Gravitational Waves do not Propagate at Light Speed, And Mercury's Perihelion Precession Does Not Confirm General Relativity

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Abstract Einstein derived General relativity under the erroneous assumption that Retarded Gravitational and Electromagnetic Potentials are identical, and his Gravitational Radiation is actually Electromagnetic because only photons propagate at light speed. Thus, assuming that gravitation propagates at light speed, he proved that gravitation propagates at light speed.

But gravitational waves are not photons, do not propagate at light speed, and the formula for Mercury's perihelion precession, that employs light speed c, does not confirm General Relativity.

In fact, Mercury Perihelion Precession was never well-determined.

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Contents

1.

Einstein's Retarded Potential

Space-time coordinates are

$$x^{\mu} = (x, y, z, t).$$

A metric on space-time is the differential form

$$(ds)^2 = \sum_{\mu=1}^{\mu=4}\sum_{\nu=1}^{\nu=4} g_{\mu\nu} dx^{\mu} dx^{\nu} .$$

The 4×4 symmetric matrix $g_{\mu\nu}(x^{\alpha})$ is the metric tensor.

To first order we take [Einstein2],

$$g_{\mu\nu}(x^{\alpha}) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \gamma_{\mu\nu}(x^{\alpha}), \quad \text{with } \left|\gamma_{\mu\nu}(x^{\alpha})\right| \ll 1.$$

Christoffel symbols are the Gravitational Potentials.

Christoffel symbols of the 1st kind is the $4 \times 4 \times 4$ symmetric matrix

$$\Gamma_{\lambda\mu\nu}(x^{\alpha}) = \tfrac{1}{2}\left[\partial_{x_{\lambda}} g_{\mu\nu} + \partial_{x_{\mu}} g_{\lambda\nu} - \partial_{x_{\nu}} g_{\lambda\mu}\right]$$

Christoffel symbols of the 2nd kind is the $4 \times 4 \times 4$ symmetric matrix

$$\Gamma^{\tau}_{\lambda\mu}(x^{\alpha}) = g^{\tau\nu}\Gamma_{\lambda\mu\nu}$$

Following the notations in [Einstein1], the equation of motion of a material point along a geodetic in space-time is

$$\frac{d^2 x_\tau}{ds^2} = \Gamma_{\mu\nu\tau} \frac{dx_\mu}{ds} \frac{dx_\nu}{ds}. \quad \text{(22), on p.132}$$

(equation (46), p.158, is the same, except for a misprint)

On p. 158, Einstein assumes non-relativistic speed

$$v = \sqrt{\left(\frac{dx_1}{dx_4}\right)^2 + \left(\frac{dx_2}{dx_4}\right)^2 + \left(\frac{dx_3}{dx_4}\right)^2} \ll 1,$$

and concludes that

$$\left|\frac{dx_1}{ds}\right|, \left|\frac{dx_2}{ds}\right|, \left|\frac{dx_3}{ds}\right| \sim 0, \text{ while } \left|\frac{dx_4}{ds}\right| \sim 1.$$

Also, $\Gamma_{\mu\nu\tau}$ are small. Thus, equation (46) keeps only the terms with $\mu = \nu = 4$, and becomes

$$\frac{d^2 x_\tau}{ds^2} = \Gamma_{44\tau} = \tfrac{1}{2}\left[\partial_4 g_{4\tau} + \partial_4 g_{4\tau} - \partial_\tau g_{44}\right],$$

$$\sim -\partial_\tau(\tfrac{1}{2} g_{44}), \quad (67),$$

neglecting the smaller size terms.

Taking $dx_4 = ds = dt$,

$$\frac{d^2 x_\tau}{dt^2} = \Gamma_{44\tau} \sim -\nabla(\tfrac{1}{2} g_{44})$$

Thus, by Newton's law the Gravitational Potential is $\frac{1}{2} g_{44}$.

From equation (53) on p. 149, (correcting the misprint)

$$\partial_{x_\tau} T_{\mu\nu\tau} + T^{\alpha}_{\mu\beta} T^{\beta}_{\nu\alpha} = -\kappa (T_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} T).$$

Substituting

$$\mu = \nu = 4,$$

and

$$T_{44} = T = \rho = \text{matter density},$$

$$\underbrace{\partial_{x_\tau}}_{\nabla\cdot} \underbrace{T_{44\tau}}_{-\nabla(\frac{1}{2}g_{44})} + \underbrace{T^{\alpha}_{4\beta} T^{\beta}_{4\alpha}}_{\text{2nd order}} = -\kappa \underbrace{(T_{44} - \tfrac{1}{2} \underbrace{g_{44}}_{\sim 1} T)}_{\frac{1}{2}\rho}.$$

$$\nabla^2 \left(\tfrac{1}{2} g_{44} \right) = \tfrac{1}{2} \kappa \rho .$$

The Gravitational Potential is

$$\tfrac{1}{2} g_{44} = -\tfrac{1}{8\pi} \kappa \int \frac{\rho}{r} d\tau ,$$

because $\nabla^2 \left(-\tfrac{1}{8\pi} \kappa \int \frac{\rho}{r} d\tau \right) = -\tfrac{1}{8\pi} \kappa \int \underbrace{\nabla^2 \frac{1}{r}}_{-4\pi\delta(r)} \rho(\vec{r}) d\tau = \tfrac{1}{2} \kappa \rho .$

Einstein concludes with the fatally erroneous guess

"...Newton's Theory, with our chosen unit of time,

gives (for the Gravitational Potential) $-\dfrac{G}{c^2} \displaystyle\int \frac{\rho}{r} d\tau$

where $G = 6.7 \times 10^{-8}$ *is the Gravitation constant.*

By comparison, $\kappa = \frac{1}{c^2} 8\pi G$*"*

How does the speed of light get into a "Newtonian" Potential? Most likely, from the Lorentz transformations, -that Einstein did not realize-, deal with charges, and photons, Not material particle devoid of charge.

The formulas for Electromagnetic Fields that embellish [Einstein1], did not help Einstein understand Electromagnetic retarded potentials.

The CGS system, that he used, ignores the crucial

vacuum permittivity ε_0,

and vacuum permeability μ_0,

and does not recognize the exclusiveness of

$$c^2 = \frac{1}{\varepsilon_0 \mu_0},$$

to electromagnetism.

Einstein also missed the exclusiveness of a photon, to thermal, and electromagnetic radiation.

We proceed with the meaning of retarded electromagnetic potentials:

2.

Retarded Potentials and Gravito-Magnetism

In Electrostatics, we assume an Electric Field $\vec{E}(r)$, derived from an Electric Potential $\phi(r)$, so that

$$\vec{E} = -\nabla\phi,$$

and generated by a charge distribution with density $\rho(r)$ so that

$$\nabla \cdot \underbrace{\vec{E}}_{-\nabla\phi} = \frac{\rho}{\varepsilon_0},$$

where ε_0 is the Electric Permittivity of the vacuum. Thus,

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0},$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r)}{r} dV.$$

In Electrodynamics, we assume Magnetic Induction $\vec{B}(\vec{r},t)$, derived from a Magnetic Vector Potential $\vec{A}(\vec{r},t)$, so that

$$\vec{B} = \vec{\nabla} \times \vec{A},$$

and Electric Field $\vec{E}(\vec{r},t)$, derived from an Electric Potential

$\phi(\vec{r},t)$, so that

$$\vec{E} = -\nabla\phi - \partial_t \vec{A}.$$

Then,

$$\nabla \cdot \underbrace{\vec{E}}_{-\nabla\phi - \partial_t \vec{A}} = \frac{\rho}{\varepsilon_0},$$

$$\nabla^2\phi + \partial_t \nabla \cdot \vec{A} = -\frac{\rho}{\varepsilon_0}.$$

Assuming Lorentz Condition, $\nabla \cdot \vec{A} = -\varepsilon_0\mu_0\partial_t\phi$, where μ_0 is the Magnetic Permeability of the vacuum

$$\nabla^2\phi - \underbrace{\varepsilon_0\mu_0}_{\frac{1}{c^2}}\partial_t^2\phi = -\frac{\rho}{\varepsilon_0}.$$

This is an electromagnetic wave equation for ϕ, with propagation speed

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}.$$

Then, the potential $\phi(\vec{r},t)$ at $\vec{r}$, at time t, is the summation on contributions at $\vec{\xi}$, at the retarded time $t - \frac{r}{c}$,

$$\phi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{\xi}, t - \frac{r}{c})}{\left|\vec{r} - \vec{\xi}\right|} dV_{\vec{\xi}}.$$

For Gravitation to follow from this formulation, we have to assume a Gravito-Magnetic Field $\vec{B}_G$, derived from a Gravito-Magnetic $\vec{A}_G$, that was never detected, so that

$$\vec{B}_G = \vec{\nabla} \times \vec{A}_G,$$

and a Gravitational Field $\vec{E}_G(\vec{r},t)$, derived from a Potential $\phi_G(\vec{r},t)$, so that

$$\vec{E}_G = -\nabla\phi_G - \partial_t\vec{A}_G.$$

Then we would need a Gravitational Lorentz Condition

$$\nabla \cdot \vec{A}_G = -\varepsilon_G\mu_G\partial_t\phi_G,$$

where we would have to give meaning to

ε_G, and μ_G,

and explain how

$$\frac{1}{\sqrt{\varepsilon_G\mu_G}} = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = c.$$

The relation $\frac{1}{\varepsilon_0\mu_0} = c^2$ is exclusive to Electro-Magnetics.

In Gravitation, ε_0, and μ_0, have no parallel, and at most we can assume that Gravitational Waves exist, and propagate at some speed v_G.

Einstein's Gravitational waves propagate at light speed because he assumed so.

3.

Einstein's Gravitational Waves

Keeping $\kappa = \frac{8\pi G}{c^2}$, Einstein renews his erroneous claim that Gravitational Waves propagate at light speed.

To first order he has [Einstein2],

$$g_{\mu\nu}(x^\alpha) \sim \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \gamma_{\mu\nu}(x^\alpha), \qquad \text{(1) on p.201}$$

where $\left|\gamma_{\mu\nu}(x^\alpha)\right| \ll 1$.

To first order, the Field Equations are

$$\sum_\alpha \left[\frac{\partial^2 \gamma_{\mu\alpha}}{\partial x_\nu \partial x_\alpha} + \frac{\partial^2 \gamma_{\nu\alpha}}{\partial x_\mu \partial x_\alpha} - \frac{\partial^2 \gamma_{\mu\nu}}{\partial x_\alpha^2}\right] - \frac{\partial^2 \sum_\alpha \gamma_{\alpha\alpha}}{\partial x_\mu \partial x_\nu} \sim -2\kappa(T_{\mu\nu} - \tfrac{1}{2}\delta_{\mu\nu})\sum_\alpha T_{\alpha\alpha}, \quad (2)$$

where κ is understood as $\kappa = \frac{8\pi G}{c^2}$ from [Einstein1].

He substitutes

$$\gamma_{\mu\nu} = \gamma'_{\mu\nu} + \psi\delta_{\mu\nu}, \qquad (3)$$

where

$$\sum_\nu \partial_{x_\nu} \gamma'_{\mu\nu} = 0, \qquad (4)$$

$$\sum_{\alpha} \gamma'_{\alpha\alpha} = -2\psi, \qquad (5)$$

and obtains

$$\sum_{\alpha} \frac{\partial^2 \gamma'_{\mu\nu}}{\partial x_{\alpha}^2} = 2\kappa T_{\mu\nu}. \qquad (6)$$

He concludes with

"... the $\gamma'_{\mu\nu}$ are the retarded potentials

$$\gamma'_{\nu\mu} = -\tfrac{1}{2\pi}\kappa \int \frac{T_{\mu\nu}(x_0, y_0, z_0, t-r)}{r} dV_0 \qquad (9)"$$

And sums up on page 206,

" It follows from (6) and (9) that gravitational fields always propagate with velocity 1, that is, with the speed of light."

But

$\kappa = \dfrac{8\pi G}{c^2}$ **was established erroneously in [Einstein1].**

4.

The Meaning of Propagation at Light speed

The radiation quantum that propagates at light speed is the photon. It is unique to electromagnetics, specifically, to Black Body thermal radiation.

The photon is a charge-less packet of energy $h\nu$, with equivalent mass $\frac{h\nu}{c^2}$. It is emitted from charged particles, and carries the Electromagnetic Field.

No other particle has the photon characteristic that in the vacuum it travels at light speed. Other particles that have been presumed to travel at light speed, such as gluons have never been detected, and their speeds have never been measured.

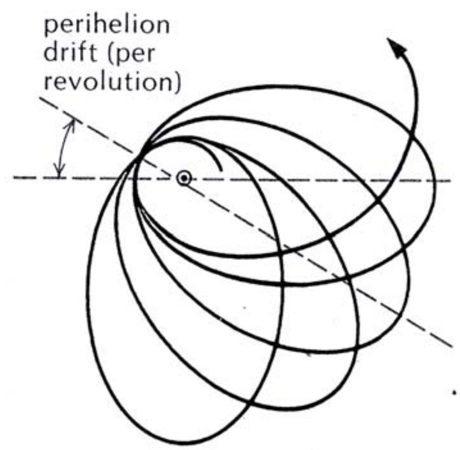
To say that Gravitational waves propagate at the speed of light, is to say that the quantum of gravitational radiation is a photon, and that charged electrons, that emit photons, are the same as uncharged mass particles that emit gravitons.

Consequently, Einstein's misunderstanding of the crucial role of vacuum permittivity, ε_0, and vacuum permeability, μ_0, in determining the speed of electromagnetic waves, annuls his claim that his theory is confirmed by the precession of the perihelion of Mercury.

5.
The Precession of the Perihelion of Mercury

Einstein claimed that the unexplained part in the observed precession of the perihelion of the planet Mercury around the sun is explained by his General Relativity.

A planet elliptical orbit rotates slowly in the direction of its motion and its perihelion encircles the sun.



Einstein proposed that the unexplained precession in radians per revolution is

$$24\pi^3 \frac{a^2}{T^2 c^2 (1-e^2)},$$

where

a = half the major axis of the ellipse (in centimeters)

e = eccentricity

c = light speed in the vacuum (in centimeters)

T = period of a revolution (in seconds)

Substituting $c^2 = \dfrac{8\pi G}{\kappa}$, the unexplained precession is

$$3\pi^2 \frac{a^2}{T^2 G(1-e^2)}\kappa$$

According to Einstein, it equals the unexplained precession of the perihelion of Mercury by $43''$ per hundred years.

But $\kappa = \dfrac{8\pi G}{c^2}$ is based on the erroneous guess that retarded gravitational and electromagnetic potentials are identical.

So much for this confirmation of General Relativity.

Nevertheless, perhaps, the correct precession formula is

$$24\pi^3 \frac{a^2}{T^2 v_G^2(1-e^2)}$$

where v_G = average speed of gravitational waves.

Then, if the $43''$ per hundred years can be trusted, we could compute v_G.

But the $43''$ value is highly speculative.

By the Wikipedia's "Tests of General Relativity", the observed perihelion precession of Mercury is $574''$.

By unspecified arguments, Gravitational pull of other planets accounts for $531''$, and $43''$ is unaccounted for.

Since the certainty of these claims is unknown, we have to consider them in terms of statistical confidence.

Note that 97% confidence in $531''$, allows for 3% error in $531''$ which is $15.93''$. But that means a 37% error in $43''$ which allows only 63% confidence in the $43''$.

Note that 95% confidence in $531''$ allows for $26.55''$ error, and only 38% confidence in the $43''$.

By obtaining the $43''$ with erroneous κ, Einstein's General Relativity establishes with 100% confidence that the unaccounted for perihelion precession of Mercury is NOT $43''$.

In a 11/28/1919 letter to the London Times, Einstein submitted that had any of his tests been wrong, the whole theory would be beyond repair, and would have to be given up:

> *"The chief attraction of the theory lies in its logical completeness. If a single one of the conclusions drawn from it proves wrong, it must be given up; to modify it without destroying the whole structure*

seems to be impossible"

He must have been aware of his unsubstantiated guess that Gravitational and Electromagnetic retarded potentials are identical, and of the speculative perihelion precession of Mercury that never confirmed his Theory.

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