The Invariance of the Energy-Momentum Under the Lorentz Transformation

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Abstract: For 50 years, throughout his life, in writings, and in lectures, Einstein erroneously argued that the constancy of the speed of light in any reference frame implies the Invariance of the Distance-Difference

$$(dx)^2 - (cdt)^2$$

which implies the Lorentz Transformation.

Einstein never noticed that the Distance-Difference Invariance is equivalent to the Invariance of the Energy-Momentum-Difference,

$$(dE)^2 - (cdp)^2,$$

and both are equivalent to the Lorentz Transformation.

The Invariance principles, and the Lorentz Transformation are fundamental principles of Physics, and cannot be the result of a trivial postulate such as light speed constancy.

Thus, the meaning of the Lorentz Transformation as a fundamental principle of Physics remained unknown to the

physics community.

For generations, physicists like Michelson rejected the Lorentz-Fitzgerald Contraction, ignored Einstein and kept getting a null result for the speed of the earth in the Aether, distrusting the Lorentz Transformation that predicted it.

Missing the equivalence of the invariance principles, Einstein could not explain the distance invariance, and his derivation of the Lorentz transformation did not make it more credible than Lorentz contraction explanation.

Einstein did not see that the Distance-Difference Invariance is the principle that had to be postulated, as fundamental physics does not follow from trivial postulates such as the constancy of light speed.

Presenting the Lorentz Transformation as the result of the trivial constancy of light-speed, discredited the Lorentz Transformation, and kept it least believable.

Being equivalent to the Energy Invariance principle, the Lorentz Transformation is a fundamental principle of Physics that constancy of light speed, and indices of tensors cannot produce.

With the Aether being irrelevant, the importance of the Michelson-Morley Experiment is in confirming the Lorentz Transformation, and establishing it as equivalent to Energy-Momentum-Difference Invariance in Space-Time

The experiment validates the formulas of velocity transformations under Lorentz Transformations, and confirms the Lorentz Transformation.

We apply the Energy-Momentum Invariance under Lorentz Transformation to Mass Energy, and to Electromagnetic Energy.

The Mass Energy Form $E^2 - (cp)^2$ is Invariantly equal to $(m_0c^2)^2$.

The Electromagnetic Energy Form $\mathcal{E}^2 - (c\mathcal{P})^2$ in free space is Invariantly equal to $\frac{1}{4}\varepsilon_0^2(E_z^2 - c^2B_y^2)^2$.

And the Electromagnetic Energy-Momentum Difference $(d\mathcal{E})^2 - (cd\mathcal{P})^2$ in free space is Invariantly equal to

$$\varepsilon_0^2\left[E_z^2 - c^2B_y^2\right]\left[(dE_z)^2 - c^2(dB_y)^2\right].$$

These Invariances are fundamental Energy Principles.

keywords Michelson-Morley, Interferometer, Aether, Photon, Planck's Radiation Law, Lorentz Transformations, Velocity Transformations, Special Relativity, Lorentz-Fitzgerald Contraction,

Galilean Transformations, Space-Time Energy Invariance, Space-Time Momentum Invariance, Space-Time Distance Invariance, Invariant Space-Time Momentum Axiom,

Physics Astronomy Classification Scheme 03.30.+

Contents

0.

The Hypothetical Aether

Wave propagation in Fluids, is explained by the harmonic motion of the fluid molecules. The harmonic motion of each molecule in the wave cross-section causes the harmonic motion of the molecule next to it, and the sinusoidal profile that we call wave propagation is virtual. The fluid does not propagate in any "wavy" way. A disturbance propagates in the fluid with a sinusoidal cross-section.

Electrodynamics was visualized by analogy with the Mechanics of Fluids. And Electromagnetic wave propagation was modeled after wave propagation in fluids. Then, the Aether was hypothesized as a medium that permeates the universe and enables electro-magnetic disturbance to propagate, similarly to the way a disturbance propagates in fluids.

However, what we call Electromagnetic waves are made of diluted material. The 1902 Planck's Radiation Law established that electromagnetic energy is made of packets of energy

$$h\nu\,,$$

which according to mass-energy equivalence, have mass

$$m = \frac{h\nu}{c^2}.$$

The hypothetical Aether is irrelevant to the motion of the photons, if not slow them down, or absorb them.

The existence of photons has been well-established by Planck's radiation law, by the photoelectric effect, by the Spectrum of the Hydrogen Atom, and by the Compton effect.

That existence makes the Aether irrelevant.

Nevertheless, in 1887, this was not known to Michelson and Morley.

To prove, or disprove the existence of Aether. Michelson, and Morley set up an experiment to detect the speed of the earth, in an immobile Aether.

1.

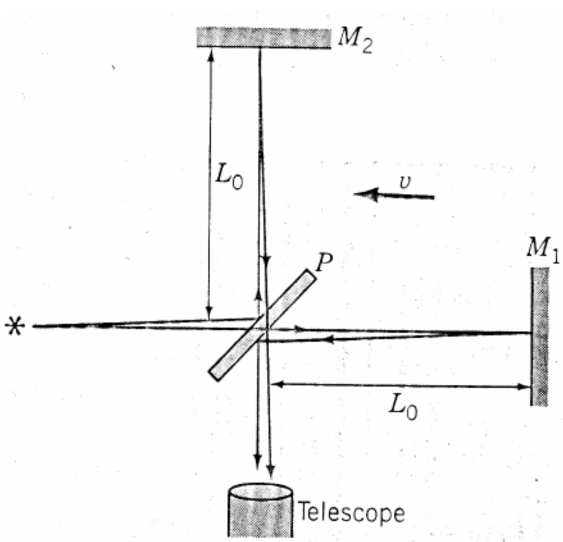
The Michelson-Morley Experiment to Detect the Aether

The motion of the earth in the Aether at speed v generates an "Aether wind" with velocity $-v$.

A light beam that travels time T_1, parallel to the earth's motion, interferes with a light-beam that travels time T_2, perpendicular to the motion.

The time-travel difference results in a wave-phase difference, and an interference pattern at the telescope.

A rotation of the Interferometer will shift the interference fringes, by an amount determined by v.



Michelson-Morley estimates for the time difference $T_1 - T_2$, were Galilean.

The speed of the beam to the mirror M_1, was taken as $c - v$

The speed of the beam returning from M_1, was taken as $c + v$

The Galilean distance in both directions is L_0,

and the Galilean time interval is

$$T_1 = \frac{L_0}{c - v} + \frac{L_0}{c + v} = \frac{2L_0}{c} \frac{1}{1 - \frac{v^2}{c^2}}$$

In a right angle triangle where c is the hypotenuse, and the Aether wind speed is one side, the speed of the beam to the mirror M_2, and back is $\sqrt{c^2 - v^2}$,

the Galilean distance in both directions is L_0,

and the Galilean time interval is

$$T_2 = \frac{L_0}{\sqrt{c^2 - v^2}} + \frac{L_0}{\sqrt{c^2 - v^2}} = \frac{2L_0}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The Galilean time difference is

$$T_1 - T_2 = \frac{2L_0}{c} \left(\frac{1}{1 - \frac{v^2}{c^2}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right).$$

For $v \ll c$,

$$T_1 - T_2 \approx \frac{2L_0}{c} \left\{ (1 + \tfrac{v^2}{c^2}) - (1 + \tfrac{1}{2}\tfrac{v^2}{c^2}) \right\} = \frac{L_0}{c} \frac{v^2}{c^2}.$$

The measured shift indicated

$$v \approx 0\,,$$

contradicting the fact that $v \approx 30\text{km/sec}$

This established no immobile Aether.

That did not sit well with the Aether believers, and the struggle over the hypothetical Aether started.

That struggle continues to these days, regardless of the irrelevance of Aether.

We aim to show that the importance of the Michelson-Morley Experiment is in confirming the Lorentz Transformation, and establishing it as the fundamental Energy Invariance principle of Space-Time Physics

2.

Lorentz Tailored Transformation, and the Fitzgerald Contraction

To support the hypothetical Aether in spite of Michelson-Morley null result, Lorentz tailored his coordinate transformation formula, and Fitzgerald used it to suggest that in the direction of the motion the distance L_0 contracts by a factor of

$$\sqrt{1-\frac{v^2}{c^2}}.$$

Consequently,

$$T_1 = \frac{L_0\sqrt{1-\frac{v^2}{c^2}}}{c-v} + \frac{L_0\sqrt{1-\frac{v^2}{c^2}}}{c+v} = \frac{2L_0}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}},$$

and

$$T_1 - T_2 = \frac{2L_0}{c}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right) = 0.$$

Feynman [Feynman, I-15-5] notes that

> *…Although the contraction hypothesis successfully accounted for the negative result of the experiment, it was open to the objection that it was invented for the express purpose of explaining away the difficulty, and was too artificial…*

Nevertheless, Feynman proceeds to justify the Lorentz Contraction with arguments adapted from Einstein's Relativity.

We proceed to interpret the Michelson-Morley Experiment in terms of Lorentz Coordinate Transformations.

3.

Michelson-Morley Experiment Under Lorentz Transformation

In the interferometer system S',

the speed of the Aether wind is $-v$.

The speed of the beam to the mirror M_1 is

$$u_x' = c.$$

Thus, the speed of the beam to the mirror M_1 in the observer system S is

$$u_x = \frac{c - v}{1 + \frac{c(-v)}{c^2}} = c.$$

The distance traveled to M_1 in the observer system S is

$$\frac{L_0 - v\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Therefore, the beam travel-time to M_1 in the observer system S is

$$\frac{L_0 - v\Delta t'}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

The speed of the beam returning from the mirror M_1 is

$$u_x' = -c.$$

Thus, the speed of the beam returning from the mirror M_1 in the observer system S is

$$u_x = \frac{-c - v}{1 + \frac{(-c)(-v)}{c^2}} = -c.$$

The distance traveled from M_1 in the observer system S is

$$\frac{-L_0 - v\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Therefore, the beam travel-time return from M_1 in the observer system S is

$$\frac{L_0 + v\Delta t'}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

Consequently, the beam time interval to go to M_1, and return from M_1, in the observer system S is

$$T_1 = \frac{L_0 - v\Delta t'}{c\sqrt{1 - \frac{v^2}{c^2}}} + \frac{L_0 + v\Delta t'}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{2L_0}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

This contradicts Michelson-Morley's $\frac{2L_0}{c}\frac{1}{1 - \frac{v^2}{c^2}}$.

The speed of the beam to the mirror M_2 is

$$u_y' = c.$$

Thus, the speed of the beam to the mirror M_2 in the observer system S is

$$u_y = \frac{c}{1+\frac{c(-v)}{c^2}}\sqrt{1-\frac{v^2}{c^2}} = c\frac{\sqrt{c^2-v^2}}{c-v}.$$

The distance traveled to M_2 in the observer system S is L_0

Therefore, the beam travel-time to M_2 in the observer system S is

$$\frac{L_0}{c\sqrt{c^2-v^2}}(c-v)$$

The speed of the beam returning from the mirror M_2 is

$$u_y^{'} = -c.$$

Thus, the speed of the beam returning from the mirror M_2 in the observer system S is

$$u_y = \frac{-c}{1+\frac{(-c)(-v)}{c^2}}\sqrt{1-\frac{v^2}{c^2}} = -c\frac{\sqrt{c^2-v^2}}{c+v}.$$

The distance traveled from M_2 in the observer system S is $-L_0$

Therefore, the beam travel-time from M_2 in the observer system S is

$$\frac{L_0}{c\sqrt{c^2-v^2}}(c+v)$$

Consequently, the beam time interval to go to M_2, and return from M_2, in the observer system S is

$$T_2 = \frac{L_0}{c\sqrt{c^2 - v^2}}(c - v) + \frac{L_0}{c\sqrt{c^2 - v^2}}(c + v)$$

$$= \frac{2L_0}{c\sqrt{1 - \frac{v^2}{c^2}}}.$$

This agrees with Michelson-Morley's $\frac{2L_0}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$.

Finally, the time difference is

$$T_1 - T_2 = \frac{2L_0}{c}\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = 0$$

That is, the time difference does not depend on the speed of the Aether wind, and no shift should be expected.

Thus, the Aether believers claim that Michelson-Morley Experiment does not rule out Aether.

And they miss the big picture. The irrelevant Aether was only the lead to the fact that the Lorentz Transformation is equivalent to the Invariance of Energy in Space-Time.

What Feynman did not notice was that the light-speed Axiom that

the speed of light is c *in any frame.*

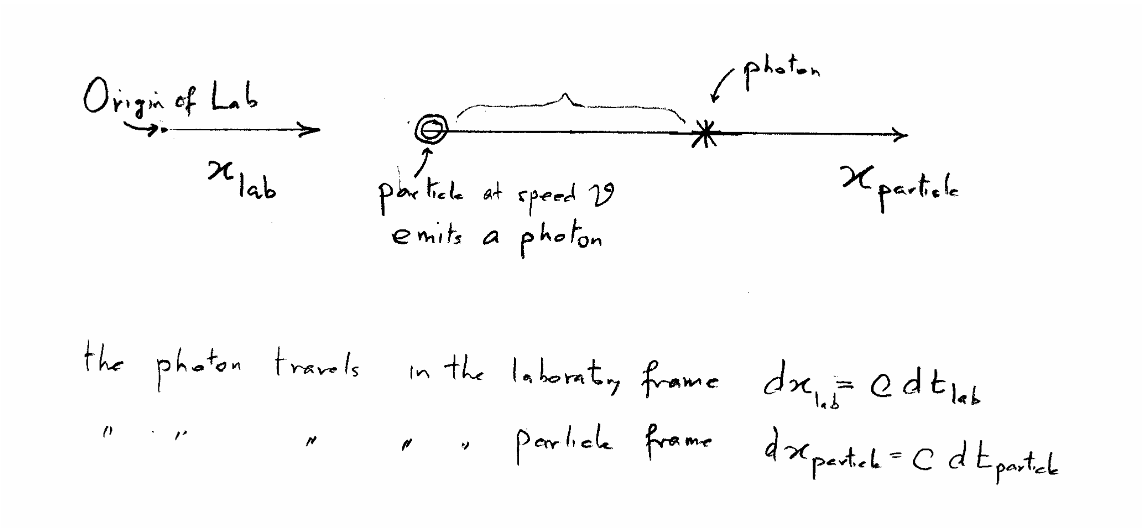
is insufficient to imply the Lorentz Transformation, and the Invariance of Energy in Space-Time.

We proceed to the setup in which Einstein failed to derive the Lorentz Transformation.

4.

Light Speed Constancy $\not\Rightarrow$ Distance-Difference Invariance

A particle moves at speed $v < c$ in the x direction, passes through the Lab's origin, and emits[1] a **photon** that moves with speed c, in the x direction.



In the Lab frame, the photon at time

$$t_{lab}$$

is at location

$$x_{lab}$$

and its speed is

$$c = \frac{dx_{lab}}{dt_{lab}}$$

Therefore,

[1] Emission requires an accelerating electron/proton. Here, the photon is bouncing off the particle

$$cdt_{lab} = dx_{lab}$$

In the particle frame, the photon at time

$$t_{par}$$

is at location

$$x_{par}$$

and its speed is

$$c = \frac{dx_{par}}{dt_{par}}$$

Therefore,

$$cdt_{par} = dx_{par}$$

Thus,

$$(dx_{par})^2 - (cdt_{par})^2 = 0 = (dx_{lab})^2 - (cdt_{lab})^2 .$$

This does not imply that the distance-difference

$$(dx)^2 + \left(icdt\right)^2$$

has invariant length in all frames.□

If such invariance is necessary, it needs to be postulated.

Then, the equivalent to it, the Lorentz Transformation cannot be derived.

In his eagerness to derive the Lorentz Transformation, Einstein missed the meaning of it as equivalent to Energy Invariance in Space-Time.

As such the Lorentz Transformation is a fundamental Physics principle that cannot be derived

However, Einstein's original argument was Galilean.

5.

Einstein's Galilean Argument

In his 1905 paper "*Electrodynamics of Moving Bodies*"[2],

Einstein has the

stationary frame (x, t) ,

and

the moving-along-the-x-axis frame (ξ, τ).

He denotes

$$x' = x - vt,$$

defines

$$\tau = \tau(x - vt, t),$$

And applies the principle of the constancy of the speed of light.

At time $\tau_0 = \tau(0, t)$,

light ray is sent from the origin of the moving frame, towards x'

At time $\tau_1 = \tau\left(x', t + \frac{x'}{c - v}\right)$,

the light is reflected from x' back to the moving-frame origin

At time $\tau_2 = \tau\left(0, t + \frac{x'}{c - v} + \frac{x'}{c + v}\right)$,

the light arrives at the moving-frame origin.

Then, we have

$$\frac{\tau_0 + \tau_2}{2} = \tau_1,$$

[2] In "Einstein the principle of Relativity", Dover, 1952, p.43

$$\frac{1}{2}\left\{\tau(0,t)+\tau\left(0,t+\frac{x'}{c-v}+\frac{x'}{c+v}\right)\right\}=\tau\left(x',t+\frac{x'}{c-v}\right)$$

We need to go no further. Because the Galilean terms

$$\frac{x'}{c-v}, \text{ and } \frac{x'}{c+v}$$

indicate that Einstein was using Galilean arguments, to derive the Lorentz Transformation.

Einstein Did Not repeat this argument in his later writings.

Instead, he replaced it with an "It must be" argument.

6.

Einstein's "It Must Be" Argument

In his "*Manuscript on Special Relativity*"[3]

Einstein insists that if two differential forms are equal at one instance, they are equal at all instances. He wrote,

> *"...the equations*
>
> $$x^2 - c^2t^2 = 0$$
>
> *and*
>
> $$x'^2 - c^2t'^2 = 0$$
>
> *must be equivalent.*
>
> *The transformation equations must be so constituted that the second equation turns in to the first if*
>
> $$x',\ t'$$
>
> *are replaced by their expressions in terms of*
>
> $$x,\ t.$$
>
> *The transformation must make the equation*
>
> $$\lambda^2(x^2 - c^2t^2) = x^2 - c^2t'^2$$
>
> *into an identity, where* λ^2 *is not zero.*
>
> λ^2 *must be independent of* x, *and* t.
>
> *Else, the right hand side divided by* λ^2 *will not be homogeneous complete function of second order in* x, t, *after the substitution is carried out.*

[3] "The collected papers of Albert Einstein, Volume 4, p. 32, Princeton.

We will show later that from physical point of view, the only case deserving consideration is $\lambda^2 = 1.$″

Had Einstein's Arguments here been solid, they would have been the greatest physical insight ever.

Not comprehending the Lorentz Transformation equivalence to the Energy-Difference Invariance in Space-Time, Einstein was trying to show that the equivalent Distance-Difference Invariance follows from the trivial constancy of the speed of light in all frames.

We proceed to show what Einstein missed:

Energy-Difference Invariance implies the Lorentz Transformation.

7.

Energy-Difference Invariance $\Rightarrow$ Lorentz Transformation

7.1 Time-Space Energy

A particle moves at speed $v < c$, in the x direction, passes through the Lab's origin, and reflects a photon which moves at speed c, in the x direction.

Assume that the photons' speed is c in any uniformly moving reference frames.

The photon speed is

$$c = \frac{dx}{dt}$$

$$= \frac{\frac{dp}{dt}dx}{dp}$$

$$= \frac{Fdx}{dp}$$

$$= \frac{dE}{dp}.$$

Therefore,

$$dE = cdp,$$

$$\left(dE\right)^2 = (cdp)^2$$

$$\left(dE\right)^2 + (icdp)^2 = 0.$$

The differentials

$$dE, \quad \text{and} \quad cdp$$

are the components of the infinitesimal Energy Vector

$$\begin{pmatrix} dE \\ icdp \end{pmatrix}$$

Its squared length is

$$(dE)^2 + (icdp)^2.$$

7.2 Energy-Difference Invariance Principle

$$\left(dE\right)^2 + (icdp)^2 \text{ is invariant}$$

in any frame related by Linear Transformation

7.3 Distance-Difference Invariance Principle

$$(dx)^2 + \left(icdt\right)^2 \text{ is invariant}$$

in any frame related by Linear Transformation

7.4 Energy-Difference Invariance $\Rightarrow$ Lorentz Transformation

Proof:

Assuming that the Energy-Difference transforms linearly,

$$\begin{bmatrix} dE_{par} \\ cdp_{par} \end{bmatrix} = \begin{bmatrix} \gamma_1 & \beta_1 \\ \beta_2 & \gamma_2 \end{bmatrix} \begin{bmatrix} dE_{lab} \\ cdp_{lab} \end{bmatrix},$$

That is,

$$dE_{par} = \gamma_1 dE_{lab} + \beta_1 cdp_{lab},$$

$$cdp_{par} = \beta_2 dE_{lab} + \gamma_2 cdp_{lab}.$$

By the Energy-Difference Invariance,

$$(dE_{lab})^2 - (cdp_{lab})^2 = (dE_{par})^2 - (cdp_{par})^2$$

$$= (\gamma_1 dE_{lab} + \beta_1 cdp_{lab})^2 - (\beta_2 dE_{lab} + \gamma_2 cdp_{lab})^2$$

$$= (\gamma_1 dE_{lab})^2 + 2\gamma_1\beta_1 cdE_{lab}dp_{lab} + (\beta_1 cdp_{lab})^2$$

$$-(\beta_2 dE_{lab})^2 - 2\beta_2\gamma_2 cdE_{lab}dp_{lab} - (\gamma_2 cdp_{lab})^2$$

Equating Coefficients,

$$\gamma_1^2 - \beta_2^2 = 1,$$

$$\beta_1^2 - \gamma_2^2 = -1 \Rightarrow \gamma_2^2 - \beta_1^2 = 1,$$

$$\gamma_1\beta_1 - \beta_2\gamma_2 = 0 \quad \Rightarrow \quad \gamma_1\beta_1 = \beta_2\gamma_2.$$

We obtain

$$\gamma_1 = \gamma_2 \equiv \gamma,$$

$$\beta_1 = \beta_2 \equiv b,$$

or

$$\gamma_1 = -\gamma_2 \equiv \gamma,$$

$$\beta_1 = -\beta_2 \equiv b,$$

so that in either case

$$\gamma^2 - b^2 = 1.$$

Now, for $\delta E_{par} = 0$,

$$0 = \gamma\delta E_{lab} + bc\delta p_{lab},$$

$$\frac{bc}{\gamma} = -\frac{\delta E_{lab}}{\delta p_{lab}} = -\frac{\delta x_{lab}}{\delta t_{lab}} = -\upsilon,$$

$$b = -\gamma\frac{\upsilon}{c}$$

Substituting in $\gamma^2 - b^2 = 1$,

$$\gamma^2 - \gamma^2 \frac{\upsilon^2}{c^2} = 1,$$

$$\gamma^2 (1 - \frac{\upsilon^2}{c^2}) = 1,$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}.$$

$\underline{\upsilon < c}$,

7.5 $$\begin{bmatrix} dE_{par} \\ cdp_{par} \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} \begin{bmatrix} 1 & -\frac{\upsilon}{c} \\ -\frac{\upsilon}{c} & 1 \end{bmatrix} \begin{bmatrix} dE_{lab} \\ cdp_{lab} \end{bmatrix}$$

In the inverse transformation, $-\upsilon$ replaces υ

7.6 $$\begin{bmatrix} dE_{lab} \\ cdp_{lab} \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} \begin{bmatrix} 1 & \frac{\upsilon}{c} \\ \frac{\upsilon}{c} & 1 \end{bmatrix} \begin{bmatrix} dE_{par} \\ cdp_{par} \end{bmatrix}$$

The Distance-Difference transforms by

7.7 $$\begin{bmatrix} dx_{par} \\ cdt_{par} \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} \begin{bmatrix} 1 & -\frac{\upsilon}{c} \\ -\frac{\upsilon}{c} & 1 \end{bmatrix} \begin{bmatrix} dx_{lab} \\ cdt_{lab} \end{bmatrix}$$

In the inverse transformation, $-\upsilon$ replaces υ,

7.8
$$\begin{bmatrix} dx_{lab} \\ cdt_{lab} \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \begin{bmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{bmatrix} \begin{bmatrix} dx_{par} \\ cdt_{par} \end{bmatrix}.$$

8.

Lorentz Transformation ⇒ Energy-Difference Invariance

Proof:

$$(dE_{par})^2 - (cdp_{par})^2 = \gamma^2 \left\{ \left[dE_{lab} - \upsilon dp_{lab} \right]^2 - \left[-\frac{\upsilon}{c} dE_{lab} + cdp_{lab} \right]^2 \right\}$$

$$= \gamma^2 \left\{ (dE_{lab})^2 \left(1 - \frac{\upsilon^2}{c^2} \right) - (cdp_{lab})^2 \left(1 - \frac{\upsilon^2}{c^2} \right) \right\}$$

$$= \underbrace{\gamma^2 \left(1 - \frac{\upsilon^2}{c^2} \right)}_{1} \left\{ (dE_{lab})^2 - (cdp_{lab})^2 \right\}$$

$$= (dE_{lab})^2 - (cdp_{lab})^2 . \square$$

Thus,

8.1 The Lorentz Transformation is Equivalent to the Energy-Difference Invariance

9.

Lorentz Transformation $\Rightarrow$ Distance-Difference Invariance

Proof:

$$(dx_{lab})^2 - c^2(dt_{lab})^2 = \gamma^2\left\{\left[dx_{par} + \upsilon dt_{par}\right]^2 - \left[\frac{\upsilon}{c}dx_{par} + cdt_{par}\right]^2\right\}$$

$$= \gamma^2\left\{(dx_{par})^2\left(1 - \frac{\upsilon^2}{c^2}\right) - c^2(dt_{par})^2\left(1 - \frac{\upsilon^2}{c^2}\right)\right\}$$

$$= \underbrace{\gamma^2\left(1 - \frac{\upsilon^2}{c^2}\right)}_{1}\left\{(dx_{par})^2 - c^2(dt_{par})^2\right\}$$

$$= (dx_{par})^2 - c^2(dt_{par})^2 . \square$$

Thus,

9.1 The Lorentz Transformation is Equivalent to the Distance-Difference Invariance

Consequently,

9.2 The Lorentz Transformation is a Rotation of the Distance-Difference in Space-Time

9.3 Distance-Difference Invariance, and Energy-Difference Invariance are Equivalent to the Lorentz Transformation, and hence, to each other

10.

Momentum-Difference Invariance

Instead of the Energy-Difference Vector, we can use the Space time Momentum-Difference Vector

$$\begin{pmatrix} \frac{1}{c}dE \\ idp \end{pmatrix}$$

Its squared length is

$$(\frac{1}{c}dE)^2 + (idp)^2.$$

10.2 Momentum-Difference Invariance Principle

$$\left(\frac{1}{c}dE\right)^2 + (idp)^2 \textit{ is invariant}$$

in any frame related by Linear Transformation

We derive similarly to 7.4,

10.3 Momentum-Difference Invariance $\Rightarrow$

Lorentz Transformation

Then,

10.4

$$\begin{bmatrix} \frac{1}{c}dE_{par} \\ dp_{par} \end{bmatrix} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\begin{bmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{c}dE_{lab} \\ dp_{lab} \end{bmatrix}$$

In the inverse transformation, $-\upsilon$ replaces υ

10.5
$$\begin{bmatrix} \frac{1}{c} dE_{lab} \\ dp_{lab} \end{bmatrix} = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} \begin{bmatrix} 1 & \frac{\upsilon}{c} \\ \frac{\upsilon}{c} & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{c} dE_{par} \\ dp_{par} \end{bmatrix}$$

10.6 The Lorentz Transformation is Equivalent to the Momentum-Difference Invariance

Consequently,

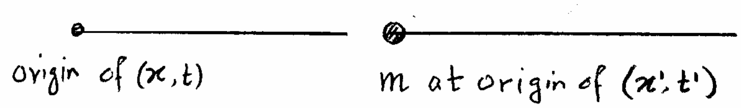
10.7 The Lorentz Transformation is a Rotation of the Momentum-Difference in Space-Time

10.8 The Distance-Difference Invariance, the Energy-Difference Invariance, and the Momentum-Difference Invariance are Equivalent to the Lorentz Transformation, and hence, to each other

11.

The Mass Energy Form is Invariantly $(m_0 c^2)^2$

The origin of (x', t') is moving at speed v:



In (x', t'), the mass has speed zero,

Mass Momentum,

$$p' = 0,$$

and Mass Energy

$$E' = m_0 c^2.$$

Its Mass Energy-Momentum form is

$$(E')^2 - (cp')^2 = (m_0 c^2)^2$$

In (x, t), the mass is moving at speed

$$v,$$

has mass Momentum

$$p = (m_0 \gamma) v,$$

and mass Energy

$$E = (m_0 \gamma) c^2,$$

Its mass Energy-Momentum Form is

$$(E)^2 - (cp)^2 = (m_0 \gamma c^2)^2 - (c m_0 \gamma v)^2$$

$$= (m_0 c^2)^2 \frac{c^2}{c^2 - v^2} - c^2 (m_0)^2 \frac{c^2}{c^2 - v^2} v^2$$

$$= (m_0 c^2)^2 \frac{1}{c^2 - v^2} (c^2 - v^2)$$

$$= (m_0 c^2)^2 = (E')^2 - (cp')^2 . \square$$

That is,

$$\boxed{(E)^2 - (cp)^2 = (m_0 c^2)^2}.$$

The Mass Energy-Momentum Form is invariantly equal to $(m_0 c^2)^2$.

12.

The Mass Energy Difference is Invariantly Zero

The Energy Difference is

$$dE = d\left[(m_0\gamma)c^2\right] = 0$$

The Momentum Difference is

$$dp = d\left[(m_0\gamma)v\right] = 0$$

The Mass Energy-Momentum difference is

$$(dE)^2 - (cdp)^2 = 0 . \square$$

13.

The Electromagnetic Energy Form is Invariantly $\frac{1}{4}\varepsilon_0^2(E_z^2 - c^2 B_y^2)^2$

The Lab (x, y, z) and the Lab (x', y', z'), have parallel frames.

The origin of the (x', y', z') lab moves at speed

$$v_x = v \text{ along the } x \text{ axis in } (x, y, z).$$

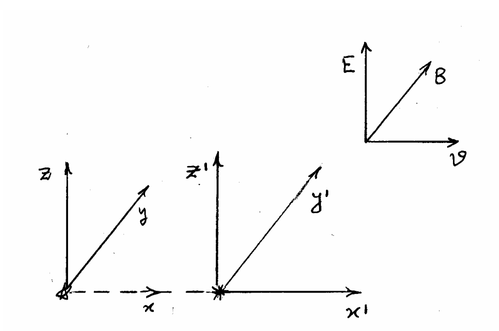
The Electromagnetic Radiation propagates in the x direction,

The Magnetic Induction is solely along y. In (x, y, z) it is

$$B_y, \quad \text{And} \quad B_x = B_z = 0.$$

The Electric Field is solely along z. In (x, y, z) it is

$$E_z, \quad \text{And } E_x = E_y = 0.$$



In (x', y', z') the Electromagnetic Fields are

$$E_{x'} = E_x = 0,$$

$$E_{y'} = \gamma(E_y - vB_z) = 0,$$

$$E_{z'} = \gamma(E_z + vB_y)$$

$$B_{x'} = B_x = 0$$

$$B_{y'} = \gamma(B_y + \frac{v}{c^2}E_z)$$

$$B_{z'} = \gamma(B_z - \frac{v}{c^2}E_y) = 0$$

13.1 The Electromagnetic Energy-Momentum Form

The electromagnetic energy in (x, y, z) is

$$\mathcal{E} = \frac{1}{2}\varepsilon_0 E_z^2 + \frac{1}{2\mu_0}B_y^2.$$

Since $\varepsilon_0\mu_0 c^2 = 1$,

$$\mathcal{E} = \frac{1}{2}\varepsilon_0(E_z^2 + c^2 B_y^2).$$

The Electromagnetic Momentum in the vacuum in (x, y, z) is

$$\mathcal{P} = \frac{1}{c^2}\vec{E} \times \vec{H},$$

$$= \frac{1}{\mu_0 c^2}E_z B_y,$$

$$= \varepsilon_0 E_z B_y$$

The Electromagnetic Energy-Momentum form in (x, y, z) is

$$\mathcal{E}^2 - (c\mathcal{P})^2 = \varepsilon_0^2\left\{\frac{1}{4}(E_z^2 + c^2B_y^2)^2 - c^2E_z^2B_y^2\right\}$$

$$\boxed{\mathcal{E}^2 - (c\mathcal{P})^2 = \frac{1}{4}\varepsilon_0^2(E_z^2 - c^2B_y^2)^2}.$$

We specified B_y, and E_z to clarify that $\vec{B}$ is non-zero only along the y axis, and $\vec{E}$ is non-zero only along the z axis.

13.2 The Electromagnetic Energy-Momentum Form is Invariant Under the Lorentz Transformation

The Electromagnetic Energy-Momentum form in (x', y', z') is

$$(\mathcal{E}')^2 - (c\mathcal{P}')^2 = \frac{1}{4}\varepsilon_0^2(E_{z'}^2 - c^2 B_{y'}^2)^2$$

$$= \frac{1}{4}\varepsilon_0^2\left(\gamma^2[E_z + vB_y]^2 - c^2\gamma^2[B_y + \frac{v}{c^2}E_z]^2\right)^2$$

$$= \frac{1}{4}\varepsilon_0^2\left(\gamma^2[E_z^2 + 2E_z vB_y + v^2 B_y^2] - \gamma^2[B_y^2 c^2 + 2E_z vB_y + \frac{v^2}{c^2}E_z^2]\right)^2$$

$$= \frac{1}{4}\varepsilon_0^2\left(\underbrace{\gamma^2(1 - \frac{v^2}{c^2})}_{1}E_z^2 - \underbrace{\gamma^2(c^2 - v^2)}_{c^2}B_y^2\right)^2$$

$$= \frac{1}{4}\varepsilon_0^2\left(E_z^2 - c^2 B_y^2\right)^2 = \mathcal{E}^2 - (c\mathcal{P})^2.$$

The Electromagnetic Energy-Momentum Form is invariantly equal to

$$\frac{1}{4}\varepsilon_0^2\left(E_z^2 - c^2 B_y^2\right)^2. \square$$

14.

The Electromagnetic Energy Difference is Invariantly

$$\varepsilon_0^2\left[E_z^2 - c^2B_y^2\right]\left[(dE_z)^2 - c^2(dB_y)^2\right]$$

The Lab (x, y, z) and the Lab (x', y', z'), have parallel frames.

The origin of the (x', y', z') lab moves at speed

$v_x = v$ along the x axis in (x, y, z).

The Magnetic Induction Difference in (x, y, z) is

dB_y along the y axis; $dB_x = dB_z = 0$.

The Electric Field Difference in (x, y, z) is

dE_z along the z axis; $dE_x = dE_y = 0$.

In (x', y', z') the Electromagnetic Fields Differences are

$$dE_{x'} = dE_x = 0,$$

$$dE_{y'} = \gamma(dE_y - vdB_z) = 0,$$

$$\boxed{dE_{z'} = \gamma(dE_z + vdB_y)}$$

$$dB_{x'} = dB_x = 0$$

$$\boxed{dB_{y'} = \gamma(dB_y + \frac{v}{c^2}dE_z)}$$

$$dB_{z'} = \gamma(dB_z - \frac{v}{c^2}dE_y) = 0$$

14.1 The Electromagnetic Energy-Momentum Difference

The electromagnetic energy Difference in (x,y,z) is

$$d\mathcal{E} = \frac{1}{2}\varepsilon_0 d(E_z^2) + \frac{1}{2\mu_0}d(B_y^2).$$

Since $\varepsilon_0\mu_0 c^2 = 1$,

$$d\mathcal{E} = \frac{1}{2}\varepsilon_0\left[d(E_z^2) + c^2 d(B_y^2)\right]$$

$$= \varepsilon_0\left[E_z dE_z + c^2 B_y dB_y\right]$$

The Electromagnetic Momentum in the vacuum in (x,y,z) is

$$d\mathcal{P} = \frac{1}{c^2}d(\vec{E}\times\vec{H}),$$

$$= \frac{1}{\mu_0 c^2}d(E_z B_y)$$

$$= \varepsilon_0\{E_z dB_y + B_y dE_z\}$$

The Electromagnetic Energy-Momentum Difference in (x,y,z) is

$$(d\mathcal{E})^2 - (cd\mathcal{P})^2 = \varepsilon_0^2\left\{\left[E_z dE_z + c^2 B_y dB_y\right]^2 - c^2\left[E_z dB_y + B_y dE_z\right]^2\right\},$$

$$= \varepsilon_0^2\left[E_z^2 - c^2 B_y^2\right]\left[(dE_z)^2 - c^2(dB_y)^2\right].$$

That is,

$$\boxed{(d\mathcal{E})^2 - (cd\mathcal{P})^2 = \varepsilon_0^2\left[E_z^2 - c^2 B_y^2\right]\left[(dE_z)^2 - c^2(dB_y)^2\right]}$$

14.2 The Electromagnetic Energy-Momentum is Invariant under Lorentz Transformation

The Electromagnetic Energy-Momentum Difference in (x', y', z') is

$$(d\mathcal{E}')^2 - (cd\mathcal{P}')^2 = \varepsilon_0^2\Big[E_{z'}^2 - c^2 B_{y'}^2\Big]\Big[(dE_{z'})^2 - (dB_{y'})^2\Big]$$

$$= \varepsilon_0^2\left(\gamma^2[E_z + vB_y]^2 - c^2\gamma^2[B_y + \frac{v}{c^2}E_z]^2\right)\times$$

$$\times\left(\gamma^2[dE_z + vdB_y]^2 - c^2\gamma^2[dB_y + \frac{v}{c^2}dE_z]^2\right)$$

$$= \varepsilon_0^2\gamma^2\left([E_z + vB_y]^2 - [cB_y + \frac{v}{c}E_z]^2\right)\times$$

$$\times\gamma^2\left([dE_z + vdB_y]^2 - [cdB_y + \frac{v}{c}dE_z]^2\right)$$

$$= \varepsilon_0^2\gamma^2\left(\left[1 - \frac{v^2}{c^2}\right]E_z^2 - (c^2 - v^2)B_y^2\right)\times$$

$$\times\gamma^2\left(\left[1 - \frac{v^2}{c^2}\right](dE_z)^2 - (c^2 - v^2)(dB_y)^2\right)$$

$$= \varepsilon_0^2\left(\underbrace{\gamma^2\left[1 - \frac{v^2}{c^2}\right]}_{1}E_z^2 - \underbrace{\frac{c^2}{c^2 - v^2}(c^2 - v^2)}_{c^2}B_y^2\right)\times$$

$$\times\left(\underbrace{\gamma^2\left[1 - \frac{v^2}{c^2}\right]}_{1}(dE_z)^2 - \underbrace{\frac{c^2}{c^2 - v^2}(c^2 - v^2)}_{c^2}(dB_y)^2\right)$$

$$= \varepsilon_0^2\Big[E_z^2 - c^2 B_y^2\Big]\Big[(dE_z)^2 - c^2(dB_y)^2\Big] = (d\mathcal{E})^2 - (cd\mathcal{P})^2.$$

The Electromagnetic Energy-Momentum Difference is invariantly equal to

$$\varepsilon_0^2\left[E_z^2 - c^2 B_y^2\right]\left[(dE_z)^2 - c^2 (dB_y)^2\right].\square$$

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