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[Gauge Institute Journal](#)

Integration

[Lebesgue Integration](#) H. Vic Dannon

Abstract:

We show that

1. the Riemann integral may exist when the Lebesgue integral does not
2. the Fundamental Theorem of Calculus may hold for Riemannian Integration but not for Lebesgue integration of the same function
3. The same limit function of Integrable functions may be Riemann-Integrable but not Lebesgue integrable.

Therefore, contrary to Common belief, Riemann Integration is more general than Lebesgue Integration.

Furthermore, we show that a convergent sequence of measurable functions may have a non-measurable limit function.

This cast a doubt on the validity of the main results of Lebesgue theory.

Consequently, Riemannian Integration is not only a superior Integration Theory. It is the only Integration Theory that we have.

Riemannian Integration H. Vic Dannon

Abstract: Riemann's Theory of Integration appeared in Riemann's paper "On the Representation of a Function by a Trigonometric Series".

There, Riemann

1. presents his Oscillation Conditions for integrability,
2. establishes the integrability of his example of a series of functions with infinitely many discontinuities in any interval of real numbers, and
3. obtains a sequence of necessary and sufficient conditions for the integration of a singular function.

These fundamental results constitute the Riemannian Integration Theory that we present here

In a forthcoming article, we show that Infinitesimal Calculus allows integration of functions that are not Riemann Integrable.

Keywords: Riemann, Integration, Integrability, Oscillation, Discontinuity, Singularity,

2000 Math Subject Classification: 26A42, 26A30, 26A15,