

In a Universe where light speed is the maximal speed, a traveler going in a straight line will end at his starting point. But in a Universe where speeds Faster than light are allowed, there is a lot of travel space.

Then, going in a straight line, at speeds faster than light, will not get the traveler back home. Most likely, he will not be seen again till the universe will start its contraction towards a total collapse.

1.

Lorentz Transformation

A particle moves in the x direction, and emits a photon in the x direction when it passes through the Lab's origin.

In the Lab, the particle moves with speed v, and is represented

by the event coordinates

$$\left(ict, x\right),$$

or by the momentum coordinates

$$\left(\tfrac{i}{c}E, p\right),$$

where c is the speed of light.

In the particle's frame, the event coordinates are

$$\left(ic\tau, \xi\right),$$

and the momentum coordinates are

$$\left(\tfrac{i}{c}E_\tau, p_\xi\right).$$

Michelson experiments confirmed that the speed of light is the same in all uniformly moving reference frames.

In the Lab frame, the photon speed is

$$c = \frac{dx}{dt} = \frac{\frac{dp}{dt}dx}{dp} = \frac{Fdx}{dp} = \frac{dE}{dp} \quad \Rightarrow cdp = dE$$

In the particle frame,

$$c = \frac{d\xi}{d\tau} = \frac{\frac{dp_\xi}{d\tau} d\xi}{dp_\xi} = \frac{F_\xi d\xi}{dp_\xi} = \frac{dE_\tau}{dp_\xi} \quad \Rightarrow cdp_\xi = dE_\tau$$

Therefore,

$$(dE_\tau)^2 - (cdp_\xi)^2 = 0 = (dE)^2 - (cdp)^2$$

Assuming that to first order, the infinitesimals transform by

$$\begin{bmatrix} idE_\tau \\ cdp_\xi \end{bmatrix} = \begin{bmatrix} \gamma_1 & \beta_1 \\ \beta_2 & \gamma_2 \end{bmatrix} \begin{bmatrix} idE \\ cdp \end{bmatrix},$$

we have

$$(dE_\tau)^2 - (cdp_\xi)^2 =$$

$$(\gamma_1 dE)^2 - 2i\gamma_1\beta_1 cdEdp - (\beta_1 cdp)^2 + (\beta_2 dE)^2 - 2i\beta_2\gamma_2 cdEdp - (\gamma_2 cdp)^2$$

Equating Coefficients,

$$\gamma_1^2 + \beta_2^2 = 1,$$

$$\beta_1^2 + \gamma_2^2 = 1,$$

$$\gamma_1\beta_1 = -\beta_2\gamma_2.$$

We obtain

$$\gamma_1 = \gamma_2 \equiv \gamma,$$

$$\beta_1 = -\beta_2 \equiv b,$$

or

$$\gamma_1 = -\gamma_2 \equiv \gamma,$$

$$\beta_1 = \beta_2 \equiv b,$$

so that in either case

$$\gamma^2 + b^2 = 1.$$

Now, for $\delta E_\tau = 0$,

$$i\gamma\delta E + bc\delta p = 0,$$

$$i\frac{bc}{\gamma} = \frac{\delta E}{\delta p} = \frac{\delta x}{\delta t} = v,$$

$$b = -i\gamma\tfrac{v}{c}$$

Substituting in $\gamma^2 + b^2 = 1$,

$$\gamma^2 - \gamma^2\tfrac{v^2}{c^2} = 1,$$

$$\gamma^2(1 - \tfrac{v^2}{c^2}) = 1,$$

For $\underline{v < c}$,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

For $\underline{v > c}$,

$$\gamma = \frac{-i}{\sqrt{\frac{v^2}{c^2} - 1}}.$$

For $\underline{v = c}$,

$$b = -i\gamma$$

but substituting in $\gamma^2 + b^2 = 1$,

$$\gamma^2 - \gamma^2 = 1,$$

This contradiction means that for $v = c$, the transformation does not exist.

2.

Orientation Preserving Mapping

For $\underline{\gamma_1 = \gamma_2 \equiv \gamma}$,and $\underline{\beta_1 = -\beta_2 \equiv b}$,the transformation is

$$\begin{bmatrix} idE_\tau \\ cdp_\xi \end{bmatrix} = \begin{bmatrix} \gamma & -i\gamma\frac{v}{c} \\ i\gamma\frac{v}{c} & \gamma \end{bmatrix}\begin{bmatrix} idE \\ cdp \end{bmatrix}.$$

The Matrix of the transformation

$$L = \begin{bmatrix} \gamma & -i\gamma\frac{v}{c} \\ i\gamma\frac{v}{c} & \gamma \end{bmatrix},$$

is

- Hermitian, because $L = \overline{L}^T = L^\dagger$
- Orthogonal, because its columns are perpendicular
- Rotation, because it is orthogonal
- Proper Rotation, preserving the orientation of the frame since $\det A = \gamma^2(1 - \frac{v^2}{c^2}) = 1$.

The Inverse Transformation has the Matrix

$$L^{-1} = \begin{bmatrix} \gamma & i\gamma\frac{v}{c} \\ -i\gamma\frac{v}{c} & \gamma \end{bmatrix} = L^T$$

Hence, L is

- Unitary as $LL^T = I$
- Normal as $LL^T = L^T L$

For $\underline{v < c}$,

$$L = \begin{bmatrix} \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}} & -i\dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\dfrac{v}{c} \\ i\dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\dfrac{v}{c} & \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \end{bmatrix}$$

For $\underline{v > c}$,

$$L = \begin{bmatrix} \dfrac{-i}{\sqrt{\frac{v^2}{c^2} - 1}} & -i\dfrac{-i}{\sqrt{\frac{v^2}{c^2} - 1}}\dfrac{v}{c} \\ i\dfrac{-i}{\sqrt{\frac{v^2}{c^2} - 1}}\dfrac{v}{c} & \dfrac{-i}{\sqrt{\frac{v^2}{c^2} - 1}} \end{bmatrix} = \begin{bmatrix} \dfrac{-i}{\sqrt{\frac{v^2}{c^2} - 1}} & \dfrac{-1}{\sqrt{\frac{v^2}{c^2} - 1}}\dfrac{v}{c} \\ \dfrac{1}{\sqrt{\frac{v^2}{c^2} - 1}}\dfrac{v}{c} & \dfrac{-i}{\sqrt{\frac{v^2}{c^2} - 1}} \end{bmatrix}$$

Either way, $\det L = 1$, and the orientation of the frame is preserved under L.

For $\underline{v = c}$, the particle is the emitted photon, the Lorentz factor $\gamma(v)$ is undefined, and there is no transformation from the particle frame to the Lab frame.

3.

Orientation Reversing Mapping

For $\underline{\gamma_1 = -\gamma_2 \equiv \gamma}$, and $\underline{\beta_1 = \beta_2 \equiv b}$,

$$\begin{bmatrix} idE_\tau \\ cdp_\xi \end{bmatrix} = \begin{bmatrix} \gamma & -i\gamma\frac{v}{c} \\ -i\gamma\frac{v}{c} & -\gamma \end{bmatrix} \begin{bmatrix} idE \\ cdp \end{bmatrix}$$

The Matrix of the transformation

$$B = \begin{bmatrix} \gamma & -i\gamma\frac{v}{c} \\ -i\gamma\frac{v}{c} & -\gamma \end{bmatrix}$$

is

- Hermitian, because $B = \overline{B}^T = B^\dagger$
- Orthogonal, because its columns are perpendicular
- Rotation, because it is orthogonal
- Improper Rotation, reversing the orientation of the frame since $\det B = -\gamma^2(1 - \frac{v^2}{c^2}) = -1$.

The Inverse Transformation has the Matrix

$$B^{-1} = \begin{bmatrix} \gamma & -i\gamma\frac{v}{c} \\ -i\gamma\frac{v}{c} & -\gamma \end{bmatrix} = B^T = B.$$

Hence, B is

- Unitary as $BB^T = I$
- Normal as $BB^T = B^T B$

Thus, the analysis of B is similar to that of L, and without loss of generality, we can restrict our faster than light analysis to L, that preserves the orientation of the frame with no axes reversal.

4.

Arc Transformation

The infinitesimal arc-length

$$dx_{\mu}dx^{\mu} = -(cdt)^2 + dx_j dx_j$$

is Lorentz invariant.

Since

$$\frac{dE}{cdp} = \frac{dx}{cdt},$$

$\begin{pmatrix} icdt \\ dx \end{pmatrix}$ is rotated by the Lorentz matrix in space so that

$$\begin{bmatrix} icd\tau \\ d\xi \end{bmatrix} = \begin{bmatrix} \gamma & -i\gamma \frac{v}{c} \\ i\gamma \frac{v}{c} & \gamma \end{bmatrix} \begin{bmatrix} icdt \\ dx \end{bmatrix}.$$

That is, Arc-length transforms similarly to momentum.

For $\underline{v < c}$

$$d\tau = \frac{dt - \frac{v}{c^2}dx}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$d\xi = \frac{dx - vdt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

For $\underline{v > c}$

$$d\tau = i\frac{\frac{v}{c^2}dx - dt}{\sqrt{\frac{v^2}{c^2} - 1}}$$

$$d\xi = i\frac{vdt - dx}{\sqrt{\frac{v^2}{c^2} - 1}}$$

To obtain the Inverse Transformation, replace v with $-v$.

Then,

$$\begin{bmatrix} icdt \\ dx \end{bmatrix} = \begin{bmatrix} \gamma & i\gamma\frac{v}{c} \\ -i\gamma\frac{v}{c} & \gamma \end{bmatrix}\begin{bmatrix} icd\tau \\ d\xi \end{bmatrix}$$

For $\underline{v < c}$

$$dt = \frac{d\tau + \frac{v}{c^2}d\xi}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$dx = \frac{d\xi + vd\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

For $\underline{v > c}$

$$dt = i\frac{-\frac{v}{c^2}d\xi - d\tau}{\sqrt{\frac{v^2}{c^2} - 1}}$$

$$dx = i\frac{-vd\tau - d\xi}{\sqrt{\frac{v^2}{c^2} - 1}}$$

5.

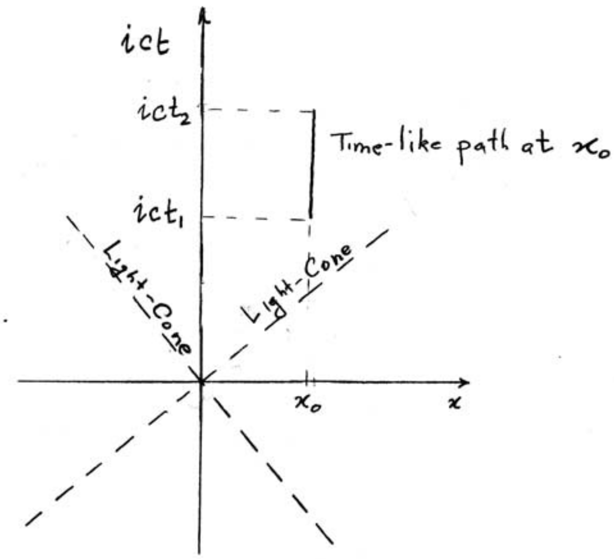
$v < c$, and Time-Like Path

If $\underline{dx_\mu dx^\mu < 0}$, then

$$v < c,$$

and the path is inside the light cone.

Then, there is a Lorentz transformation of the Lab into an inertial frame where both events occur at the same point, at two different times causally ordered.



Time ordering of events on **Time-like path** is causal:

$v < c$ mandates

$$dp_\mu dp^\mu > 0.$$

Since

$$dp_{\mu}dp^{\mu} = -(\tfrac{1}{c}dE)^2 + dp_j dp_j$$

is Lorentz invariant, it remains positive, and there is a Lorentz transformation of the Lab into an inertial frame where only momentum changes along the path.

The arc-lengths $-c^2t^2 + x^2$, and $-c^2\tau^2 + \xi^2$ lie on the same hyperbola in the (ict, x) plane.

The arc-lengths $-c^2t^2 + x^2 + y^2$, and $-c^2\tau^2 + \xi^2 + \eta^2$ lie on the same two sheet hyperboloid generated by revolving $-c^2t^2 + x^2 + y^2$ about the ict axis in the $\left(ict, x, y\right)$ space.

The arc-lengths $-c^2t^2 + x^2 + y^2 + z^2$, and $-c^2\tau^2 + \xi^2 + \eta^2 + \zeta^2$ lie on the same two sheet hyper-surface generated by revolving $-c^2t^2 + x^2 + y^2 + z^2$ about the ict axis in the $\left(ict, x, y, z\right)$ space.

The time ordering of two events is the same under any Lorentz transformation to another frame.

6.

$v = c$, and Light-Like Path

If $\underline{dx_\mu dx^\mu = 0}$, then

$$v = c,$$

and the path is on the light cone.

The light path is causal, and has

$$dp_\mu dp^\mu = 0.$$

7.

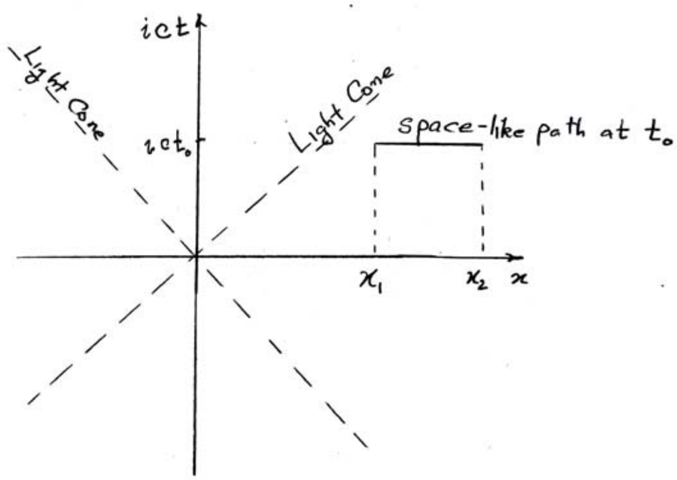
$v > c$, and Space-Like Path

If $\underline{dx_\mu dx^\mu > 0}$, then

$$\underline{v > c},$$

and the path is out of the light cone.

Then, there is a Lorentz Transformation to an inertial frame where the two events occur at the same time, at two different points. Only distance changes along the Path.



Time-ordering of events on **space-like path** depends on the frames, and has

$$dp_\mu dp^\mu < 0.$$

Since $dp_\mu dp^\mu$ is Lorentz invariant, it stays negative, and there is a Lorentz Transformation to an inertial frame where only energy changes along the path.

The arc-lengths $-c^2t^2 + x^2$, and $-c^2\tau^2 + \xi^2$ lie on the same hyperbola in the (ict, x) plane.

The arc-lengths $-c^2t^2 + x^2 + y^2$, and $-c^2\tau^2 + \xi^2 + \eta^2$ lie on the same one sheet hyperboloid generated by revolving $-c^2t^2 + x^2 + y^2$ about the ict axis in the $\left(ict, x, y\right)$ space.

The arc-lengths $-c^2t^2 + x^2 + y^2 + z^2$, and $-c^2\tau^2 + \xi^2 + \eta^2 + \zeta^2$ lie on the same one sheet hyper-surface generated by revolving $-c^2t^2 + x^2 + y^2 + z^2$ about the ict axis in the $\left(ict, x, y, z\right)$ space.

The time ordering of two events can be changed.

8.

Contraction of a Moving Ruler

A ruler moves with velocity v in the x direction.

The ruler has the Lab coordinates

$$(ict, x),$$

and its own frame coordinates

$$(ic\tau, \xi).$$

The origins of both frames coincide at time $t = 0$. Then,

$$\Delta\xi = \gamma\left(\Delta x - v\Delta t\right).$$

In the Lab, at any one time, $\Delta t = 0$, and

$$\Delta x\Big|_{\Delta t = o} = \Delta\xi \,/\, \gamma = \Delta\xi\sqrt{1 - \frac{v^2}{c^2}}.$$

For $\underline{v < c}$ $\Delta\xi$ look shorter in the Lab frame by $\beta(v) = \sqrt{1 - \frac{v^2}{c^2}}$.

For $\underline{v = c}$ $\beta = 0$, and the ruler will shrink to length zero.

For $\underline{v > c}$ $\beta(v) = i\sqrt{\frac{v^2}{c^2} - 1}$, may be larger or smaller than i

but the ruler will not be seen.

When a cigar shaped space-ship flies away from us, its length contracts. To the space travelers, we, the observers contract. We both contract in the other party view.

As its speed reaches the speed of light, the cigar shrinks to a point. To the space travelers, the observers become points. We both become points to each other.

When the ship speed is greater than light speed, it disappears from our view, and we disappear from their view. We both become invisible to each other.

9.

Retardation of a Moving Clock

A clock moves with velocity v in the x direction.

The clock has the Lab coordinates

$$(ict, x),$$

and its own frame coordinates

$$(ic\tau, \xi).$$

The origins of both frames coincide at time $t = 0$. Then, from section 4,

$$\Delta t = \gamma\left(\Delta\tau + \frac{v}{c^2}\Delta\xi\right).$$

In the clock frame, at any one location $\Delta\xi = 0$. Thus, in the Lab,

$$\Delta t\Big|_{\Delta\xi=0} = \gamma\Delta\tau = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}}$$

For $\underline{v < c}$ A clock time $\Delta\tau$ will look retarded in the Lab by

$$\gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.$$

For $\underline{v}$ almost c, the clock time will be infinitely retarded.

For $\underline{v > c}$ $\gamma(v) = \dfrac{-i}{\sqrt{\frac{v^2}{c^2} - 1}}$, may be larger or smaller than $-i$

but the clock will not be seen.

The **Proper Time** increment

$$\delta\tau \equiv \frac{\delta t}{\gamma(u)} = \delta t\sqrt{1 - \frac{u^2}{c^2}},$$

satisfies

$$(\delta\tau)^2 = \left(1 - \tfrac{u^2}{c^2}\right)(\delta t)^2$$

$$= -\tfrac{1}{c^2}\left[(ic\delta t)^2 + (u\delta t)^2\right]$$

$$= -\tfrac{1}{c^2}\left[(ic\delta t)^2 + dx_i dx_i\right]$$

$$= -\tfrac{1}{c^2} dx_\mu dx_\mu .$$

Therefore,

- $\delta\tau$ is invariant under Lorentz rotation
- $\delta\tau$ is not an exact differential, and $\tau = \int_{t_1}^{t_2} dt\sqrt{1 - \frac{u^2}{c^2}}$ is path dependent.

When a space-ship flies away from us, its clock looks to us retarded. To the space travelers, our clock retards. Either clock retards in the other party view.

As the ship's speed reaches the speed of light, its clock stops. To the space travelers, our clocks stop. Either clock stops in the other party view.

When the ship speed is greater than light speed, its clock is invisible to us, and our clock is invisible to them. Either clock becomes invisible to the other party.

10.

Velocity Transformation

A particle moves with velocity u_ξ in the frame $(ic\tau, \xi)$, whose origin moves with velocity v, in the x direction, in the Lab frame (ict, x).

The particle velocity in the moving frame is

$$u_\xi = \frac{d\xi}{d\tau} = \frac{\gamma(v)\left(dx - vdt\right)}{\gamma(v)\left(dt - \frac{v}{c^2}dx\right)} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{u - v}{1 - \frac{uv}{c^2}}.$$

For $\underline{u < c}$ $\quad u_\xi = \dfrac{u - v}{1 - \frac{uv}{c^2}} = \dfrac{\frac{u}{c} - \frac{v}{c}}{1 - \frac{uv}{c^2}} c < c$,

because $0 < (1 - \frac{u}{c})(1 + \frac{v}{c}) = 1 - \frac{uv}{c^2} - \frac{u}{c} + \frac{v}{c} \quad \Rightarrow \dfrac{\frac{u}{c} - \frac{v}{c}}{1 - \frac{uv}{c^2}} < 1$

For $\underline{u = c}$ $\quad u_\xi = \dfrac{c - v}{1 - \frac{cv}{c^2}} = \dfrac{c - v}{c - v} c = c$.

For $\underline{u > c}$ $\quad u_\xi = \dfrac{u - v}{1 - \frac{uv}{c^2}} = \dfrac{\frac{u}{c} - \frac{v}{c}}{1 - \frac{uv}{c^2}} c > c$,

because $0 > (1 - \frac{u}{c})(1 + \frac{v}{c}) = 1 - \frac{uv}{c^2} - \frac{u}{c} + \frac{v}{c} \quad \Rightarrow \dfrac{\frac{u}{c} - \frac{v}{c}}{1 - \frac{uv}{c^2}} > 1$.

The Lab particle velocity is obtained by replacing v with $-v$

$$u = \frac{dx}{dt} = \frac{\gamma(v)\Big(d\xi + vd\tau\Big)}{\gamma(v)\Big(d\tau + \frac{v}{c^2}d\xi\Big)} = \frac{u_\xi + v}{1 + \frac{u_\xi v}{c^2}}$$

For $\underline{u_\xi < c}$ $\quad u = \frac{u_\xi + v}{1 + \frac{u_\xi v}{c^2}} = \frac{\frac{u_\xi}{c} + \frac{v}{c}}{1 + \frac{u_\xi v}{c^2}} c < c$,

because $0 < (1 - \frac{u_\xi}{c})(1 - \frac{v}{c}) = 1 + \frac{u_\xi v}{c^2} - \frac{u_\xi}{c} - \frac{v}{c} \Rightarrow \frac{\frac{u_\xi}{c} + \frac{v}{c}}{1 + \frac{u_\xi v}{c^2}} < 1$

For $\underline{u_\xi = c}$ $\quad u = \frac{c + v}{1 + \frac{cv}{c^2}} = \frac{c + v}{c + v} c = c$.

For $\underline{u_\xi > c}$ $\quad u = \frac{u_\xi + v}{1 + \frac{u_\xi v}{c^2}} = \frac{\frac{u_\xi}{c} + \frac{v}{c}}{1 + \frac{u_\xi v}{c^2}} c > c$,

because $0 > (1 - \frac{u_\xi}{c})(1 - \frac{v}{c}) = 1 + \frac{u_\xi v}{c^2} - \frac{u_\xi}{c} - \frac{v}{c} \Rightarrow \frac{\frac{u_\xi}{c} + \frac{v}{c}}{1 + \frac{u_\xi v}{c^2}} > 1$

11.

Plane-Velocity Transformation

A particle has velocity (u_ξ, u_η) in the frame $(ic\tau, \xi, \eta)$, that moves with velocity v, in the x direction, in the Lab frame (ict, x, y).

The particle velocities in the moving frame are

$$u_\xi = \frac{d\xi}{d\tau} = \frac{\gamma(v)\Big(dx - vdt\Big)}{\gamma(v)\Big(dt - \frac{v}{c^2}dx\Big)} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{u_x - v}{1 - \frac{vu_x}{c^2}},$$

$$u_\eta = \frac{d\eta}{d\tau} = \frac{dy}{\gamma(v)(dt - \frac{v}{c^2}dx)} = \frac{\frac{dy}{dt}}{\gamma(v)(1 - \frac{v}{c^2}\frac{dx}{dt})} = \frac{u_y}{\gamma(v)(1 - \frac{vu_x}{c^2})}$$

By replacing v with $-v$, the particle velocities in the Lab are

$$u_x = \frac{dx}{dt} = \frac{\gamma(v)\Big(d\xi + vd\tau\Big)}{\gamma(v)\Big(d\tau + \frac{v}{c^2}d\xi\Big)} = \frac{u_\xi + v}{1 + \frac{vu_\xi}{c^2}}$$

$$u_y = \frac{dy}{dt} = \frac{d\eta}{\gamma(v)\Big(d\tau + \frac{v}{c^2}d\xi\Big)} = \frac{u_\eta}{\gamma(v)\Big(1 + \frac{v}{c^2}u_\xi\Big)}$$

12.

Plane-Force Transformation

A particle has velocity (u_ξ, u_η), and momentum $(i\frac{1}{c}E_\tau, p_\xi, p_\eta)$ in the frame $(ic\tau, \xi, \eta)$, that moves with velocity v, in the x direction, in the Lab frame (ict, x, y).

The Forces on the particle in the moving frame are

$$F_\xi = \frac{dp_\xi}{d\tau} = \frac{\gamma(v)\left(-\frac{v}{c^2}dE + dp_x\right)}{\gamma(v)\left(dt - \frac{v}{c^2}dx\right)} = \frac{-\frac{v}{c^2}\frac{dE}{dt} + \frac{dp_x}{dt}}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{-\frac{v}{c^2}F_x u_x + F_x}{1 - \frac{v}{c^2}u_x} = F_x$$

$$F_\eta = \frac{dp_\eta}{d\tau} = \frac{dp_y}{\gamma(v)(dt - \frac{v}{c^2}dx)} = \frac{\frac{dp_y}{dt}}{\gamma(v)(1 - \frac{v}{c^2}\frac{dx}{dt})} = \frac{F_y}{\gamma(v)(1 - \frac{vu_x}{c^2})}.$$

If the particle moves only in the y direction, its x-velocity in the Lab is $u_x = v$, and

$$F_\eta = \frac{F_y}{\gamma(v)(1 - \frac{vv}{c^2})} = \frac{F_y}{\gamma(v)}$$

13.

Mass Transformation

The frame $(ic\tau, \xi, \eta)$, aligned with the Lab frame (ict, x, y), moves with speed v along the x axis.

A particle with mass m_ξ, moving in the y direction with velocity

$$u_\eta$$

collides with another m_ξ particle moving in the y direction with

$$-u_\eta.$$

In the Lab, the second particle has mass m, and velocities

$$u_x = 0 \quad \Rightarrow u_\xi = -v$$

$$u_y = \frac{-u_\eta}{\gamma(v)(1+\frac{v}{c^2}u_\xi)} = \frac{-u_\eta}{\gamma(v)(1+\frac{v}{c^2}(-v))} = \frac{-u_\eta}{\gamma(v)}.$$

After the collision

$$u_\eta \to -U_\eta,$$

and

$$\frac{-u_\eta}{\gamma(v)} \to \frac{U_\eta}{\gamma(v)}.$$

By momentum conservation in either frame, in the y direction,

$$m_\xi u_\eta + m\frac{-u_\eta}{\gamma(v)} = m_\xi(-U_\eta) + m\frac{U_\eta}{\gamma(v)}.$$

Therefore, the Lorentz mass observed in the Lab is

$$m = \gamma(v)m_\xi = \frac{m_\xi}{\sqrt{1-\frac{v^2}{c^2}}}.$$

For a particle moving with velocity u in the Lab, m_0 is the rest mass of the particle in its frame.

The Lab mass is

$$m(u) = \gamma(u)m_0.$$

For $\underline{u < c}$, the particle will look heavier in the Lab by the factor

$$\gamma(u) = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}.$$

For $\underline{u = c}$, the particles are photons. If they bounce at the collision and remain photons, their momentum conservation is

$$m_\xi c + m(-c) = m_\xi(-c) + m(c),$$

$$m_{\text{photon}} = (m_\xi)_{\text{photon}}.$$

That is, the photon's mass is the same in all frames. It is

$$m_{\text{photon}} = \frac{p_{\text{photon}}}{c} = \frac{\frac{1}{c}E_{photon}}{c} = \frac{h\nu}{c^2}.$$

For $u > c$, $\gamma(u)$ is imaginary. The particle cannot be seen.

Its mass is the imaginary

$$m = i\frac{m_0}{\sqrt{\frac{u^2}{c^2} - 1}}$$

14.

Energy and Mass

$$dm(u) = m_0 d(1 - \tfrac{u^2}{c^2})^{-\frac{1}{2}}$$

$$= m_0 \tfrac{1}{2c^2}(1 - \tfrac{u^2}{c^2})^{-\frac{3}{2}} d(u^2)$$

$$= \tfrac{1}{2}\tfrac{1}{c^2 - u^2} \underbrace{m_0 \underbrace{(1 - \tfrac{u^2}{c^2})^{-\frac{1}{2}}}_{\gamma(u)}}_{m(u)} d(u^2).$$

Hence,

$$(c^2 - u^2)dm(u) = \tfrac{1}{2} m(u) d(u^2).$$

Then,

$$dE(u) = \frac{d\vec{p}(u)}{dt} \cdot d\vec{x}$$

$$= d\vec{p}(u) \cdot \vec{u}$$

$$= d\big(m(u)\vec{u}\big) \cdot \vec{u}$$

$$= dm(u)\underbrace{(\vec{u} \cdot \vec{u})}_{u^2} + \underbrace{\tfrac{1}{2} m(u) d(\vec{u} \cdot \vec{u})}_{(c^2 - u^2)dm(u)}$$

$$= c^2 dm(u).$$

Integrating,

$$\underbrace{E(u) - E(0)}_{\text{kinetic Energy T}} = c^2 \int_{\mu=0}^{\mu=u} dm(\mu) = [\underbrace{m(u)}_{m_0\gamma(u)} - m_0]c^2,$$

$$E(u) = m_0 \gamma(u) c^2,$$

$$E(0) = m_0 c^2$$

For $\underline{u < c}$,

$$\gamma(u) = (1 - \tfrac{u^2}{c^2})^{-\frac{1}{2}}$$

$$= 1 + \tfrac{1}{2}\tfrac{u^2}{c^2} + \tfrac{1}{2}\tfrac{3}{2}\left(\tfrac{u^2}{c^2}\right)^2 + \tfrac{1}{2}\tfrac{3}{2}\tfrac{5}{2}\left(\tfrac{u^2}{c^2}\right)^3 + \ldots$$

$$E(u) = m_0 c^2 \gamma(u)$$

$$= m_0 c^2 + \underbrace{\tfrac{1}{2} m_0 u^2 + \tfrac{1}{2}\tfrac{3}{2}\frac{m_0}{c^2} u^4 + \tfrac{1}{2}\tfrac{3}{2}\tfrac{5}{2}\frac{m_0}{c^4} u^6 + \ldots}_{\text{T=Kinetic Energy}}$$

For $\underline{u > c}$,

$$\gamma(u) = -i(\tfrac{u^2}{c^2} - 1)^{-\frac{1}{2}}$$

$$= -i\tfrac{c}{u}(1 - \tfrac{c^2}{u^2})^{-\frac{1}{2}}$$

$$= -i\tfrac{c}{u}\{1 + \tfrac{1}{2}\tfrac{c^2}{u^2} + \tfrac{1}{2}\tfrac{3}{2}\left(\tfrac{c^2}{u^2}\right)^2 + \tfrac{1}{2}\tfrac{3}{2}\tfrac{5}{2}\left(\tfrac{c^2}{u^2}\right)^3 + \ldots\}$$

$$E(u) = m_0 c^2 \gamma(u)$$

$$= -i m_0 c^2 \tfrac{c}{u}(1 - \tfrac{c^2}{u^2})^{-\frac{1}{2}}$$

$$= -i m_0 c^2 \{\tfrac{c}{u} + \tfrac{1}{2}\tfrac{c^3}{u^3} + \tfrac{1}{2}\tfrac{3}{2}\tfrac{c^5}{u^5} + \tfrac{1}{2}\tfrac{3}{2}\tfrac{5}{2}\tfrac{c^7}{u^7} + \ldots\}$$

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