

American Mathematical Monthly claims that in a Circular Motion, $\vec{r}'' = 0$, there is no Gravitational Force, the Earth does not orbit the Sun, and the Sun's Mass is Zero

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Introduction

In a circular motion,

$$\begin{aligned}r^2(t) &= \text{constant}, \\2rr' &= 0, \\(r')^2 + rr'' &= 0, \\r'' &= -\frac{(r')^2}{r} = -\frac{v^2}{r}.\end{aligned}$$

In Mechanics, the radial force on an orbiting mass m is

$$mr'' = m\frac{v^2}{r}$$

In Gravitation, the centripetal force on the earth balances with the Force between the Sun and the Earth.

In the Hydrogen Atom, the centripetal force on the electron balances with the Force between the electron and the Proton.

The American Mathematical Monthly [Saari, p.409], claims that

“If $\vec{r}(t)$ is the vector position of a planet with mass m ,

relative to the sun with mass M ,

$r = |\vec{r}|$ is the vector’s length, and

G is the gravitational constant,

then, Newton’s inverse-square force law is

$$m\vec{r}'' = -G \frac{Mm}{r^2} \frac{\vec{r}}{r}, \quad (1)$$

where \vec{r}'' is the acceleration.

To find the radial acceleration r'' , differentiate the scalar product $r^2 = (\vec{r}, \vec{r})$ twice. Using

$$\vec{r}^2 \vec{v}^2 = (\vec{r} \cdot \vec{v})^2 + (\vec{r} \times \vec{v})^2 = r^2 v^2 + r^2 v_{rot}^2 \quad (*)$$

where $\vec{v} = \dot{\vec{r}}$ is the velocity, and v_{rot} is the rotational velocity, it follows by substitution into equation 1 that the scalar acceleration satisfies

$$mr'' = -G \frac{Mm}{r^2} + \frac{mv_{rot}^2}{r} \quad (2)$$

Earth orbit is essentially circular, which means that

$$r'' \approx 0, \quad (**)$$

or (from equation 2) that the sun’s mass is

$$M \approx \frac{rv_{rot}^2}{G} \quad (3).”$$

1.

Any of the Claims is False!

1.1 Equation 1 says The Gravitational Force is Zero

Proof:

The Monthly does not know that

$$r'' = 0 \Leftrightarrow \vec{r}'' = 0 \Leftrightarrow m\vec{r}'' = 0$$

Hence, by (**), equation (1) says

$$G \frac{Mm}{r^2} \frac{\vec{r}}{r} = -m\vec{r}'' = 0$$

The Gravitational Force is Zero. \square

1.2 Equation (*) says

$$v_{rot} = 0$$

The Earth does not orbit the Sun

Proof:

The Monthly does not know that

$$r^2 r'^2 = r^2 v^2 = \vec{r}'^2 \vec{v}^2.$$

Substituting in equation (*),

$$\vec{r}'^2 \vec{v}^2 = \vec{r}'^2 \vec{v}^2 + r^2 v_{rot}^2,$$

$$r^2 v_{rot}^2 = 0.$$

$$v_{rot} = 0$$

The earth does not orbit the Sun. \square

1.3 equation (*) says

$$v = 0$$

The Earth does not move, let alone orbit the Sun

Proof:

The Monthly does not know that

$$v = v_{rot}$$

Hence, equation (*) says

$$v = 0,$$

The Earth does not move, let alone orbit the Sun. \square

1.4 Equation (3) says

The Sun's Mass is Zero

Proof:

By 1.2, $v_{rot} = 0 \Rightarrow M \approx \frac{rv_{rot}^2}{G} = 0. \square$

2.

The American Mathematical Monthly Response

To the fact that the radial acceleration is NEVER zero,
the Editor responded with

“The paper was thoroughly refereed by two experts.

Our acceptance rate at the monthly is well below 10%”

Experts in tossing away well over 90% of the submissions?

The author responded with

*“Let me remind you that the radius of circular motion
must be a constant...*

*Also let me remind you that any derivative of a constant,
including the second derivative (which is the scalar
acceleration) is zero...”*

A function of time is a constant?

*“Let me suggest that you look at the material prior to this
comment to learn how to convert vector acceleration
which is not zero, into scalar acceleration which is zero
for circular motion.”*

More of the same?

References

[Saari], Donald Saari, “*Mathematics and the Dark Matter Puzzle*” *American Mathematical Monthly*, Volume 122, No. 5, May 2015, pp.407-423.

[Giancoli] Douglas Giancoli, *Physics for scientists and Engineers*, 2nd Edition, Prentice Hall, 1989, p. 99.

[Spiegel], Murray Spiegel, *Theoretical Mechanics*, Schaum’s Outline, p.116.