Faster Than Light Motion

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Abstract We show here that Lorentz-like transformations can be defined and applied to faster than light motion.

Faster than light motion is avoided by the special theory of relativity. Lorentz transformations apply only to motions with speeds smaller than light speed. At speeds faster than light, Lorentz transformations lead to contradictions, and paradoxes.

This led physicists to conclude that superluminal speeds do not exist, although Cherenkov radiation proves that faster than light speeds exist.

We construct Lorentz-like transformations, and extend Special Relativity to Faster than Light motions.

Physics & Astronomy Classification Scheme: 03.30.+p; 04; 11.30.Cp; 13.40.Dk; 95.30.Sf; 95.35.+d; 98.80.Jk;

Contents

Faster than Light

0.1 Faster Than Light Neutrinos

Faster than light Neutrinos are detected by the Cherenkov Radiation that they emit in water.

The interaction of Faster Than Light neutrinos with nucleus of atoms in water molecules generates bursts of Cherenkov Radiation, that are counted by photo-multiplier tubes that surround the water tank.

Photon's speed decreases by third in water, and particles that can move in water faster than photons, can move in the air faster than photons.

Particles Faster than Light in the Air are invisible to us, but in Matter they may move slower than photons in the air, and become visible.

electrically charged particles leave behind them an optical wake of Cherenkov Radiation: A blue glow of radiation on the verge of becoming invisible.

That optical wake is similar to the sonic shock wave created by a jet plane faster than Sound.

0.2 Lorentz Transformations, and Faster than Light

Faster than Light motions are beyond the special theory of relativity. Lorentz transformation applies only to motions with speeds smaller than light speed. Applying Lorentz transformations to speeds faster than light, leads to contradictions, and paradoxes.

This led physicists to conclude that superluminal speeds do not exist, although Cherenkov radiation proves that faster than light speeds exist.

We show here that Lorentz transformation definition depends on whether the motion is slower, or faster than light. And Lorentz transformations can be defined to apply to systems moving at superluminal speeds.

We proceed with the construction of these transformations, and with the extension of Special Relativity to Faster than Light motions.

1.

Lorentz Transformations for Slower than Light Motion

A particle moves at speed $v < c$ in the x direction, passes through the Lab's origin, and emits a photon, a quantum of radiation that moves with speed c, in the x direction.

That radiation signal, that may be invisible to us, will be detected in a radiation detector.

We will assume that the speed of the radiation signal is the same in all uniformly moving reference frames.

In the Lab frame, the signal speed is

$$c = \frac{dx}{dt}$$

$$= \frac{\frac{dp}{dt} dx}{dp}$$

$$= \frac{F dx}{dp}$$

$$= \frac{dE}{dp}.$$

Therefore,

$$c dp = dE$$

In the particle frame,

$$c = \frac{d\xi}{d\tau}$$

$$= \frac{\frac{dp_\xi}{d\tau} d\xi}{dp_\xi}$$

$$= \frac{F_\xi d\xi}{dp_\xi}$$

$$= \frac{dE_\tau}{dp_\xi}$$

Hence,

$$cdp_\xi = dE_\tau$$

Therefore,

$$(dE_\tau)^2 - (cdp_\xi)^2 = 0 = (dE)^2 - (cdp)^2.$$

Thus, the infinitesimal momentum

$$\left(\frac{i}{c} dE\right)^2 + (dp)^2 \text{ is Lorentz invariant.}$$

Since

$$\frac{dE}{cdp} = \frac{dx}{cdt},$$

distance transforms similarly to momentum, and the infinitesimal distance

$$-c^2(dt)^2 + (dx)^2 \quad \text{is Lorentz invariant}$$

In the Lab, the event coordinates are

$$\left(ict, x\right),$$

and the momentum coordinates are

$$\left(\frac{i}{c}E, p\right).$$

In the particle's frame, the event coordinates are

$$\left(ic\tau, \xi\right),$$

and the momentum coordinates are

$$\left(\frac{i}{c}E_\tau, p_\xi\right).$$

Assuming that the infinitesimal energies transform linearly

$$\begin{bmatrix} dE_\tau \\ cdp_\xi \end{bmatrix} = \begin{bmatrix} \gamma_1 & \beta_1 \\ \beta_2 & \gamma_2 \end{bmatrix} \begin{bmatrix} dE \\ cdp \end{bmatrix},$$

we have

$$dE_\tau = \gamma_1 dE + \beta_1 cdp,$$

$$cdp_\xi = \beta_2 dE + \gamma_2 cdp,$$

$$\begin{aligned}(dE)^2 - (cdp)^2 &= (dE_\tau)^2 - (cdp_\xi)^2 \\ &= (\gamma_1 dE + \beta_1 cdp)^2 - (\beta_2 dE + \gamma_2 cdp)^2 \\ &= (\gamma_1 dE)^2 + 2\gamma_1\beta_1 cdEdp + (\beta_1 cdp)^2 \\ &\quad -(\beta_2 dE)^2 - 2\beta_2\gamma_2 cdEdp - (\gamma_2 cdp)^2\end{aligned}$$

Equating Coefficients,

$$\gamma_1^2 - \beta_2^2 = 1,$$

$$\beta_1^2 - \gamma_2^2 = -1 \Rightarrow \gamma_2^2 - \beta_1^2 = 1,$$

$$\gamma_1\beta_1 - \beta_2\gamma_2 = 0 \quad \Rightarrow \quad \gamma_1\beta_1 = \beta_2\gamma_2.$$

We obtain

$$\gamma_1 = \gamma_2 \equiv \gamma,$$

$$\beta_1 = \beta_2 \equiv b,$$

or

$$\gamma_1 = -\gamma_2 \equiv \gamma,$$

$$\beta_1 = -\beta_2 \equiv b,$$

so that in either case

$$\gamma^2 - b^2 = 1.$$

Now, for $\delta E_\tau = 0$,

$$0 = \gamma\delta E + bc\delta p,$$

$$\frac{bc}{\gamma} = -\frac{\delta E}{\delta p} = -\frac{\delta x}{\delta t} = -v,$$

$$b = -\gamma\frac{v}{c}$$

Substituting in $\gamma^2 - b^2 = 1$,

$$\gamma^2 - \gamma^2\frac{v^2}{c^2} = 1,$$

$$\gamma^2(1 - \frac{v^2}{c^2}) = 1,$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

$\underline{v < c}$,

1.1
$$\begin{bmatrix} dE_\tau \\ cdp_\xi \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \begin{bmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{bmatrix} \begin{bmatrix} dE \\ cdp \end{bmatrix}$$

In the inverse transformation, $-v$ replaces v

1.2
$$\begin{bmatrix} dE \\ cdp \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \begin{bmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{bmatrix} \begin{bmatrix} dE_\tau \\ cdp_\xi \end{bmatrix}$$

The distance transforms by

1.3
$$\begin{bmatrix} d\xi \\ cd\tau \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \begin{bmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{bmatrix} \begin{bmatrix} dx \\ cdt \end{bmatrix}$$

In the inverse transformation, $-v$ replaces v,

1.4
$$\begin{bmatrix} dx \\ cdt \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \begin{bmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{bmatrix} \begin{bmatrix} d\xi \\ cd\tau \end{bmatrix}.$$

1.5 The infinitesimal Distance is Lorentz Invariant

Proof:

$$\begin{aligned}(dx)^2 - c^2(dt)^2 &= \gamma^2 \left\{ \left[d\xi + \upsilon d\tau \right]^2 - \left[\frac{\upsilon}{c} d\xi + c d\tau \right]^2 \right\} \\ &= \gamma^2 \left\{ (d\xi)^2 \left(1 - \frac{\upsilon^2}{c^2} \right) - c^2 (d\tau)^2 \left(1 - \frac{\upsilon^2}{c^2} \right) \right\} \\ &= \underbrace{\gamma^2 \left(1 - \frac{\upsilon^2}{c^2} \right)}_{1} \left\{ (d\xi)^2 - c^2 (d\tau)^2 \right\} \\ &= (d\xi)^2 - c^2 (d\tau)^2 . \square\end{aligned}$$

2.

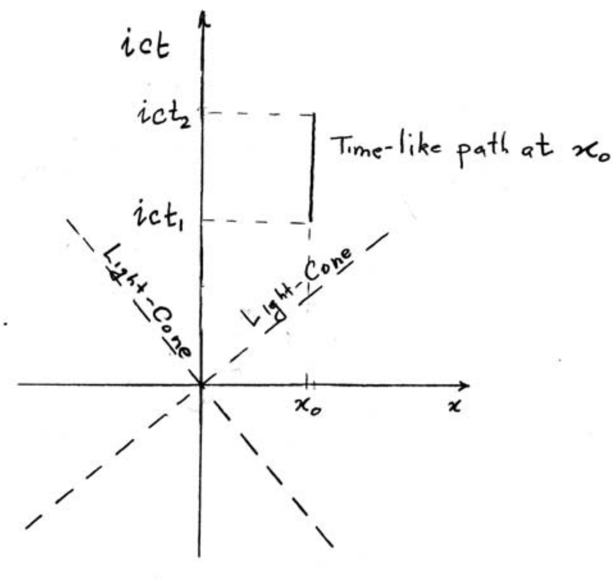
$v < c$, and Time-Like Path

If $v < c$, then

$$-c^2(dt)^2 + (dx)^2 = \left[-c^2 + v^2\right](dt)^2 < 0$$

That is, the infinitesimal distance is negative, and the path is inside the radiation cone.

Then, there is a Lorentz transformation of the Lab into an inertial frame where both events occur at the same point, at two different times causally ordered.



Time ordering of events on **Time-like path** is causal:

$v < c \Rightarrow$ the infinitesimal momentum is positive

$$-\left(\frac{1}{c}dE\right)^2 + (dp)^2 > 0\,.$$

Since $dp_\mu dp^\mu$ is Lorentz invariant, it remains positive, and there is a Lorentz transformation of the Lab into an inertial frame where only momentum changes along the path.

The arc-lengths $-c^2t^2 + x^2 + y^2$, and $-c^2\tau^2 + \xi^2 + \eta^2$ lie on the same two sheet hyperboloid generated by revolving $-c^2t^2 + x^2 + y^2$ about the ict axis in the $\left(ict, x, y\right)$ space.

The arc-lengths $-c^2t^2 + x^2 + y^2 + z^2$, and $-c^2\tau^2 + \xi^2 + \eta^2 + \zeta^2$ lie on the same two sheet hyper-surface generated by revolving $-c^2t^2 + x^2 + y^2 + z^2$ about the ict axis in the $\left(ict, x, y, z\right)$ space.

The time ordering of two events is the same under any Lorentz transformation to another frame. That is,

2.1 Time-like Path is Causal

3.

Orientation Preserving Lorentz Transformation for $v < c$

For

$$\gamma_1 = \gamma_2 \equiv \gamma ,$$

$$\beta_1 = \beta_2 \equiv b = -\gamma \frac{v}{c},$$

we obtained

$$\begin{bmatrix} dE_\tau \\ cdp_\xi \end{bmatrix} = \gamma \begin{bmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{bmatrix} \begin{bmatrix} dE \\ cdp \end{bmatrix}.$$

In the coordinates (iE_τ, cp_ξ), we rewrite

$$\begin{bmatrix} idE_\tau \\ cdp_\xi \end{bmatrix} = \underbrace{\gamma_\nu \begin{bmatrix} 1 & -i\frac{v}{c} \\ i\frac{v}{c} & 1 \end{bmatrix}}_{L} \begin{bmatrix} idE \\ cdp \end{bmatrix}.$$

The Lorentz Transformation L is

- Hermitian, because $L = \overline{L}^T = L^\dagger$
- Orthogonal, because its columns are perpendicular
- Rotation, because it is orthogonal
- Proper Rotation, preserving the orientation of the

frame, since $\det L = \gamma^2 \left(1 - \frac{v^2}{c^2}\right) = 1$.

The inverse transformation is

$$L^{-1} = \gamma \begin{bmatrix} 1 & i\frac{v}{c} \\ -i\frac{v}{c} & 1 \end{bmatrix} = L^T.$$

Hence L is

- Unitary since $LL^T = I$
- Normal since $LL^T = L^T L$

4.

Orientation Reversing Mapping for $\upsilon < c$

For

$$\gamma_1 = -\gamma_2 \equiv \gamma,$$

$$\beta_1 = -\beta_2 \equiv b = -\gamma \frac{\upsilon}{c},$$

we obtained

$$\begin{bmatrix} dE_\tau \\ cdp_\xi \end{bmatrix} = \gamma \begin{bmatrix} 1 & -\frac{\upsilon}{c} \\ \frac{\upsilon}{c} & -1 \end{bmatrix} \begin{bmatrix} dE \\ cdp \end{bmatrix}.$$

In the coordinates (iE_τ, cp_ξ), we rewrite

$$\begin{bmatrix} idE_\tau \\ cdp_\xi \end{bmatrix} = \gamma \underbrace{\begin{bmatrix} 1 & -i\frac{\upsilon}{c} \\ -i\frac{\upsilon}{c} & -1 \end{bmatrix}}_{B} \begin{bmatrix} idE \\ cdp \end{bmatrix}$$

B is

- Hermitian, because $B = \overline{B}^T = B^\dagger$
- Orthogonal, because its columns are perpendicular
- Rotation, because it is orthogonal
- Improper Rotation, reversing the orientation of the frame since

$$\det B = \begin{vmatrix} \gamma & -i\gamma \frac{v}{c} \\ -i\gamma \frac{v}{c} & -\gamma \end{vmatrix} = -\gamma^2 + \gamma^2 \frac{v^2}{c^2} = -\gamma^2 \left(1 - \frac{v^2}{c^2}\right) = -1 .$$

The inverse transformation is

$$B^{-1} = \begin{bmatrix} \gamma & -i\gamma \frac{v}{c} \\ -i\gamma \frac{v}{c} & -\gamma \end{bmatrix} = B^T = B .$$

Hence, B is

- Unitary as $BB^T = I$
- Normal as $BB^T = B^T B$

Thus, the analysis of B is similar to that of L.

5.

Lorentz Transformation for $\upsilon < c$, is Inadequate for $\upsilon > c$

For $\upsilon > c$,

$$\frac{1}{\sqrt{1-\frac{\upsilon^2}{c^2}}} = \frac{-i}{\sqrt{\frac{\upsilon^2}{c^2}-1}},$$

$$\begin{bmatrix} dE_\tau \\ cdp_\xi \end{bmatrix} = \frac{-i}{\sqrt{\frac{\upsilon^2}{c^2}-1}} \begin{bmatrix} 1 & -\frac{\upsilon}{c} \\ -\frac{\upsilon}{c} & 1 \end{bmatrix} \begin{bmatrix} dE \\ cdp \end{bmatrix}$$

In the inverse transformation, $-\upsilon$ replaces υ

$$\begin{bmatrix} dE \\ cdp \end{bmatrix} = \frac{-i}{\sqrt{\frac{\upsilon^2}{c^2}-1}} \begin{bmatrix} 1 & \frac{\upsilon}{c} \\ \frac{\upsilon}{c} & 1 \end{bmatrix} \begin{bmatrix} dE_\tau \\ cdp_\xi \end{bmatrix}.$$

5.1 *The energy transformations*

$$dE = \frac{-i}{\sqrt{\frac{\upsilon^2}{c^2}-1}} \left(dE_\tau + \upsilon dp_\xi \right)$$

and

$$cdp = \frac{-i}{\sqrt{\frac{v^2}{c^2}-1}}\left(\frac{v}{c}dE_\tau + cdp_\xi\right)$$

are not physical.

For the distance,

$$\begin{bmatrix} d\xi \\ cd\tau \end{bmatrix} = \frac{-i}{\sqrt{\frac{v^2}{c^2}-1}}\begin{bmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{bmatrix}\begin{bmatrix} dx \\ cdt \end{bmatrix}.$$

In the inverse transformation, $-v$ replaces v,

$$\begin{bmatrix} dx \\ cdt \end{bmatrix} = \frac{-i}{\sqrt{\frac{v^2}{c^2}-1}}\begin{bmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{bmatrix}\begin{bmatrix} d\xi \\ cd\tau \end{bmatrix}.$$

5.2 *The distance transformations*

$$dx = \frac{-i}{\sqrt{\frac{v^2}{c^2}-1}}\left(d\xi + vd\tau\right)$$

and

$$cdt = \frac{-i}{\sqrt{\frac{v^2}{c^2}-1}}\left(\frac{v}{c}d\xi + cd\tau\right)$$

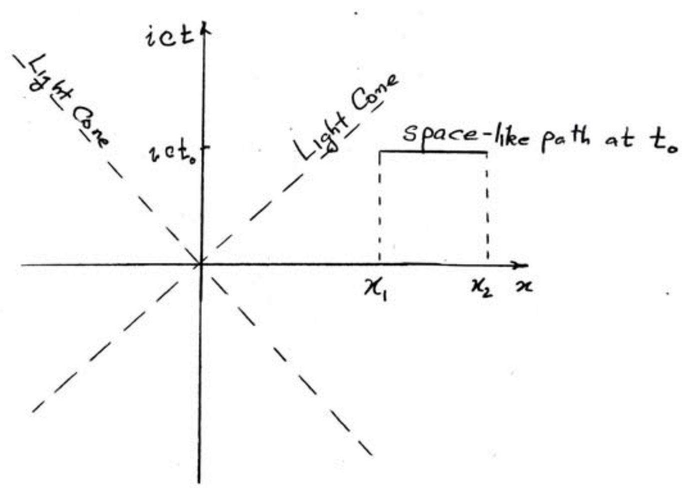
are not physical.

6.

Lorentz Transformation and Time Paradox at $v > c$

If $v > c$, then $dx_{\mu}dx^{\mu} > 0$,
and the path is out of the radiation cone.

Then, there is a Lorentz Transformation to an inertial frame where the two events occur at the same time, at two different points. Only distance changes along the Path.



Time-ordering of events on **space-like path** depends on the frames, and has $dp_{\mu}dp^{\mu} < 0$.

Since $dp_{\mu}dp^{\mu}$ is Lorentz invariant, it stays negative, and there is a Lorentz Transformation to an inertial frame where only energy changes along the path.

6.1 *For $v > c$, transformations 5.1, and 5.2 can change the time ordering of two events, and violate causality.*

In other words, there is a Lorentz Transformation for motion at $v < c$, given by 5.1, and 5.2, that can transfer an inertial system moving at $v > c$, where a chicken lays an egg, into an inertial system in which the egg is laid before the chicken is born. Thus, space-like-path may lead to a time paradox.

We seek here a Lorentz-like transformation that when applied to an inertial system moving at $v > c$, will avoid a time paradox.

7.

Lorentz-like Transformations for $\upsilon > c$

For motions faster than light, we need Lorentz-like transformations that have physical meaning, and avoid time paradoxes,

We obtain such formula by exchanging υ, and c in the Lorentz transformation formulas. That is, we will use the factor

$$\frac{1}{\sqrt{1-\frac{c^2}{\upsilon^2}}} = \frac{1}{\frac{c}{\upsilon}\sqrt{\frac{\upsilon^2}{c^2}-1}}$$

and the transformation matrix

$$\begin{bmatrix} 1 & -\frac{c}{\upsilon} \\ -\frac{c}{\upsilon} & 1 \end{bmatrix}$$

Then, the energy transforms by

7.1
$$\begin{bmatrix} dE_\tau \\ cdp_\xi \end{bmatrix} = \frac{1}{\frac{c}{\upsilon}\sqrt{\frac{\upsilon^2}{c^2}-1}} \begin{bmatrix} 1 & -\frac{c}{\upsilon} \\ -\frac{c}{\upsilon} & 1 \end{bmatrix} \begin{bmatrix} dE \\ cdp \end{bmatrix}$$

$$= \frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c} & -1 \\ -1 & \frac{\upsilon}{c} \end{bmatrix} \begin{bmatrix} dE \\ cdp \end{bmatrix}$$

The distance transforms similarly by

7.2 $$\begin{bmatrix} cd\tau \\ d\xi \end{bmatrix} = \frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c} & -1 \\ -1 & \frac{\upsilon}{c} \end{bmatrix} \begin{bmatrix} cdt \\ dx \end{bmatrix}$$

Replacing υ with $-\upsilon$

7.3 $$\begin{bmatrix} cdt \\ dx \end{bmatrix} = \frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \begin{bmatrix} -\frac{\upsilon}{c} & -1 \\ -1 & -\frac{\upsilon}{c} \end{bmatrix} \begin{bmatrix} cd\tau \\ d\xi \end{bmatrix}$$

This Lorentz-like transformation is valid because

7.4 The infinitesimal Distance is Lorentz-like Invariant

<u>*Proof*</u>:

$$-c^2(dt)^2 + (dx)^2 = \frac{1}{\frac{\upsilon^2}{c^2} - 1} \left\{ -\left[-d\xi - \frac{\upsilon}{c} cd\tau \right]^2 + \left[-\frac{\upsilon}{c} d\xi - \upsilon d\tau \right]^2 \right\}$$

$$= \frac{1}{\frac{v^2}{c^2}-1}\left\{-c^2(d\tau)^2\left(\frac{v^2}{c^2}-1\right)+(d\xi)^2\left(\frac{v^2}{c^2}-1\right)\right\}$$

$$= \underbrace{\frac{1}{\frac{v^2}{c^2}-1}\left(\frac{v^2}{c^2}-1\right)}_{1}\left\{-c^2(d\tau)^2+(d\xi)^2\right\}$$

$$= -c^2(d\tau)^2+(d\xi)^2 . \square$$

8.

Orientation Preserving for Motion at $\upsilon > c$

we obtained

$$\begin{bmatrix} dE_\tau \\ cdp_\xi \end{bmatrix} = \frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c} & -1 \\ -1 & \frac{\upsilon}{c} \end{bmatrix} \begin{bmatrix} dE \\ cdp \end{bmatrix}.$$

In the coordinates (iE_τ, cp_ξ), we rewrite

$$\begin{bmatrix} idE_\tau \\ cdp_\xi \end{bmatrix} = \underbrace{\frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c} & -i \\ i & \frac{\upsilon}{c} \end{bmatrix}}_{L_{\upsilon > c}} \begin{bmatrix} idE \\ cdp \end{bmatrix}.$$

The Lorentz-like Transformation $L_{\upsilon > c}$ is

- Hermitian, because $L = \overline{L}^T = L^\dagger$
- Orthogonal, because its columns are perpendicular
- Rotation, because it is orthogonal
- Proper Rotation, preserving the orientation of the frame, since $\det L_{\upsilon > c} = \frac{1}{\frac{\upsilon^2}{c^2} - 1} \left(\frac{\upsilon^2}{c^2} - 1 \right) = 1.$

The inverse transformation is

$$L_{\upsilon>c}^{-1} = \frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c} & i \\ -i & \frac{\upsilon}{c} \end{bmatrix} = L_{\upsilon>c}^{T}.$$

Hence $L_{\upsilon>c}$ is

- Unitary since $L_{\upsilon>c} L_{\upsilon>c}^{T} = I$
- Normal since $L_{\upsilon>c} L_{\upsilon>c}^{T} = L_{\upsilon>c}^{T} L_{\upsilon>c}$

9.

Motion at Signal Speed $v \approx c$

For $v \approx c$,

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx \frac{1}{0}.$$

Thus, the transformation does not exist. The particle is the emitted radiation quantum, and there is no transformation from the particle frame to the Lab frame.

10.

$v = c$, and Radiation-Like Path

If $\underline{v = c}$, then

$$dx_{\mu}dx^{\mu} = 0.$$

and the path is on the radiation cone.

The radiation path is causal, and has

$$dp_{\mu}dp^{\mu} = 0.$$

11.

Energy Transformation

11.1 *For* $\upsilon < c$,

$$\Delta E_{\tau} = \frac{1 - \frac{\upsilon}{c}}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} \Delta E .$$

For $\upsilon > c$,

$$\Delta E_{\tau} = \frac{\frac{\upsilon}{c} - 1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \Delta E ,$$

Proof:

For $\upsilon < c$,

$$\begin{bmatrix} dE_{\tau} \\ cdp_{\xi} \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} \begin{bmatrix} 1 & -\frac{\upsilon}{c} \\ -\frac{\upsilon}{c} & 1 \end{bmatrix} \begin{bmatrix} dE \\ cdp \end{bmatrix}$$

$$dE_{\tau} = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} \left(dE - \frac{\upsilon}{c} cdp \right)$$

$$= \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} \left(dE - \frac{\upsilon}{c} dE \right)$$

$$= \frac{1 - \frac{\upsilon}{c}}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} dE \,.$$

$$\Delta E_\tau = \frac{1 - \frac{\upsilon}{c}}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} \Delta E \,. \square$$

For $\upsilon > c$,

$$\begin{bmatrix} dE_\tau \\ cdp_\xi \end{bmatrix} = \frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \begin{bmatrix} \frac{\upsilon}{c} & -1 \\ -1 & \frac{\upsilon}{c} \end{bmatrix} \begin{bmatrix} dE \\ cdp \end{bmatrix},$$

$$dE_\tau = \frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \left(\frac{\upsilon}{c} dE - cdp \right)$$

$$= \frac{\frac{\upsilon}{c} - 1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} dE \,. \square$$

12.

Contraction of a Moving Ruler

A ruler moves with velocity v in the x direction, and is detected by its emission of radiation

The ruler has the Lab coordinates

$$(ict, x),$$

and its own frame coordinates

$$(ic\tau, \xi).$$

12.1 *If* $v < c$,

$$\Delta x\Big|_{\Delta t=0} = \Delta\xi\sqrt{1-\frac{v^2}{c^2}}.$$

the ruler length appears shorter in the Lab frame by

$$\sqrt{1-\frac{v^2}{c^2}}.$$

If $v > c$,

$$\Delta x\Big|_{\Delta t=0} = \Delta\xi\sqrt{1-\frac{c^2}{v^2}},$$

the ruler length appears shorter in the Lab frame by

$$\sqrt{1-\frac{c^2}{v^2}}.$$

Proof:

$\underline{v < c}$,

The origins of both frames coincide at time $t = 0$. Then,

$$\Delta\xi = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\left(\Delta x - v\Delta t\right).$$

In the Lab, at any one time, $\Delta t = 0$, and

$$\Delta x\big|_{\Delta t=0} = \Delta\xi\sqrt{1 - \frac{v^2}{c^2}}\,.\square$$

$\underline{v > c}$

The origins of both frames coincide at time $t = 0$. Then,

$$\Delta\xi = \frac{1}{\sqrt{\frac{v^2}{c^2} - 1}}\left(\frac{v}{c}\Delta x - c\Delta t\right).$$

In the Lab, at any one time, $\Delta t = 0$, and

$$\Delta\xi = \frac{1}{\sqrt{\frac{v^2}{c^2} - 1}}\frac{v}{c}\Delta x\big|_{\Delta t=o}$$

$$\Delta x\big|_{\Delta t=0} = \frac{c}{v}\Delta\xi\sqrt{\frac{v^2}{c^2} - 1}\,,$$

$$= \Delta\xi\sqrt{1 - \frac{c^2}{v^2}}\,.\square$$

13.

Retardation of a Moving Clock

A clock moves with velocity υ in the x direction.

The clock has the Lab coordinates

$$(ict, x),$$

and its own frame coordinates

$$(ic\tau, \xi).$$

13.1

For $\upsilon < c$,

$$\Delta t\Big|_{\Delta\xi=0} = \frac{\Delta\tau}{\sqrt{1-\dfrac{\upsilon^2}{c^2}}}.$$

the clock will be retarded in the Lab by $\dfrac{1}{\sqrt{1-\dfrac{\upsilon^2}{c^2}}}$.

For $\upsilon > c$,

$$\Delta t\Big|_{\Delta\xi=0} = -\frac{\Delta\tau}{\sqrt{1-\dfrac{c^2}{\upsilon^2}}}.$$

the clock will be retarded in the Lab by

$$\left|\frac{\Delta t\Big|_{\Delta\xi=0}}{\Delta\tau}\right| = \frac{1}{\sqrt{1-\dfrac{c^2}{\upsilon^2}}}.$$

Proof:

$v < c$

The origins of both frames coincide at time $t = 0$. Then,

$$\Delta t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\left(\Delta\tau + \frac{v}{c^2}\Delta\xi\right).$$

In the clock frame, at any one location $\Delta\xi = 0$. Thus, in the Lab,

$$\Delta t\Big|_{\Delta\xi=0} = \frac{\Delta\tau}{\sqrt{1 - \frac{v^2}{c^2}}}.\square$$

$v > c$

The origins of both frames coincide at time $t = 0$. Then,

$$c\Delta t = \frac{1}{\sqrt{\frac{v^2}{c^2} - 1}}\left(-\frac{v}{c}c\Delta t - \Delta\xi\right).$$

In the clock frame, at any one location $\Delta\xi = 0$. Thus, in the Lab,

$$\Delta t\Big|_{\Delta\xi=0} = -\frac{v}{c}\frac{1}{\sqrt{\frac{v^2}{c^2} - 1}}\Delta\tau = \frac{-1}{\sqrt{1 - \frac{c^2}{v^2}}}\Delta\tau \ .\square$$

14.

Velocities at $\upsilon < c$

A particle moves with velocity u_ξ in the frame $(ic\tau, \xi)$.

The frame moves with velocity $\upsilon < c$, in the x direction, in the Lab frame (ict, x).

The particle emits radiation signals, and is observed by detecting these signals.

14.1 *The particle velocity in the moving frame is*

$$u_\xi = \frac{u_x - \upsilon}{1 - \frac{u_x \upsilon}{c^2}}$$

Proof: $u_\xi = \dfrac{d\xi}{d\tau}$,

$$= \frac{\gamma(\upsilon)\left(dx - \upsilon dt\right)}{\gamma(\upsilon)\left(dt - \frac{\upsilon}{c^2}dx\right)},$$

$$= \frac{\frac{dx}{dt} - \upsilon}{1 - \frac{\upsilon}{c^2}\frac{dx}{dt}} = \frac{u_x - \upsilon}{1 - \frac{u_x \upsilon}{c^2}}.\square$$

14.2 $\upsilon < c$ *and* $u_x < c \Rightarrow u_\xi < c$

Proof:

$$0 < \left(1 - \frac{u_x}{c}\right)\left(1 + \frac{\upsilon}{c}\right) = 1 - \frac{u_x \upsilon}{c^2} - \frac{u_x}{c} + \frac{\upsilon}{c}$$

$$\frac{\frac{u_x}{c} - \frac{\upsilon}{c}}{1 - \frac{u_x \upsilon}{c^2}} < 1$$

$$u_\xi = \frac{u_x - \upsilon}{1 - \frac{u_x \upsilon}{c^2}}$$

$$= \frac{\frac{u_x}{c} - \frac{\upsilon}{c}}{1 - \frac{u_x \upsilon}{c^2}} c < c . \square$$

14.3 $\upsilon < c$ *and* $u_x = c \Rightarrow u_\xi = c$.

Proof: $u_\xi = \dfrac{u_x - \upsilon}{1 - \frac{u_x \upsilon}{c^2}}$

$$= \frac{c - \upsilon}{1 - \frac{c\upsilon}{c^2}}$$

$$= \frac{c - \upsilon}{c - \upsilon} c = c . \square$$

14.4 $\upsilon < c$ *and* $u_x > c \Rightarrow u_\xi > c$

Proof:

$$0 > \left(1 - \frac{u}{c}\right)\left(1 + \frac{\upsilon}{c}\right) = 1 - \frac{u\upsilon}{c^2} - \frac{u}{c} + \frac{\upsilon}{c},$$

$$\frac{\frac{u_x}{c} - \frac{\upsilon}{c}}{1 - \frac{u_x \upsilon}{c^2}} > 1\,.$$

Therefore,

$$u_\xi = \frac{u_x - \upsilon}{1 - \frac{u_x \upsilon}{c^2}}$$

$$= \frac{\frac{u_x}{c} - \frac{\upsilon}{c}}{1 - \frac{u_x \upsilon}{c^2}} c > c\,.\square$$

14.5 *The particle velocity in the Lab is*

$$u_x = \frac{u_\xi + \upsilon}{1 + \frac{u_\xi \upsilon}{c^2}}$$

Proof: Replacing υ with $-\upsilon$

$$u_x = \frac{dx}{dt}$$

$$= \frac{\gamma(\upsilon)\left(d\xi + \upsilon d\tau\right)}{\gamma(\upsilon)\left(d\tau + \frac{\upsilon}{c^2} d\xi\right)}$$

$$= \frac{\dfrac{d\xi}{d\tau} + \upsilon}{1 + \dfrac{\upsilon}{c^2}\dfrac{d\xi}{d\tau}}$$

$$= \frac{u_\xi + \upsilon}{1 + \dfrac{u_\xi \upsilon}{c^2}}.\square$$

14.6 $\upsilon < c$ *and* $u_\xi < c \Rightarrow u_x < c$

Proof:

$$0 < \left(1 - \frac{u_\xi}{c}\right)\left(1 - \frac{\upsilon}{c}\right) = 1 + \frac{u_\xi \upsilon}{c^2} - \frac{u_\xi}{c} - \frac{\upsilon}{c}$$

$$\frac{\dfrac{u_\xi}{c} + \dfrac{\upsilon}{c}}{1 + \dfrac{u_\xi \upsilon}{c^2}} < 1$$

$$u_x = \frac{u_\xi + \upsilon}{1 + \dfrac{u_\xi \upsilon}{c^2}},$$

$$= \frac{\dfrac{u_\xi}{c} + \dfrac{\upsilon}{c}}{1 + \dfrac{u_\xi \upsilon}{c^2}} c < c.\square$$

14.7 $\upsilon < c \;\; and \;\; u_\xi = c \Rightarrow u_x = c$

Proof: $u_x = \dfrac{c+\upsilon}{1+\dfrac{c\upsilon}{c^2}}$

$$= \frac{c+\upsilon}{c+\upsilon}c = c$$

14.8 $\upsilon < c \;\; and \;\; u_\xi > c \Rightarrow u_x > c$

Proof:

$$0 > \left(1 - \frac{u_\xi}{c}\right)\left(1 - \frac{\upsilon}{c}\right) = 1 + \frac{u_\xi \upsilon}{c^2} - \frac{u_\xi}{c} - \frac{\upsilon}{c},$$

$$\frac{\dfrac{u_\xi}{c} + \dfrac{\upsilon}{c}}{1 + \dfrac{u_\xi \upsilon}{c^2}} > 1.$$

Therefore

$$u_x = \frac{u_\xi + \upsilon}{1 + \dfrac{u_\xi \upsilon}{c^2}},$$

$$= \frac{\dfrac{u_\xi}{c} + \dfrac{\upsilon}{c}}{1 + \dfrac{u_\xi \upsilon}{c^2}} c > c. \square$$

14.9 *For* $\upsilon < c$

$$u_{\xi} < c \Leftrightarrow u_{x} < c,$$

$$u_{x} > c \Leftrightarrow u_{\xi} > c,$$

$$u_{\xi} = c \Leftrightarrow u_{x} = c$$

15.

Velocities at $\upsilon > c$

A particle moves with velocity u_ξ in the frame $(ic\tau, \xi)$.

The frame moves with velocity $\upsilon > c$, in the x direction, in the Lab frame (ict, x). The particle emits radiation signals, and is observed by detecting these signals.

15.1 *The particle velocity in the moving frame is*

$$u_\xi = \frac{\upsilon - u_x}{\frac{\upsilon u_x}{c^2} - 1}$$

<u>*Proof*</u>: $u_\xi = \frac{d\xi}{d\tau}$,

$$= \frac{\frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}}\left(-dx + \upsilon dt\right)}{\frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}}\left(\frac{\upsilon}{c^2}dx - dt\right)},$$

$$= \frac{-\frac{dx}{dt} + \upsilon}{\frac{\upsilon}{c^2}\frac{dx}{dt} - 1}$$

$$= \frac{-\frac{dx}{dt} + \upsilon}{\frac{\upsilon}{c^2}\frac{dx}{dt} - 1}$$

$$= \frac{-u_x + \upsilon}{\frac{\upsilon}{c^2}u_x - 1} . \square$$

15.2 $\upsilon > c$ *and* $u_x < c \Rightarrow u_\xi > c$

Proof:

$$0 > \left(\frac{u_x}{c} - 1\right)\left(1 + \frac{\upsilon}{c}\right) = -1 + \frac{u_x \upsilon}{c^2} + \frac{u_x}{c} - \frac{\upsilon}{c}$$

$$\frac{\frac{\upsilon}{c} - \frac{u_x}{c}}{\frac{u_x \upsilon}{c^2} - 1} > 1$$

$$u_\xi = c\frac{\frac{\upsilon}{c} - \frac{u_x}{c}}{\frac{u_x \upsilon}{c^2} - 1} > c . \square$$

15.3 $\upsilon > c$ *and* $u_x = c \Rightarrow u_\xi = c$.

Proof: $u_\xi = c\frac{\frac{\upsilon u_x}{c^2} - 1}{\frac{\upsilon}{c} - \frac{u_x}{c}}$

$$= c\frac{\frac{\upsilon c}{c^2} - 1}{\frac{\upsilon}{c} - \frac{c}{c}} = c\,.\square$$

15.4 $\upsilon > c$ *and* $u_x > c \Rightarrow u_\xi < c$

Proof:

$$0 < \left(\frac{u_x}{c} - 1\right)\left(1 + \frac{\upsilon}{c}\right) = -1 + \frac{u_x \upsilon}{c^2} + \frac{u_x}{c} - \frac{\upsilon}{c}$$

$$\frac{\frac{\upsilon}{c} - \frac{u_x}{c}}{\frac{\upsilon u_x}{c^2} - 1} < 1$$

$$u_\xi = c\frac{\frac{\upsilon}{c} - \frac{u_x}{c}}{\frac{\upsilon u_x}{c^2} - 1} < c\,.\square$$

15.5 *The particle velocity in the Lab is* $u_x = c\dfrac{\frac{\upsilon}{c} + \frac{u_\xi}{c}}{\frac{u_\xi \upsilon}{c^2} + 1}$

Proof: Replacing υ with $-\upsilon$

$$u_x = \frac{dx}{dt}$$

$$= \frac{\frac{1}{\sqrt{\frac{\upsilon^2}{c^2}-1}}\left(-d\xi - \upsilon d\tau\right)}{\frac{1}{\sqrt{\frac{\upsilon^2}{c^2}-1}}\left(-\frac{\upsilon}{c^2}d\xi - d\tau\right)}$$

$$= \frac{\frac{d\xi}{d\tau} + \upsilon}{\frac{\upsilon}{c^2}\frac{d\xi}{d\tau} + 1}$$

$$= c\frac{\frac{u_\xi}{c} + \frac{\upsilon}{c}}{\frac{u_\xi \upsilon}{c^2} + 1}. \square$$

15.6 $\upsilon > c \quad and \quad u_\xi > c \Rightarrow u_x < c$

<u>*Proof*</u>: $0 < \left(1 - \frac{u_\xi}{c}\right)\left(1 - \frac{\upsilon}{c}\right) = 1 + \frac{u_\xi \upsilon}{c^2} - \frac{u_\xi}{c} - \frac{\upsilon}{c}$

$$\frac{\frac{u_\xi}{c} + \frac{\upsilon}{c}}{\frac{u_\xi \upsilon}{c_\nu^2} + 1} < 1$$

$$u = c\frac{\frac{u_\xi \upsilon}{c^2}+1}{\frac{u_\xi}{c}+\frac{\upsilon}{c}} < c. \square$$

15.7 $\upsilon > c$ *and* $u_\xi = c \Rightarrow u_x = c$

Proof: $u_x = c\frac{\frac{c\upsilon}{c^2}+1}{\frac{c}{c}+\frac{\upsilon}{c}} = c. \square$

15.8 $\upsilon > c$ *and* $u_\xi < c \Rightarrow u_x > c$

Proof: $0 > \left(1-\frac{u_\xi}{c}\right)\left(1-\frac{\upsilon}{c}\right) = 1 + \frac{u_\xi \upsilon}{c^2} - \frac{u_\xi}{c} - \frac{\upsilon}{c}$,

$$\frac{\frac{u_\xi}{c}+\frac{\upsilon}{c}}{\frac{u_\xi \upsilon}{c^2}+1} > 1$$

$$u_x = c\frac{\frac{u_\xi}{c}+\frac{\upsilon}{c}}{\frac{u_\xi \upsilon}{c^2}+1} > c. \square$$

15.9 *For* $\upsilon > c$

$$u_\xi < c \Leftrightarrow u_x > c\,,$$

$$u_x < c \Leftrightarrow u_\xi > c\,,$$

$$u_\xi = c \Leftrightarrow u_x = c$$

16.

Plane-Velocities at $\upsilon < c$

A particle has velocity (u_ξ, u_η) in the frame $(ic\tau, \xi, \eta)$.

That frame moves with velocity $\upsilon < c$, in the x direction, in the Lab frame (ict, x, y).

16.1 *The particle velocities in the moving frame are*

$$u_\xi = \frac{u_x - \upsilon}{1 - \frac{\upsilon u_x}{c^2}}$$

$$u_\eta = \frac{u_y\sqrt{1 - \frac{\upsilon^2}{c^2}}}{1 - \frac{\upsilon u_x}{c^2}}$$

Proof: $u_\xi = \dfrac{d\xi}{d\tau}$,

$$= \frac{\gamma(\upsilon)\left(dx - \upsilon dt\right)}{\gamma(\upsilon)\left(dt - \frac{\upsilon}{c^2}dx\right)}$$

$$= \frac{\frac{dx}{dt} - \upsilon}{1 - \frac{\upsilon}{c^2}\frac{dx}{dt}},$$

$$= \frac{u_x - \upsilon}{1 - \frac{\upsilon u_x}{c^2}}.$$

$$u_\eta = \frac{d\eta}{d\tau}$$

$$= \frac{dy}{\gamma(\upsilon)(dt - \frac{\upsilon}{c^2}dx)}$$

$$= \frac{\frac{dy}{dt}\sqrt{1 - \frac{\upsilon^2}{c^2}}}{1 - \frac{\upsilon}{c^2}\frac{dx}{dt}}$$

$$= \frac{u_y\sqrt{1 - \frac{\upsilon^2}{c^2}}}{1 - \frac{\upsilon u_x}{c^2}}.\square$$

Replacing υ with $-\upsilon$,

16.2 *The particle velocities in the Lab are*

$$u_x = \frac{u_\xi + \upsilon}{1 + \dfrac{\upsilon u_\xi}{c^2}}$$

$$u_y = \frac{u_\eta \sqrt{1 - \dfrac{\upsilon^2}{c^2}}}{1 + \dfrac{\upsilon u_\xi}{c^2}}$$

17.

Plane-Velocities at $\upsilon > c$

A particle has velocity (u_ξ, u_η) in the frame $(ic\tau, \xi, \eta)$.

That frame moves with velocity $\upsilon > c$, in the x direction, in the Lab frame (ict, x, y).

17.1 *The particle velocities in the moving frame are*

$$u_\xi = \frac{\upsilon - u_x}{\frac{u_x \upsilon}{c^2} - 1}$$

$$u_\eta = \frac{u_y}{\left(\frac{u_x \upsilon}{c^2} - 1\right)} \sqrt{\frac{\upsilon^2}{c^2} - 1}$$

<u>*Proof*</u>: $u_\xi = \dfrac{d\xi}{d\tau}$,

$$= \frac{\frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}}\left(-dx + \upsilon dt\right)}{\frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}}\left(\frac{\upsilon}{c^2} dx - dt\right)}$$

$$= \frac{-\dfrac{dx}{dt} + \upsilon}{\dfrac{\upsilon}{c^2}\dfrac{dx}{dt} - 1}$$

$$= c\,\frac{\dfrac{\upsilon}{c} - \dfrac{u_x}{c}}{\dfrac{u_x \upsilon}{c^2} - 1}. \square$$

$$u_\eta = \frac{d\eta}{d\tau}$$

$$= \frac{dy}{\dfrac{1}{\sqrt{\dfrac{\upsilon^2}{c^2} - 1}}\left(\dfrac{\upsilon}{c^2}dx - dt\right)}$$

$$= \frac{\dfrac{dy}{dt}}{\left(\dfrac{\upsilon}{c^2}\dfrac{dx}{dt} - 1\right)}\sqrt{\frac{\upsilon^2}{c^2} - 1}$$

$$= \frac{u_y}{\left(\dfrac{u_x \upsilon}{c^2} - 1\right)}\sqrt{\frac{\upsilon^2}{c^2} - 1}. \square$$

Replacing υ with $-\upsilon$,

17.2 *The particle velocities in the Lab are*

$$u_x = \frac{\upsilon + u_\xi}{\frac{u_\xi \upsilon}{c^2} + 1}$$

$$u_y = -\frac{u_\eta}{\frac{u_\xi \upsilon}{c^2} + 1}\sqrt{\frac{\upsilon^2}{c^2} - 1}$$

18.

Plane-Force at $\upsilon < c$

A particle has velocity (u_ξ, u_η), and momentum $\left(i\frac{1}{c}E_\tau, p_\xi, p_\eta\right)$

in the frame $(ic\tau, \xi, \eta)$.

That frame moves with velocity $\upsilon < c$, in the x direction, in the Lab frame (ict, x, y).

The Forces on the particle in the moving frame are

$$F_\xi = \frac{dp_\xi}{d\tau}$$

$$= \frac{\gamma(\upsilon)\left(-\frac{\upsilon}{c^2}dE + dp_x\right)}{\gamma(\upsilon)\left(dt - \frac{\upsilon}{c^2}dx\right)}$$

$$= \frac{-\frac{\upsilon}{c^2}\frac{dE}{dt} + \frac{dp_x}{dt}}{1 - \frac{\upsilon}{c^2}\frac{dx}{dt}}$$

$$= \frac{-\frac{\upsilon}{c^2}F_x u_x + F_x}{1 - \frac{\upsilon}{c^2}u_x} = F_x$$

$$F_{\eta} = \frac{dp_{\eta}}{d\tau}$$

$$= \frac{dp_y}{\gamma(\upsilon)(dt - \frac{\upsilon}{c^2}dx)}$$

$$= \frac{\frac{dp_y}{dt}}{1 - \frac{\upsilon}{c^2}\frac{dx}{dt}}\sqrt{1 - \frac{\upsilon^2}{c^2}}$$

$$= \frac{F_y}{1 - \frac{\upsilon u_x}{c^2}}\sqrt{1 - \frac{\upsilon^2}{c^2}}$$

If the particle moves only in the y direction, its x-velocity in the Lab is $u_x = v$, and

$$F_{\eta} = \frac{F_y\sqrt{1 - \frac{\upsilon^2}{c^2}}}{1 - \frac{\upsilon\upsilon}{c^2}} = \frac{F_y}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}.$$

19.

Plane-Force at $\upsilon > c$

A particle has velocity (u_ξ, u_η), and momentum $\left(i\frac{1}{c}E_\tau, p_\xi, p_\eta\right)$

in the frame $(ic\tau, \xi, \eta)$.

That frame moves with velocity $\upsilon > c$, in the x direction, in the Lab frame (ict, x, y).

19.1 *The Forces on the particle in the moving frame are*

$$F_\xi = \frac{\frac{\upsilon}{c} - \frac{u_x}{c}}{\frac{\upsilon u_x}{c^2} - 1} F_x$$

$$F_\eta = \sqrt{\frac{\upsilon^2}{c^2} - 1}\, \frac{1}{\frac{\upsilon u_x}{c^2} - 1} F_y$$

<u>*Proof*</u>:

$$F_\xi = \frac{dp_\xi}{d\tau}$$

$$= \frac{\frac{1}{\sqrt{\frac{\upsilon^2}{c^2}-1}}\left(-\frac{1}{c}dE + \frac{\upsilon}{c}dp_x\right)}{\frac{1}{\sqrt{\frac{\upsilon^2}{c^2}-1}}\left(\frac{\upsilon}{c^2}dx - dt\right)}$$

$$= \frac{-\frac{1}{c}\frac{dE}{dt} + \frac{\upsilon}{c}\frac{dp_x}{dt}}{\frac{\upsilon}{c^2}\frac{dx}{dt} - 1}$$

$$= \frac{-\frac{1}{c}\frac{F_x dx}{dt} + \frac{\upsilon}{c}F_x}{\frac{u_x \upsilon}{c^2} - 1}$$

$$= \frac{\frac{\upsilon}{c} - \frac{u_x}{c}}{\frac{\upsilon u_x}{c^2} - 1}F_x . \square$$

$$F_\eta = \frac{dp_\eta}{d\tau}$$

$$= \frac{dp_y}{\frac{1}{\sqrt{\frac{\upsilon^2}{c^2}-1}}\left(\frac{\upsilon}{c^2}dx - dt\right)}$$

$$= \frac{\frac{dp_y}{dt}}{\frac{\upsilon}{c^2}\frac{dx}{dt} - 1}\sqrt{\frac{\upsilon^2}{c^2} - 1}$$

$$= \sqrt{\frac{\upsilon^2}{c^2} - 1}\,\frac{1}{\frac{\upsilon u_x}{c^2} - 1}F_y . \square$$

If the particle moves only in the y direction, its x-velocity in the Lab is $u_x = v$, and

$$F_\eta = \sqrt{\frac{\upsilon^2}{c^2} - 1}\,\frac{1}{\frac{\upsilon^2}{c^2} - 1}F_y = \frac{1}{\sqrt{\frac{\upsilon^2}{c^2} - 1}}F_y .$$

20.

Mass Conversion into Photons' Energy at $v \ll c$

20.1 *At* $v \ll c$, *the mass* m *converts into Photons' energy*

$$E = mc^2$$

Proof: Following [Einstein1], and [Einstein2],

A body at rest in the (x, y, z) system, has the energy

$$E_0.$$

In the (ξ, η, ζ) system, which is moving along the x axis with velocity $v < c$, the body has the energy

$$H_0.$$

Hence, the kinetic energy of the body is

$$T_0 = H_0 - E_0$$

Let the body radiate light energy $\frac{1}{2}L$ in the direction of x, and $\frac{1}{2}L$ in the opposite direction. Then, it has energy E_1, in the (x, y, z) system, and energy H_1 in the (ξ, η, ζ) system.

Hence, the kinetic energy of the body after radiation is

$$T_1 = H_1 - E_1.$$

By energy conservation in the (x, y, z) system,

$$E_0 = E_1 + \tfrac{1}{2}L + \tfrac{1}{2}L,$$

$$= E_1 + L$$

By energy conservation in the (ξ, η, ζ) system,

$$H_0 = H_1 + \tfrac{1}{2}L\frac{1-\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} + \tfrac{1}{2}L\frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= H_1 + L\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

The change in the kinetic energy is

$$T_0 - T_1 = (H_0 - E_0) - (H_1 - E_1)$$

$$= (H_0 - H_1) - (E_0 - E_1)$$

$$= L\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - L.$$

Assuming $v / c \ll 1$,

$$= L\left[1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{1}{2}\frac{3}{2}\left(\frac{v^2}{c^2}\right)^2 + \frac{1}{2}\frac{3}{2}\frac{5}{2}\left(\frac{v^2}{c^2}\right)^3 + \ldots\right] - L$$

$$= \frac{1}{2}\frac{L}{c^2}v^2 + \frac{1}{2}\frac{3}{2}L\left(\frac{v}{c}\right)^4 + \frac{1}{2}\frac{3}{2}\frac{5}{2}L\left(\frac{v}{c}\right)^6 + \ldots$$

Assuming $\upsilon / c \ll 1$, and keeping terms up to order 2,

$$\approx \frac{1}{2}\frac{L}{c^2}\upsilon^2$$

The change in kinetic energy is due to the loss of mass that is radiated as light. That is,

$$T_0 - T_1 = \tfrac{1}{2}(\Delta m)\upsilon^2.$$

Therefore,

$$\frac{1}{2}(\Delta m)\upsilon^2 \approx \frac{1}{2}\frac{L}{c^2}\upsilon^2,$$

$$(\Delta m)c^2 \approx L.$$

20.2 Each Radiation has its Mass-Energy Equivalence

Einstein concludes with

> *"If the body releases radiation energy, its mass decreases by* $\frac{L}{c^2}$ *"*

This is true only for electromagnetic radiation that propagates at light speed c. For neutrino radiation, its speed replaces the light speed.

Einstein proceeds with

> *"the fact that the energy withdrawn from the body becomes energy of radiation rather than some other*

kind of energy makes no difference"

In fact, each energy has its own equivalence formula:

For photon radiation that propagates at speed c, the mass Δm converts into photon energy $(\Delta m)c^2$.

For neutrino radiation that propagates at speed c_ν, we have shown that Δm converts into neutrino energy $(\Delta m)c_\nu^2$.

21.

Mass Moving at $v < c$

21.1 *The frame* $(ic\tau, \xi, \eta)$ *is aligned with the Lab frame* (ict, x, y)*, and moves with speed* v *along the* x *axis.*

A mass m_ξ *in* $(ic\tau, \xi, \eta)$*, measures in the Lab as*

$$m = \frac{m_\xi}{\sqrt{1 - \frac{v^2}{c^2}}}$$

<u>*Proof*</u>: A mass m_ξ, moving in the y direction with velocity

$$u_\eta$$

collides with a mass m_ξ moving in the $-y$ direction with

$$-u_\eta.$$

In the Lab, the second mass has mass m, and velocities

$$\underbrace{u_x}_{0} = \frac{v + u_\xi}{\frac{u_\xi v}{c^2} + 1} \quad \Rightarrow u_\xi = -v$$

$$u_y = \frac{-u_\eta}{1 + \frac{\upsilon u_\xi}{c^2}} \sqrt{1 - \frac{\upsilon^2}{c^2}},$$

$$= \frac{-u_\eta}{1 + \frac{\upsilon(-\upsilon)}{c^2}} \sqrt{1 - \frac{\upsilon^2}{c^2}},$$

$$= \frac{-u_\eta}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}$$

After the collision,

$$u_\eta \to -U_\eta.$$

Hence,

$$\frac{-u_\eta}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} \to \frac{U_\eta}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}.$$

By momentum conservation in the y direction,

$$m_\xi u_\eta + m \frac{-u_\eta}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} = m_\xi(-U_\eta) + m \frac{U_\eta}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}.$$

Therefore, the mass measured in the Lab is

$$m = \frac{m_\xi}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}. \square$$

21.2 *If a particle moves with velocity* $u < c$ *in the Lab, and if* m_0 *is the rest mass of the particle in its frame.*

Then, the particle's mass in the Lab is

$$m(u) = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}}.$$

The particle will be heavier in the Lab by

$$\frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}.$$

22.

Mass Moving at $\upsilon > c$

22.1 *The frame* $(ic\tau, \xi, \eta)$ *is aligned with the Lab frame* (ict, x, y)*, and moves with speed* υ *along the* x *axis.*

A mass m_ξ *in* $(ic\tau, \xi, \eta)$*, measures in the Lab as*

$$m = \frac{m_\xi}{\sqrt{\frac{\upsilon^2}{c^2} - 1}}$$

<u>*Proof*</u>: A mass m_ξ, moving in the y direction with velocity

$$u_\eta$$

collides with a mass m_ξ moving in the $-y$ direction with

$$-u_\eta.$$

In the Lab, the second mass has mass m_x, and velocities

$$\underbrace{u_x}_{0} = \frac{\upsilon + u_\xi}{\frac{u_\xi \upsilon}{c^2} + 1} \quad \Rightarrow u_\xi = -\upsilon$$

$$u_y = -\frac{-u_\eta}{\frac{u_\xi \upsilon}{c^2} + 1}\sqrt{\frac{\upsilon^2}{c^2} - 1}$$

$$= \frac{u_\eta}{\frac{(-\upsilon)\upsilon}{c^2} + 1}\sqrt{\frac{\upsilon^2}{c^2} - 1},$$

$$= \frac{-u_\eta}{\frac{\upsilon^2}{c^2} - 1}\sqrt{\frac{\upsilon^2}{c^2} - 1},$$

$$= \frac{-u_\eta}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}$$

After the collision,

$$u_\eta \to -U_\eta.$$

Hence,

$$\frac{-u_\eta}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \to \frac{U_\eta}{\sqrt{\frac{\upsilon^2}{c^2} - 1}}.$$

By momentum conservation in the y direction,

$$m_\xi u_\eta + m\frac{-u_\eta}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} = m_\xi(-U_\eta) + m\frac{U_\eta}{\sqrt{\frac{\upsilon^2}{c^2} - 1}}.$$

Therefore, the mass measured in the Lab is

$$m = \frac{m_\xi}{\sqrt{\frac{v^2}{c^2} - 1}}. \square$$

22.2 *If a particle moves with velocity* $u > c$ *in the Lab, and if* m_0 *is the rest mass of the particle in its frame.*

Then, the particle's mass in the Lab is

$$m(u) = \frac{m_0}{\sqrt{\frac{u^2}{c^2} - 1}}.$$

The particle will be heavier in the Lab by

$$\frac{1}{\sqrt{\frac{u^2}{c^2} - 1}}.$$

23.

Photon's Energy

23.1 The Photon's Mass is the same in all frames

Proof: For $u = c$, the particles are photons.

If they bounce at the collision and remain neutrinos,

their momentum conservation is

$$m_\xi c + m(-c) = m_\xi(-c) + m(c),$$

$$m_{\text{photon}} = (m_\xi)_{\text{photon}}.$$

Thus, the photon's mass is the same in all frames.

23.2 $$E_{\text{photon}} = m_{\text{photon}} c^2$$

Proof:

$$m_{\text{photon}} = \frac{p_{\text{photon}}}{c}$$

$$= \frac{\frac{1}{c}E_{\text{photon}}}{c}$$

$$= \frac{E_{\text{photon}}}{c^2}.\square$$

24.

Mass Conversion into Photons' Energy for $u < c$

24.1 *At speed* $u < c$, *the mass* $m(u) = \dfrac{1}{\sqrt{1 - \dfrac{u^2}{c^2}}} m_0$

converts into the Photons' Energy $E(u) = \dfrac{1}{\sqrt{1 - \dfrac{u^2}{c^2}}} m_0 c^2$.

Hence, $m(u_2) - m(u_1)$ *converts into* $\left[m(u_2) - m(u_1)\right] c^2$.

<u>*Proof*</u>:

$$dm(u) = m_0 d\left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$$

$$= m_0(-\tfrac{1}{2})\left(1 - \frac{u^2}{c^2}\right)^{-\frac{3}{2}}\left(-\frac{1}{c^2}\right) d(u^2)$$

$$= \tfrac{1}{2}\frac{1}{c^2 - u^2}\underbrace{m_0\left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}}_{m(u)} d(u^2).$$

Hence,

$$(c^2 - u^2)dm(u) = \tfrac{1}{2}m(u)d(u^2).$$

Then,

$$dE(u) = \frac{d\vec{p}(u)}{dt} \cdot d\vec{x}$$

$$= d\vec{p}(u) \cdot \vec{u}$$

$$= d\Big(m(u)\vec{u}\Big) \cdot \vec{u}$$

$$= dm(u)\underbrace{(\vec{u} \cdot \vec{u})}_{u^2} + \underbrace{m(u)\vec{u} \cdot d\vec{u}}_{\frac{1}{2}m(u)d(\vec{u}\cdot\vec{u})}$$

$$= u^2 dm(u) + \underbrace{\frac{1}{2}m(u)d(u^2)}_{(c^2-u^2)dm(u)}$$

$$= u^2 dm(u) + (c^2 - u^2)dm(u)$$

$$= c^2 dm(u).$$

Integrating,

$$\underbrace{E(u) - E(0)}_{\text{kinetic Energy T}} = c^2 \int_{\mu=0}^{\mu=u} dm(\mu),$$

$$= [m(u) - m_0]c^2$$

$$= \left(\frac{m_0}{\sqrt{1 - \dfrac{u^2}{c^2}}} - m_0 \right) c^2$$

$$E(u) = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}} c^2 . \square$$

24.2 For $u \ll c$,

$$E(u) = m_0 c^2 + \underbrace{\frac{1}{2} m_0 u^2 + \frac{1}{2}\frac{3}{2}\frac{m_0}{c^2} u^4 + \frac{1}{2}\frac{3}{2}\frac{5}{2}\frac{m_0}{c^4} u^6 + \dots}_{\text{T=Kinetic Energy}}$$

$$\approx m_0 c^2 + \underbrace{\frac{1}{2} m_0 u^2}_{\text{T=Kinetic Energy}}$$

<u>*Proof*</u>:

$$E(u) = m_0 c^2 \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$$

For $u \ll c$,

$$= m_0 c^2 \left(1 + \frac{1}{2}\frac{u^2}{c^2} + \frac{1}{2}\frac{3}{2}\left(\frac{u^2}{c^2}\right)^2 + \frac{1}{2}\frac{3}{2}\frac{5}{2}\left(\frac{u^2}{c^2}\right)^3 + \dots\right)$$

$$= m_0 c^2 + \underbrace{\frac{1}{2} m_0 u^2 + \frac{1}{2}\frac{3}{2}\frac{m_0}{c^2} u^4 + \frac{1}{2}\frac{3}{2}\frac{5}{2}\frac{m_0}{c^2} u^6 + \dots}_{\text{T=Kinetic Energy}}$$

$$\approx m_0 c^2 + \underbrace{\frac{1}{2} m_0 u^2}_{\text{T=Kinetic Energy}} . \square$$

25.

Mass Conversion into Photons' Energy for $u > c$

25.1 *At speed* $u > c$, *the mass* $m(u) = \dfrac{1}{\sqrt{\dfrac{u^2}{c^2} - 1}} m_0$

converts into the Photons' Energy $E(u) = \dfrac{1}{\sqrt{\dfrac{u^2}{c^2} - 1}} m_0 c^2$.

Hence, $m(u_2) - m(u_1)$ *converts into* $\left[m(u_2) - m(u_1)\right]c^2$.

<u>*Proof*</u>:

$$dm(u) = m_0 d\left(\frac{u^2}{c^2} - 1\right)^{-\frac{1}{2}}$$

$$= m_0(-\tfrac{1}{2})\left(\frac{u^2}{c^2} - 1\right)^{-\frac{3}{2}}\left(\frac{1}{c^2}\right)d(u^2)$$

$$= \tfrac{1}{2}\frac{1}{c^2 - u^2}\underbrace{m_0\left(\frac{u^2}{c^2} - 1\right)^{-\frac{1}{2}}}_{m(u)} d(u^2).$$

Hence,

$$(c^2 - u^2)dm(u) = \tfrac{1}{2}m(u)d(u^2).$$

Then,

$$\begin{aligned} dE(u) &= \frac{d\vec{p}(u)}{dt}\cdot d\vec{x} \\ &= d\vec{p}(u)\cdot\vec{u} \\ &= d\Big(m(u)\vec{u}\Big)\cdot\vec{u} \\ &= dm(u)\underbrace{(\vec{u}\cdot\vec{u})}_{u^2} + \underbrace{m(u)\vec{u}\cdot d\vec{u}}_{\frac{1}{2}m(u)d(\vec{u}\cdot\vec{u})} \\ &= u^2 dm(u) + \underbrace{\tfrac{1}{2}m(u)d(u^2)}_{(c^2-u^2)dm(u)} \\ &= u^2 dm(u) + (c^2 - u^2)dm(u) \\ &= c^2 dm(u). \end{aligned}$$

Integrating,

$$\begin{aligned} \underbrace{E(u) - E(0)}_{\text{kinetic Energy T}} &= c^2 \int_{\mu=0}^{\mu=u} dm(\mu), \\ &= [m(u) - m_0]c^2 \\ &= \left(\frac{m_0}{\sqrt{\frac{u^2}{c^2} - 1}} - m_0\right)c^2 \end{aligned}$$

$$E(u) = \frac{m_0}{\sqrt{\frac{u^2}{c^2} - 1}} c^2 . \square$$

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