The Proton Spectrum in the Hydrogen Atom

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Abstract In the Bohr Atom, the electron with mass m_e, and charge e encircles the proton with mass $M_p \approx 1836 m_e$, and charge $-e$. The radius of the circular orbit is r.

The electric attraction between the electron and the proton

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2},$$

is balanced by the centrifugal repulsion of the electron from the center. The electron speed is v, and the centrifugal force is

$$m_e\frac{v^2}{r}.$$

As the electron accelerates towards the proton, the electron radiates photons, and looses energy to the proton. That loss is so great that the electron should spiral into the proton in a fraction of a second. And yet, it does not.

Clearly, the electron energy in its orbit does not change. But why?

Bohr proposed that the electron orbits can defy Electrodynamics

because they have angular momentums that are discrete integer multiples of $\hbar$.

Consequently, the orbits are occupied by standing waves, and no radiation takes place in them.

However, while the standing wave argument is pure hypothesis, the radiation by an accelerating charge is a fundamental fact of electrodynamics.

In 2014, we submitted[1] that the electron's lost radiation is returned by the proton. The proton must be orbiting the center to be accelerating, and to shower the electron with photons.

At the center, the proton "sees" a static electron towards which it is electrically attracted, and from which it is centrifugally repulsed, due to the circular motion.

Clearly, the Proton has its own orbits, in which it accelerates towards the electron, and showers the electron with radiation.

If ρ is the radius of the proton orbit, and V is the proton speed in that orbit, the electric attraction between the proton and the electron imagined at the center

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2},$$

is balanced by the centrifugal repulsion of the proton from the

[1] H. Vic Dannon, Radiation Equilibrium, Inertia Moments, and the Nucleus Radius in the Electron-Proton Atom Gauge Institute Journal, Vol.10 No 3, August 2014,

center

$$M_p \frac{V^2}{\rho}.$$

Assuming that the power of the energy radiated by the proton into the electron field equals the power of the energy radiated by the electron into the proton field, we showed that

The electron-proton Atom is *<u>electrodynamically stable</u>* *if and only if the electron and proton inertia moments are equal*

$$\boxed{m_e r^2 \approx M_p \rho^2}$$

This equality determines the radius of the proton's orbit, which is the Nucleus Radius of the Hydrogen Atom:

If the electron is in its first orbit with Bohr radius

$$r_1 = (5.291772481)10^{-11}\text{m}.$$

The Associated first orbit of the proton has radius

$$\rho_1 \approx r_1\sqrt{\frac{m_e}{M_p}} \approx (1.234942581)10^{-12}\text{m}.$$

The associated n^{th} orbit of the proton has radius

$$\rho_n \approx r_n\sqrt{\frac{m_e}{M_p}} \approx n^2(1.234942581)10^{-12}\text{m}$$

In 2021[2], we computed the associated orbits of the electron, and

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[2] H. Vic Dannon, "The Electron and Proton Orbits in the Hydrogen Atom" Gauge Institute Journal of Math and Physics, Volume 17, No 1, February 2021.

the proton. The periods, the angular momentums, and the energies.

Here, we compute the spectral transitions between proton (and electron) orbits.

For the Lyman Series transition from orbit 2 to orbit 1,

Electron	Proton
$E_{2\to 1} = (1634.905577)10^{-21}\text{Joul}$	$\mathcal{E}_{2\to 1} = (70{,}056.2801)10^{-21}\text{Joul}$
$\nu_{2\to 1} = (2.467381448)10^{15}\ \frac{\text{cycles}}{\text{sec}}$	$N_{2\to 1} = (105.7281646)10^{15}\ \frac{\text{cycles}}{\text{sec}}$
$\lambda_{2\to 1} = (121.5022745)\text{nm}$	$\Lambda_{2\to 1} = (2.835502339)\text{nm}$

The Proton's greater spectral energy, should make it easier to observe its spectral line $\Lambda_{2\to 1} = (2.835502339)\text{nm}$.

The Spectral lines for transition from n_2 orbit to n_1 orbit are related by

$$\boxed{\frac{\lambda_{n_2\to n_1}}{\Lambda_{n_2\to n_1}} = \frac{\mathcal{E}_1}{E_1} = \sqrt{\frac{M_p}{m_e}} \approx 42.8503524}$$

Following are the Spectral lines of the Lyman Series, the Balmer Series, the Paschen Series, the Brackett Series, and the Pfund Series

We obtained them pair by pair, and checked that they satisfy this relation.

For the **Lyman** Series, the spectral lines are

Electron	Proton
$\lambda_{2\to1} = (121.5022745)\text{nm}$	$\Lambda_{2\to1} = (2.835502339)\text{nm}$
$\lambda_{3\to1} = (102.5175441)\text{nm}$	$\Lambda_{3\to1} = (2.392455099)\text{nm}$
$\lambda_{4\to1} = (97.2018195)\text{nm}$	$\Lambda_{4\to1} = (2.268402749)\text{nm}$
$\lambda_{5\to1} = (94.92365914)\text{nm}$	$\Lambda_{5\to1} = (2.215236203)\text{nm}$
$\lambda_{6\to1} = (93.73032603)\text{nm}$	$\Lambda_{6\to1} = (2.187387519)\text{nm}$
$\lambda_{7\to1} = (93.0251789)\text{nm}$	$\Lambda_{7\to1} = (2.170931479)\text{nm}$

For the **Balmer** Series, the spectral lines are

Electron	Proton
$\lambda_{3\to2} = (656.112282)\text{nm}$	$\Lambda_{3\to2} = (15.31171266)\text{nm}$
$\lambda_{4\to2} = (486.0096097)\text{nm}$	$\Lambda_{4\to2} = (11.34200938)\text{nm}$
$\lambda_{5\to2} = (433.936694)\text{nm}$	$\Lambda_{5\to2} = (10.12679407)\text{nm}$
$\lambda_{6\to2} = (410.070176)\text{nm}$	$\Lambda_{6\to2} = (9.56982039)\text{nm}$
$\lambda_{7\to2} = (396.90743)\text{nm}$	$\Lambda_{7\to2} = (9.26264097)\text{nm}$

For the **Paschen** Series, the spectral lines are

Electron	Proton
$\lambda_{4\to3} = (1874.606521)\text{nm}$	$\Lambda_{4\to3} = (43.74775038)\text{nm}$
$\lambda_{5\to3} = (1281.469301)\text{nm}$	$\Lambda_{5\to3} = (29.90568874)\text{nm}$
$\lambda_{6\to3} = (1093.52047)\text{nm}$	$\Lambda_{6\to3} = (25.51952106)\text{nm}$
$\lambda_{7\to3} = (1004.671932)\text{nm}$	$\Lambda_{7\to3} = (23.44605997)\text{nm}$

For the **Brackett** Series, the spectral line are,

Electron	Proton
$\lambda_{5\to4} = (4050.075816)\text{nm}$	$\Lambda_{5\to4} = (94.51674465)\text{nm}$
$\lambda_{6\to4} = (2624.449129)\text{nm}$	$\Lambda_{6\to4} = (61.24685053)\text{nm}$
$\lambda_{7\to4} = (2164.949618)\text{nm}$	$\Lambda_{7\to4} = (50.52349623)\text{nm}$

For the **Pfund** Series, the spectral lines are

Electron	Proton
$\lambda_{6\to5} = (7455.821389)\text{nm}$	$\Lambda_{6\to5} = (173.9967345)\text{nm}$
$\lambda_{7\to5} = (4651.258945)\text{nm}$	$\Lambda_{7\to5} = (108.5465739)\text{nm}$

Keywords: Electromagnetic Radiation of Accelerated Charge, Radiation Loss, Quantized Angular Momentum, Atomic Orbits, Electron, Proton, photon, Atom, Nucleus Radius, Zero Point Energy, Bohr's Atom, Fine Structure Constant, Magnetic Energy, Electric Energy,

2000 Physics and Astronomy Classification Scheme

03.50.Kk, 03.65.-w. 41.60-m; 32; 32.10-f; 31.15.B-;

Contents

1.

Spectral lines of Associated Orbits

1.1e $$\boxed{r_1 \approx \frac{\hbar^2}{e^2}\frac{4\pi\varepsilon_0}{m_e} \approx (5.291772481)10^{-11}\text{m}}$$

Proof: $$m_e\omega_1^2 r_1 \approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_1^2}$$

$$\underbrace{m_e(\omega_1 r_1)r_1}_{\hbar}\underbrace{(\omega_1 r_1)}_{v_1} \approx \frac{1}{4\pi\varepsilon_0}e^2$$

$$\hbar\underbrace{(m_e v_1 r_1)}_{\hbar} \approx m_e r_1\frac{1}{4\pi\varepsilon_0}e^2$$

$$r_1 \approx \frac{\hbar^2}{e^2}\frac{4\pi\varepsilon_0}{m_e}$$

$$\approx \frac{1}{\underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}}_{\approx 137.035989}}\frac{\hbar}{m_e c}$$

$$= 137.035989\frac{(1.05457266)10^{-34}}{(9.1093897)10^{-31}(2.99792458)\cdot 10^8}$$

$$= (5.291772481)10^{-11}\text{m}.\square$$

1.2e $$\boxed{r_n \approx n^2 r_1 \approx n^2(5.291772481)10^{-11}\text{m}}$$

Proof:
$$n\hbar\underbrace{(m_e v_n r_n)}_{n\hbar} \approx m_e r_n \frac{1}{4\pi\varepsilon_0} e^2$$

$$r_n \approx n^2 \underbrace{\frac{\hbar^2}{e^2}\frac{4\pi\varepsilon_0}{m_e}}_{r_1}$$

$$\approx n^2(5.291772481)10^{-11}\mathrm{m}.\square$$

1.1p
$$\boxed{\rho_1 \approx r_1\sqrt{\frac{m_\mathrm{e}}{M_\mathrm{p}}} \approx (1.23494258)10^{-12}\mathrm{m}}$$

Proof: $\rho_1 \approx r_1\sqrt{\frac{m_\mathrm{e}}{M_\mathrm{p}}}$

$$\approx (5.291772481)10^{-11}\sqrt{\frac{1}{1836.152701}}$$

$$= (1.23494258)10^{-12}\mathrm{m}.\square$$

1.2p
$$\boxed{\rho_n \approx n^2\rho_1 \approx n^2(1.23494258)10^{-12}\mathrm{m}}$$

Proof: $\rho_n \approx r_n\sqrt{\frac{m_\mathrm{e}}{M_\mathrm{p}}}$

$$\approx n^2 r_1\sqrt{\frac{m_\mathrm{e}}{M_\mathrm{p}}}$$

$$= n^2\rho_1$$

$$= n^2(1.23494258)10^{-12}\text{m}.\square$$

1.3e The Electron's Kinetic Energy in its First Orbit

$$\boxed{E_1 = \frac{1}{2}m_e v_1^2 = \alpha^2 \frac{1}{2}m_e c^2 \approx (2179.874102)10^{-21}\text{Joul}=(13.6)\text{eV}}$$

Proof: $E_1 = \frac{1}{2}m_e v_1^2$

$$= \alpha^2 \frac{1}{2}m_e c^2$$

$$= \left[(7.29735308)10^{-3}\right]^2 \frac{1}{2}(9.1093897)10^{-31}\left[(2.99792458)10^{8}\right]^2$$

$$= (2179.874102)10^{-21}\text{Joul}$$

$$= \frac{(2179.874102)10^{-21}}{(160.2177)10^{-21}}\text{eV}$$

$$= (13.60570088)\text{eV}.\square$$

1.4e The Electron's Kinetic Energy in its n^{th} Orbit

$$\boxed{E_n = \frac{1}{2}m_e v_n^2 = \frac{1}{n^2}\alpha^2 \frac{1}{2}m_e c^2 \approx \frac{1}{n^2}(2179.874102)10^{-21}\text{Joul}=\frac{1}{n^2}(13.6)\text{eV}}$$

Proof: $E_n = \frac{1}{2}m_e v_n^2$

$$= \frac{1}{2}m_e\left(\frac{1}{n}v_1\right)^2$$

$$= \frac{1}{n^2}\alpha^2 \frac{1}{2} m_e c^2$$

$$= \frac{1}{n^2} E_1$$

$$= \frac{1}{n^2}(13.60570088)\text{eV}. \square$$

1.3p The Proton's Kinetic Energy in its First Orbit

$$\boxed{\mathcal{E}_1 = \frac{1}{2} M_p V_1^2 = \sqrt{\frac{M_p}{m_e}} E_1 \approx 583\text{eV} = (9.340837346)10^{-17}\text{Joul}}$$

Proof: $\mathcal{E}_1 = \frac{1}{2} M_p V_1^2$

$$= \frac{1}{2} M_p \sqrt{\frac{m_e}{M_p}} \alpha^2 c^2$$

$$= \sqrt{\frac{M_p}{m_e}} \alpha^2 \left(\frac{1}{2} m_e c^2\right)$$

$$= \sqrt{\frac{M_p}{m_e}} E_1$$

$$= \sqrt{1836.152701}(13.60570088)\text{eV}$$

$$= (583.0090774)\text{eV}$$

$$= \sqrt{1836.152701}(2179.874102)10^{-21}\text{Joul}$$

$$= (9.340837346)10^{-17}\text{Joul}. \square$$

1.4p The Proton's Kinetic Energy in its n^{th} Orbit

$$\mathcal{E}_n = \frac{1}{2}M_p V_n^2 = \frac{1}{n^2}\mathcal{E}_1 \approx \frac{1}{n^2}583\text{eV} = \frac{1}{n^2}(9.340837346)10^{-17}\text{Joul}$$

Proof: $\mathcal{E}_n = \frac{1}{2}M_p V_n^2$

$$= \frac{1}{2}M_p\left(\frac{1}{n}V_1\right)^2$$

$$= \frac{1}{n^2}\left(\frac{1}{2}M_p V_1^2\right)$$

$$= \frac{1}{n^2}\mathcal{E}_1$$

$$\approx \frac{1}{n^2}(583.0090774)\text{eV}$$

$$= \frac{1}{n^2}(9.340837346)10^{-17}\text{Joul}. \square$$

1.5 The Transition Energies between n_1 orbit and n_2 orbit

For the electron , $E_{n_2 \to n_1} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)E_1$

For the Proton, $\mathcal{E}_{n_2 \to n_1} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\mathcal{E}_1$.

1.6 The Transition Frequencies between n_1, and n_2 orbits

For the Electron, $\nu_{n_2 \to n_1} = \dfrac{E_{n_2 \to n_1}}{h} = \left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2} \right) \dfrac{E_1}{h}$

For the Proton, $N_{n_2 \to n_1} = \dfrac{\mathcal{E}_{n_2 \to n_1}}{h} = \left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2} \right) \dfrac{\mathcal{E}_1}{h}$

1.7 The spectral lines between n_1 orbit and n_2 orbit

For the Electron, $\lambda_{n_2 \to n_1} = \dfrac{c}{\nu_{n_2 \to n_1}} = \dfrac{1}{\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2} \right)} \dfrac{hc}{E_1}$

For the Proton, $\Lambda_{n_2 \to n_1} = \dfrac{c}{N_{n_2 \to n_1}} = \dfrac{1}{\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2} \right)} \dfrac{hc}{\mathcal{E}_1}$

1.8

$$\boxed{\frac{\lambda_{n_2 \to n_1}}{\Lambda_{n_2 \to n_1}} = \frac{\mathcal{E}_1}{E_1} = \sqrt{\frac{M_p}{m_e}} \approx 42.8503524}$$

2.

Lyman[3] Transitions to 1st Orbits

$\boxed{2_e \to 1_e}$ The energy of the photon emitted in the transition of the electron from its 2^{nd} orbit to its 1^{st} orbit is

$$E_{2\to 1} = E_1 - E_2$$

$$= E_1 - \tfrac{1}{2^2}E_1$$

$$= \tfrac{3}{4}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{2\to 1} = (1634.905577)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{2\to 1} = \frac{E_{2\to 1}}{h}$$

$$= \frac{(1634.905577)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{2\to 1} = (2.467381448)10^{15}\text{cycles/sec}}$$ (in ultra-violet[4])

Its wave-length is

$$\lambda_{2\to 1} = \frac{c}{\nu_{2\to 1}} = \frac{(2.99792458)10^{8}}{(2.467381448)10^{15}}$$

$$\boxed{\lambda_{2\to 1} = (121.5022745)\text{nm}}$$

[3] https://en.wikipedia.org/wiki/Lyman-alpha_line
[4] https://en.wikipedia.org/wiki/Ultraviolet#VUV

$\boxed{2_p \to 1_p}$ The energy of a photon emitted in transition of the proton from its 2^{nd} orbit to its 1^{st} orbit is

$$\mathcal{E}_{2\to 1} = \mathcal{E}_1 - \mathcal{E}_2$$

$$= \mathcal{E}_1 - \frac{1}{2^2}\mathcal{E}_1$$

$$= \frac{3}{4}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{2\to 1} = (7.00562801)10^{-17}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{2\to 1} = \frac{\mathcal{E}_{2\to 1}}{h}$$

$$= \frac{(7.00562801)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{2\to 1} = (1.057281646)10^{17}\,\text{cycles/sec}}$$ **(In the x-rays[5])**

Its wave-length is

$$\Lambda_{2\to 1} = \frac{c}{\nu_{2\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(1.057281646)10^{17}}$$

$$\boxed{\Lambda_{2\to 1} = (2.835502339)\text{nm}}$$

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[5] https://en.wikipedia.org/wiki/Orders_of_magnitude_(frequency)

$\boxed{3_e \to 1_e}$ The energy of a photon emitted in transition of the electron from the 3^{rd} orbit to the 1^{st} orbit is

$$E_{3\to 1} = E_1 - E_3$$

$$= E_1 - \frac{1}{3^2} E_1$$

$$= \frac{8}{9}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{3\to 1} = (1937.665868)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{3\to 1} = \frac{E_{3\to 1}}{h}$$

$$= \frac{(1937.665868)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{3\to 1} = (2.924303939)10^{15}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{3\to 1} = \frac{c}{\nu_{3\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(2.924303939)10^{15}}$$

$$\boxed{\lambda_{3\to 1} = (102.5175441)\text{nm}}$$

$\boxed{3_p \to 1_p}$ The energy of a photon emitted in the transition of the proton from its 3^{rd} orbit to its 1^{st} orbit is

$$\mathcal{E}_{3\to 1} = \mathcal{E}_1 - \mathcal{E}_3$$

$$= \mathcal{E}_1 - \frac{1}{3^2}\mathcal{E}_1$$

$$= \frac{8}{9}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{3\to 1} = (8.30296653)10^{-17}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{3\to 1} = \frac{\mathcal{E}_{3\to 1}}{h}$$

$$= \frac{(8.30296653)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{3\to 1} = (1.253074543)10^{17}\,\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{3\to 1} = \frac{c}{N_{3\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(1.253074543)10^{17}}$$

$$\boxed{\Lambda_{3\to 1} = (2.392455099)\text{nm}}$$

$\boxed{4_e \to 1_e}$ The energy of a photon emitted in the transition of the electron from its 4^{th} orbit to its 1^{st} orbit is

$$E_{4\to 1} = E_1 - E_4$$

$$= E_1 - \frac{1}{4^2}E_1$$

$$= \frac{15}{16}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{4\to 1} = (2043.631971)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{4\to 1} = \frac{E_{4\to 1}}{h}$$

$$= \frac{(2043.631971)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{4\to 1} = (3.084226811)10^{15}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{4\to 1} = \frac{c}{\nu_{4\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(3.084226811)10^{15}}$$

$$\boxed{\lambda_{4\to 1} = (97.2018195)\text{nm}}$$

$\boxed{4_p \to 1_p}$ The energy of a photon emitted in the transition of the proton from its 4^{th} to its 1^{st} orbit is

$$\mathcal{E}_{4\to 1} = \mathcal{E}_1 - \mathcal{E}_4$$

$$= \mathcal{E}_1 - \frac{1}{4^2}\mathcal{E}_1$$

$$= \frac{15}{16}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{4\to 1} = (8.757031626)10^{-17}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{4\to 1} = \frac{\mathcal{E}_{4\to 1}}{h}$$

$$= \frac{(8.757031626)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{4\to 1} = (1.321601546)10^{17}\,\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{4\to 1} = \frac{c}{N_{4\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(1.321601546)10^{17}}$$

$$\boxed{\Lambda_{4\to 1} = (2.268402749)\text{nm}}$$

$\boxed{5_e \to 1_e}$ The energy of a photon emitted in the transition of the electron from its 5^{th} orbit to its 1^{st} orbit is

$$E_{5\to 1} = E_1 - E_5$$

$$= E_1 - \frac{1}{5^2}E_1$$

$$= \frac{24}{25}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{5\to 1} = (2092.679138)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{5\to 1} = \frac{E_{5\to 1}}{h}$$

$$= \frac{(2092.679138)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{5\to 1} = (3.158248254)10^{15}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{5\to 1} = \frac{c}{\nu_{5\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(3.158248254)10^{15}}$$

$$..\boxed{\lambda_{5\to 1} = (94.92365914)\text{nm}}$$

$\boxed{5_p \to 1_p}$ The energy of a photon emitted in the transition of the proton from its 5^{th} orbit to its 1^{st} orbit is

$$\mathcal{E}_{5\to 1} = \mathcal{E}_1 - \mathcal{E}_5$$

$$= \mathcal{E}_1 - \frac{1}{5^2}\mathcal{E}_1$$

$$= \frac{24}{25}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{5\to 1} = (8.967203852)10^{-17}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{5\to 1} = \frac{\mathcal{E}_{5\to 1}}{h}$$

$$= \frac{(8.967203852)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{5\to 1} = (1.353320507)10^{17}\,\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{5\to 1} = \frac{c}{N_{5\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(1.353320507)10^{17}}$$

$$\boxed{\Lambda_{5\to 1} = (2.215236203)\text{nm}}$$

$\boxed{6_e \to 1_e}$ The energy of a photon emitted in the transition of the electron from its 6^{th} orbit to its 1^{st} orbit is

$$E_{6\to 1} = E_1 - E_6$$

$$= E_1 - \frac{1}{6^2} E_1$$

$$= \frac{35}{36}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{6\to 1} = (2119.322044)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{6\to 1} = \frac{E_{6\to 1}}{h}$$

$$= \frac{(2119.322044)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{6\to 1} = (3.198457433)10^{15}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{6\to 1} = \frac{c}{\nu_{6\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(3.198457433)10^{15}}$$

$$\boxed{\lambda_{6\to 1} = (93.73032603)\text{nm}}$$

$\boxed{6_p \to 1_p}$ The energy of a photon emitted in the transition of the proton from its 6^{th} orbit to its 1^{st} orbit is

$$\mathcal{E}_{6\to 1} = \mathcal{E}_1 - \mathcal{E}_6$$

$$= \mathcal{E}_1 - \frac{1}{6^2}\mathcal{E}_1$$

$$= \frac{35}{36}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{6\to 1} = (9.081369642)10^{-17}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{6\to 1} = \frac{\mathcal{E}_{6\to 1}}{h}$$

$$= \frac{(9.081369642)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{6\to 1} = (1.370550282)10^{17}\,\text{cycles/sec}}$$ **(In the x-rays)**

Its wave-length is

$$\Lambda_{6\to 1} = \frac{c}{N_{6\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(1.370550282)10^{17}}$$

$$\boxed{\Lambda_{6\to 1} = (2.187387519)\text{nm}}$$

$\boxed{7_e \to 1_e}$ The energy of a photon emitted in the transition of the electron from its 7^{th} orbit to its 1^{st} orbit is

$$E_{7\to 1} = E_1 - E_7$$

$$= E_1 - \frac{1}{7^2}E_1$$

$$= \frac{48}{49}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{7\to 1} = (2135.386875)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{7\to 1} = \frac{E_{7\to 1}}{h}$$

$$= \frac{(2135.386875)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{7\to 1} = (3.2227023)10^{15}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{7\to 1} = \frac{c}{\nu_{7\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(3.2227023)10^{15}}$$

$$\boxed{\lambda_{7\to 1} = (93.0251789)\text{nm}}$$

$\boxed{7_p \to 1_p}$ The energy of a photon emitted in the transition of the proton from its 7^{th} orbit to its 1^{st} orbit is

$$\mathcal{E}_{7\to 1} = \mathcal{E}_1 - \mathcal{E}_7$$

$$= \mathcal{E}_1 - \frac{1}{7^2}\mathcal{E}_1$$

$$= \frac{48}{49}(9.340837346)10^{-17}\text{Joul}$$

$$\boxed{\mathcal{E}_{7\to 1} = (9.150208012)10^{-17}\text{Joul}}$$

The frequency of the emitted photon is

$$N_{7\to 1} = \frac{\mathcal{E}_{7\to 1}}{h}$$

$$= \frac{(9.150208012)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{7\to 1} = (1.380939292)10^{17}\text{cycles/sec}}$$ **(In the x-rays)**

Its wave-length is

$$\Lambda_{7\to 1} = \frac{c}{N_{7\to 1}}$$

$$= \frac{(2.99792458)10^{8}}{(1.380939292)10^{17}}$$

$$\boxed{\Lambda_{7\to 1} = (2.170931479)\text{nm}}$$

3.

Balmer Transitions to 2nd Orbits

$\boxed{3_e \to 2_e}$ The energy of a photon emitted in the transition of the electron from its 3^{rd} orbit to its 2^{nd} orbit is

$$E_{3\to 2} = E_2 - E_3$$

$$= \tfrac{1}{2^2} E_1 - \tfrac{1}{3^2} E_1$$

$$= \tfrac{5}{36}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{3\to 2} = (302.7602919)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{3\to 2} = \frac{E_{3\to 2}}{h} = \frac{(30.27602919)10^{-20}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{3\to 2} = (4.569224905)10^{14}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{3\to 2} = \frac{c}{\nu_{3\to 2}}$$

$$= \frac{(2.99792458)10^{8}}{(4.569224905)10^{14}}$$

$$\boxed{\lambda_{3\to 2} = (656.112282)\text{nm}}$$

$\boxed{3_p \to 2_p}$ The energy of a photon emitted in the transition of the proton from its 3^{rd} orbit to its 2^{nd} orbit is

$$\mathcal{E}_{3\to 2} = \mathcal{E}_2 - \mathcal{E}_3$$

$$= \frac{1}{2^2}\mathcal{E}_1 - \frac{1}{3^2}\mathcal{E}_1$$

$$= \frac{5}{36}(9.340837346)10^{-17}\text{Joul}$$

$$\boxed{\mathcal{E}_{3\to 2} = (1.29733852)10^{-17}\text{Joul}}$$

The frequency of the emitted photon is

$$N_{3\to 2} = \frac{\mathcal{E}_{3\to 2}}{h}$$

$$= \frac{(1.29733852)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{3\to 2} = (1.95792897)10^{16}\text{cycles/sec}} \text{ (In the x-rays)}$$

Its wave-length is

$$\Lambda_{3\to 2} = \frac{c}{N_{3\to 2}}$$

$$= \frac{(2.99792458)10^{8}}{(1.95792897)10^{16}}$$

$$\boxed{\Lambda_{3\to 2} = (15.31171266)\text{nm}}$$

$\boxed{4_e \to 2_e}$ The energy of a photon emitted in the transition of the electron from its 4^{th} orbit to its 2^{nd} orbit is

$$E_{4\to 2} = E_2 - E_4$$

$$= \tfrac{1}{2^2}E_1 - \tfrac{1}{4^2}E_1$$

$$= \tfrac{3}{16}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{4\to 2} = (408.7263941)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{4\to 2} = \frac{E_{4\to 2}}{h}$$

$$= \frac{(40.87263941)10^{-20}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{4\to 2} = (6.168453621)10^{14}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{4\to 2} = \frac{c}{\nu_{4\to 2}}$$

$$= \frac{(2.99792458)10^{8}}{(6.168453621)10^{14}}$$

$$\boxed{\lambda_{4\to 2} = (486.0096097)\text{nm}}$$

$\boxed{4_p \to 2_p}$ The energy of a photon emitted in the transition of the proton from its 4^{th} orbit to its 2^{nd} orbit is

$$\mathcal{E}_{4\to 2} = \mathcal{E}_2 - \mathcal{E}_4$$

$$= \frac{1}{2^2}\mathcal{E}_1 - \frac{1}{4^2}\mathcal{E}_1$$

$$= \frac{3}{16}(9.340837346)10^{-17}\text{Joul}$$

$$\boxed{\mathcal{E}_{4\to 2} = (1.751407002)10^{-17}\text{Joul}}$$

The frequency of the emitted photon is

$$N_{4\to 2} = \frac{\mathcal{E}_{4\to 2}}{h}$$

$$= \frac{(1.751407002)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{4\to 2} = (2.64320411)10^{16}\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{4\to 2} = \frac{c}{N_{4\to 2}}$$

$$= \frac{(2.99792458)10^{8}}{(2.64320411)10^{16}}$$

$$\boxed{\Lambda_{4\to 2} = (11.34200938)\text{nm}}$$

$\boxed{5_e \to 2_e}$ The energy of a photon emitted in the transition of the electron from its 5^{th} orbit to its 2^{nd} orbit is

$$E_{5\to2} = E_2 - E_5$$

$$= \tfrac{1}{2^2}E_1 - \tfrac{1}{5^2}E_1$$

$$= \tfrac{21}{100}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{5\to2} = (457.7735614)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{5\to2} = \frac{E_{5\to2}}{h}$$

$$= \frac{(45.77735614)10^{-20}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{5\to2} = (6.908668055)10^{14}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{5\to2} = \frac{c}{\nu_{5\to2}}$$

$$= \frac{(2.99792458)10^{8}}{(6.908668055)10^{14}}$$

$$\boxed{\lambda_{5\to2} = (433.936694)\text{nm}}$$

$\boxed{5_p \to 2_p}$ The energy of a photon emitted in the transition of the proton from its 5^{th} orbit to its 2^{nd} orbit is

$$\mathcal{E}_{5\to 2} = \mathcal{E}_2 - \mathcal{E}_5$$

$$= \frac{1}{2^2}\mathcal{E}_1 - \frac{1}{5^2}\mathcal{E}_1$$

$$= \frac{21}{100}(9.340837346)10^{-17}\text{Joul}$$

$$\boxed{\mathcal{E}_{5\to 2} = (1.961575843)10^{-17}\text{Joul}}$$

The frequency of the emitted photon is

$$N_{5\to 2} = \frac{\mathcal{E}_{5\to 2}}{h}$$

$$= \frac{(1.961575843)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{5\to 2} = (2.960388608)10^{16}\text{cycles/sec}}$$ **(In the x-rays)**

Its wave-length is

$$\Lambda_{5\to 2} = \frac{c}{N_{5\to 2}}$$

$$= \frac{(2.99792458)10^{8}}{(2.960388608)10^{16}}$$

$$\boxed{\Lambda_{5\to 2} = (10.12679407)\text{nm}}$$

$\boxed{6_e \to 2_e}$ The energy of a photon emitted in the transition of the electron from its 6^{th} orbit to its 2^{nd} orbit is

$$E_{6\to 2} = E_2 - E_6$$

$$= \tfrac{1}{2^2}E_1 - \tfrac{1}{6^2}E_1$$

$$= \tfrac{2}{9}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{6\to 2} = (484.4164671)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{6\to 2} = \frac{E_{6\to 2}}{h}$$

$$= \frac{(48.44164671)10^{-20}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{6\to 2} = (7.310759847)10^{14}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{6\to 2} = \frac{c}{\nu_{6\to 2}}$$

$$= \frac{(2.99792458)10^{8}}{(7.310759847)10^{14}}$$

$$\boxed{\lambda_{6\to 2} = (410.070176)\text{nm}}$$

$\boxed{6_p \to 2_p}$ The energy of a photon emitted in the transition of the proton from its 6^{th} orbit to its 2^{nd} orbit is

$$\mathcal{E}_{6\to 2} = \mathcal{E}_2 - \mathcal{E}_6$$

$$= \frac{1}{2^2}\mathcal{E}_1 - \frac{1}{6^2}\mathcal{E}_1$$

$$= \frac{2}{9}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{6\to 2} = (2.075741632)10^{-17}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{6\to 2} = \frac{\mathcal{E}_{6\to 2}}{h}$$

$$= \frac{(2.075741632)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{6\to 2} = (3.132686358)10^{16}\,\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{6\to 2} = \frac{c}{N_{6\to 2}}$$

$$= \frac{(2.99792458)10^{8}}{(3.132686358)10^{16}}$$

$$\boxed{\Lambda_{6\to 2} = (9.56982039)\text{nm}}$$

$\boxed{7_e \to 2_e}$ The energy of a photon emitted in the transition of the electron from its 7^{th} orbit to its 2^{nd} orbit is

$$E_{7\to 2} = E_2 - E_7$$

$$= \tfrac{1}{2^2}E_1 - \tfrac{1}{7^2}E_1$$

$$= \tfrac{45}{196}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{7\to 2} = (500.4812989)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{7\to 2} = \frac{E_{7\to 2}}{h}$$

$$= \frac{(50.04812989)10^{-20}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{7\to 2} = (7.553208516)10^{14}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{7\to 2} = \frac{c}{\nu_{7\to 2}}$$

$$= \frac{(2.99792458)10^{8}}{(7.553208516)10^{14}}$$

$$\boxed{\lambda_{7\to 2} = (396.90743)\text{nm}}$$

$\boxed{7_p \to 2_p}$ The energy of a photon emitted in the transition of the proton from its 7^{th} orbit to its 2^{nd} orbit is

$$\mathscr{E}_{7\to 2} = \mathscr{E}_2 - \mathscr{E}_7$$

$$= \frac{1}{2^2}\mathscr{E}_1 - \frac{1}{7^2}\mathscr{E}_1$$

$$= \frac{45}{196}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathscr{E}_{7\to 2} = (2.144580003)10^{-17}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{7\to 2} = \frac{\mathscr{E}_{7\to 2}}{h}$$

$$= \frac{(2.144580003)10^{-17}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{7\to 2} = (3.23657646)10^{16}\,\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{7\to 2} = \frac{c}{N_{7\to 2}}$$

$$= \frac{(2.99792458)10^{8}}{(3.23657646)10^{16}}$$

$$\boxed{\Lambda_{7\to 2} = (9.26264097)\text{nm}}$$

4.

Paschen Transitions to 3rd Orbits

$\boxed{4_e \to 3_e}$ The energy of a photon emitted in the transition of the electron from its 4^{th} orbit to its 3^{rd} orbit is

$$E_{4\to 3} = E_3 - E_4$$

$$= \tfrac{1}{3^2}E_1 - \tfrac{1}{4^2}E_1$$

$$= \tfrac{7}{144}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{4\to 3} = (105.9661022)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{4\to 3} = \frac{E_{4\to 3}}{h}$$

$$= \frac{(10.59661022)10^{-20}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{4\to 3} = (1.599228717)10^{14}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{4\to 3} = \frac{c}{\nu_{4\to 3}} = \frac{(2.99792458)10^{8}}{(1.599228717)10^{14}}$$

$$\boxed{\lambda_{4\to 3} = (1874.606521)\text{nm}}$$

$\boxed{4_p \rightarrow 3_p}$ The energy of a photon emitted in the transition of the proton from its 4^{th} orbit to its 3^{rd} orbit is

$$\mathcal{E}_{4\rightarrow 3} = \mathcal{E}_3 - \mathcal{E}_4$$

$$= \frac{1}{3^2}\mathcal{E}_1 - \frac{1}{4^2}\mathcal{E}_1$$

$$= \frac{7}{144}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{4\rightarrow 3} = (4.540684821)10^{-18}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{4\rightarrow 3} = \frac{\mathcal{E}_{4\rightarrow 3}}{h}$$

$$= \frac{(4.540684821)10^{-18}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{4\rightarrow 3} = (6.8527514)10^{15}\,\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{4\rightarrow 3} = \frac{c}{N_{4\rightarrow 3}}$$

$$= \frac{(2.99792458)10^{8}}{(6.8527514)10^{15}}$$

$$\boxed{\Lambda_{4\rightarrow 3} = (43.74775038)\text{nm}}$$

$\boxed{5_e \to 3_e}$ The energy of a photon emitted in the transition of the electron from its 5^{th} orbit to its 3^{rd} orbit is

$$E_{5\to3} = E_3 - E_5$$

$$= \tfrac{1}{3^2}E_1 - \tfrac{1}{5^2}E_1$$

$$= \tfrac{16}{225}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{5\to3} = (155.0132695)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{5\to3} = \frac{E_{5\to3}}{h}$$

$$= \frac{(15.50132695)10^{-20}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{5\to3} = (2.339443151)10^{14}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{5\to3} = \frac{c}{\nu_{5\to3}}$$

$$= \frac{(2.99792458)10^{8}}{(2.339443151)10^{14}}$$

$$\boxed{\lambda_{5\to3} = (1281.469301)\text{nm}}$$

$\boxed{5_p \to 3_p}$ The energy of a photon emitted in the transition of the proton from its 5^{th} orbit to its 3^{rd} orbit is

$$\mathcal{E}_{5\to3} = \mathcal{E}_3 - \mathcal{E}_5$$

$$= \frac{1}{3^2}\mathcal{E}_1 - \frac{1}{5^2}\mathcal{E}_1$$

$$= \frac{16}{225}(9.340837346)10^{-17}\text{Joul}$$

$$\boxed{\mathcal{E}_{5\to3} = (6.642373224)10^{-18}\text{Joul}}$$

The frequency of the emitted photon is

$$N_{5\to3} = \frac{\mathcal{E}_{5\to3}}{h}$$

$$= \frac{(6.642373224)10^{-18}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{5\to3} = (1.002459634)10^{16}\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{5\to3} = \frac{c}{N_{5\to3}}$$

$$= \frac{(2.99792458)10^{8}}{(1.002459634)10^{16}}$$

$$\boxed{\Lambda_{5\to3} = (29.90568874)\text{nm}}$$

$\boxed{6_e \to 3_e}$ The energy of a photon emitted in the transition of the electron from its 6^{th} orbit to its 3^{rd} orbit is

$$E_{6\to 3} = E_3 - E_6$$

$$= \tfrac{1}{3^2}E_1 - \tfrac{1}{6^2}E_1$$

$$= \tfrac{1}{12}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{6\to 3} = (181.6561752)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{6\to 3} = \frac{E_{6\to 3}}{h}$$

$$= \frac{(18.16561752)10^{-20}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{6\to 3} = (2.741534943)10^{14}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{6\to 3} = \frac{c}{\nu_{6\to 3}}$$

$$= \frac{(2.99792458)10^{8}}{(2.741534943)10^{14}}$$

$$\boxed{\lambda_{6\to 3} = (1093.52047)\text{nm}}$$

$\boxed{6_p \to 3_p}$ The energy of a photon emitted in the transition of the proton from its 6^{th} orbit to its 3^{rd} orbit is

$$\mathcal{E}_{6\to3} = \mathcal{E}_3 - \mathcal{E}_6$$

$$= \frac{1}{3^2}\mathcal{E}_1 - \frac{1}{6^2}\mathcal{E}_1$$

$$= \frac{1}{12}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{6\to3} = (7.784031122)10^{-18}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{6\to3} = \frac{\mathcal{E}_{6\to3}}{h}$$

$$= \frac{(7.784031122)10^{-18}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{6\to3} = (1.174757384)10^{16}\,\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{6\to3} = \frac{c}{N_{6\to3}}$$

$$= \frac{(2.99792458)10^{8}}{(1.174757384)10^{16}}$$

$$\boxed{\Lambda_{6\to3} = (25.51952106)\text{nm}}$$

$\boxed{7_e \to 3_e}$ The energy of a photon emitted in the transition of the electron from its 7^{th} orbit to its 3^{rd} orbit is

$$E_{7\to 3} = E_3 - E_7$$

$$= \frac{1}{3^2} E_1 - \frac{1}{7^2} E_1$$

$$= \frac{40}{441}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{7\to 3} = (197.721007)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{7\to 3} = \frac{E_{7\to 3}}{h}$$

$$= \frac{(19.7721007)10^{-20}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{7\to 3} = (2.983983611)10^{14}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{7\to 3} = \frac{c}{\nu_{7\to 3}}$$

$$= \frac{(2.99792458)10^{8}}{(2.983983611)10^{14}}$$

$$\boxed{\lambda_{7\to 3} = (1004.671932)\text{nm}}$$

$\boxed{7_p \to 3_p}$ The energy of a photon emitted in the transition of the proton from its 7^{th} orbit to its 3^{rd} orbit is

$$\mathcal{E}_{7\to 3} = \mathcal{E}_3 - \mathcal{E}_7$$

$$= \frac{1}{3^2}\mathcal{E}_1 - \frac{1}{7^2}\mathcal{E}_1$$

$$= \frac{40}{441}(9.340837346)10^{-17}\text{Joul}$$

$$\boxed{\mathcal{E}_{7\to 3} = (8.472414826)10^{-18}\text{Joul}}$$

The frequency of the emitted photon is

$$N_{7\to 3} = \frac{\mathcal{E}_{7\to 3}}{h}$$

$$= \frac{(8.472414826)10^{-18}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{7\to 3} = (1.278647493)10^{16}\text{cycles/sec}}$$ **(In the x-rays)**

Its wave-length is

$$\Lambda_{7\to 3} = \frac{c}{N_{7\to 3}}$$

$$= \frac{(2.99792458)10^{8}}{(1.278647493)10^{16}}$$

$$\boxed{\Lambda_{7\to 3} = (23.44605997)\text{nm}}$$

5.

Brackett Transitions to 4th Orbits

$\boxed{5_e \to 4_e}$ The energy of a photon emitted in the transition of the electron from its 5^{th} orbit to its 4^{th} orbit is

$$E_{5\to 4} = E_4 - E_5$$

$$= \tfrac{1}{4^2}E_1 - \tfrac{1}{5^2}E_1$$

$$= \tfrac{9}{400}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{5\to 4} = (49.0471673)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{5\to 4} = \frac{E_{5\to 4}}{h}$$

$$= \frac{(49.0471673)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{5\to 4} = (7.402144345)10^{13}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{5\to 4} = \frac{c}{\nu_{5\to 4}} = \frac{(2.99792458)10^{8}}{(7.402144345)10^{13}}$$

$$\boxed{\lambda_{5\to 4} = (4050.075816)\text{nm}}$$

$\boxed{5_p \to 4_p}$ The energy of a photon emitted in the transition of the proton from its 5^{th} orbit to its 4^{th} orbit is

$$\mathscr{E}_{5\to 4} = \mathscr{E}_4 - \mathscr{E}_5$$

$$= \frac{1}{4^2}\mathscr{E}_1 - \frac{1}{5^2}\mathscr{E}_1$$

$$= \frac{9}{400}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathscr{E}_{5\to 4} = (2.101688403)10^{-18}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{5\to 4} = \frac{\mathscr{E}_{5\to 4}}{h}$$

$$= \frac{(2.101688403)10^{-18}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{5\to 4} = (3.171844937)10^{15}\,\text{cycles/sec}}$$ **(In the x-rays)**

Its wave-length is

$$\Lambda_{5\to 4} = \frac{c}{N_{5\to 4}}$$

$$= \frac{(2.99792458)10^{8}}{(3.171844937)10^{15}}$$

$$\boxed{\Lambda_{5\to 4} = (94.51674465)\text{nm}}$$

$\boxed{6_e \to 4_e}$ The energy of a photon emitted in the transition of the electron from its 6^{th} orbit to its 4^{th} orbit is

$$E_{6\to 4} = E_4 - E_6$$

$$= \frac{1}{4^2}E_1 - \frac{1}{6^2}E_1$$

$$= \frac{5}{144}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{6\to 4} = (75.69007299)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{6\to 4} = \frac{E_{6\to 4}}{h}$$

$$= \frac{(75.69007299)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{6\to 4} = (11.42306226)10^{13}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{6\to 4} = \frac{c}{\nu_{6\to 4}}$$

$$= \frac{(2.99792458)10^{8}}{(11.42306226)10^{13}}$$

$$\boxed{\lambda_{6\to 4} = (2624.449129)\text{nm}}$$

$\boxed{6_p \to 4_p}$ The energy of a photon emitted in the transition of the proton from its 6^{th} orbit to its 4^{th} orbit is

$$\mathcal{E}_{6\to 4} = \mathcal{E}_4 - \mathcal{E}_6$$

$$= \frac{1}{4^2}\mathcal{E}_1 - \frac{1}{6^2}\mathcal{E}_1$$

$$= \frac{5}{144}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{6\to 4} = (3.243346301)10^{-18}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{6\to 4} = \frac{\mathcal{E}_{6\to 4}}{h}$$

$$= \frac{(3.243346301)10^{-18}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{6\to 4} = (4.894822434)10^{15}\,\text{cycles/sec}}$$ **(In the x-rays)**

Its wave-length is

$$\Lambda_{6\to 4} = \frac{c}{N_{6\to 4}}$$

$$= \frac{(2.99792458)10^{8}}{(4.894822434)10^{15}}$$

$$\boxed{\Lambda_{6\to 4} = (61.24685053)\text{nm}}$$

$\boxed{7_e \to 4_e}$ The energy of a photon emitted in the transition of the electron from its 7^{th} orbit to its 4^{th} orbit is

$$E_{7\to 4} = E_4 - E_7$$

$$= \tfrac{1}{4^2}E_1 - \tfrac{1}{7^2}E_1$$

$$= \tfrac{33}{784}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{7\to 4} = (91.7549048)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{7\to 4} = \frac{E_{7\to 4}}{h}$$

$$= \frac{(91.7549048)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{7\to 4} = (13.84754895)10^{13}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{7\to 4} = \frac{c}{\nu_{7\to 4}}$$

$$= \frac{(2.99792458)10^{8}}{(13.84754895)10^{13}}$$

$$\boxed{\lambda_{7\to 4} = (2164.949618)\text{nm}}$$

$\boxed{7_p \to 4_p}$ The energy of a photon emitted in the transition of the proton from its 7^{th} orbit to its 4^{th} orbit is

$$\mathcal{E}_{7\to4} = \mathcal{E}_4 - \mathcal{E}_7$$

$$= \frac{1}{4^2}\mathcal{E}_1 - \frac{1}{7^2}\mathcal{E}_1$$

$$= \frac{33}{784}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{7\to4} = (3.931730005)10^{-18}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{7\to4} = \frac{\mathcal{E}_{7\to4}}{h}$$

$$= \frac{(3.931730005)10^{-18}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{7\to4} = (5.933723522)10^{15}\,\text{cycles/sec}}$$ **(In the x-rays)**

Its wave-length is

$$\Lambda_{7\to4} = \frac{c}{N_{7\to4}}$$

$$= \frac{(2.99792458)10^{8}}{(5.933723522)10^{15}}$$

$$\boxed{\Lambda_{7\to4} = (50.52349623)\text{nm}}$$

6.

Pfund Transitions to 5th Orbits

$\boxed{6_e \to 5_e}$ The energy of a photon emitted in the transition of the electron from its 6^{th} orbit to its 5^{th} orbit is

$$E_{6\to 5} = E_5 - E_6$$

$$= \tfrac{1}{5^2}E_1 - \tfrac{1}{6^2}E_1$$

$$= \tfrac{11}{900}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{6\to 5} = (26.64290569)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{6\to 5} = \frac{E_{6\to 5}}{h}$$

$$= \frac{(26.64290569)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{6\to 5} = (4.020917916)10^{13}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{6\to 5} = \frac{c}{\nu_{6\to 5}} = \frac{(2.99792458)10^{8}}{(4.020917916)10^{13}}$$

$$\boxed{\lambda_{6\to 5} = (7455.821389)\text{nm}}$$

$\boxed{6_p \to 5_p}$ The energy of a photon emitted in the transition of the proton from its 6^{th} orbit to its 5^{th} orbit is

$$\mathcal{E}_{6\to 5} = \mathcal{E}_5 - \mathcal{E}_6$$

$$= \frac{1}{5^2}\mathcal{E}_1 - \frac{1}{6^2}\mathcal{E}_1$$

$$= \frac{11}{900}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{6\to 5} = (1.141657898)10^{-18}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{6\to 5} = \frac{\mathcal{E}_{6\to 5}}{h}$$

$$= \frac{(1.141657898)10^{-18}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{6\to 5} = (1.722977497)10^{15}\,\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{6\to 5} = \frac{c}{N_{6\to 5}}$$

$$= \frac{(2.99792458)10^{8}}{(1.722977497)10^{15}}$$

$$\boxed{\Lambda_{6\to 5} = (173.9967345)\text{nm}}$$

$\boxed{7_e \to 5_e}$ The energy of a photon emitted in the transition of the electron from its 7^{th} orbit to its 5^{th} orbit is

$$E_{7\to5} = E_5 - E_7$$

$$= \frac{1}{5^2}E_1 - \frac{1}{7^2}E_1$$

$$= \frac{24}{1225}(2179.874102)10^{-21}\text{Joul}$$

$$\boxed{E_{7\to5} = (42.70773751)10^{-21}\text{Joul}}$$

The frequency of the emitted photon is

$$\nu_{7\to5} = \frac{E_{7\to5}}{h}$$

$$= \frac{(42.70773751)10^{-21}}{(6.6260755)10^{-34}}$$

$$\boxed{\nu_{7\to5} = (6.4454046)10^{13}\text{cycles/sec}}$$

Its wave-length is

$$\lambda_{7\to5} = \frac{c}{\nu_{7\to5}}$$

$$= \frac{(2.99792458)10^{8}}{(6.4454046)10^{13}}$$

$$\boxed{\lambda_{7\to5} = (4651.258945)\text{nm}}$$

$\boxed{7_p \to 5_p}$ The energy of a photon emitted in the transition of the proton from its 7^{th} orbit to its 5^{th} orbit is

$$\mathcal{E}_{7\to 5} = \mathcal{E}_5 - \mathcal{E}_7$$

$$= \frac{1}{5^2}\mathcal{E}_1 - \frac{1}{7^2}\mathcal{E}_1$$

$$= \frac{24}{1225}(9.340837346)10^{-17}\,\text{Joul}$$

$$\boxed{\mathcal{E}_{7\to 5} = (1.830041602)10^{-18}\,\text{Joul}}$$

The frequency of the emitted photon is

$$N_{7\to 5} = \frac{\mathcal{E}_{7\to 5}}{h}$$

$$= \frac{(1.830041602)10^{-18}}{(6.6260755)10^{-34}}$$

$$\boxed{N_{7\to 5} = (2.761878585)10^{15}\,\text{cycles/sec}}$$ (In the x-rays)

Its wave-length is

$$\Lambda_{7\to 5} = \frac{c}{N_{7\to 5}}$$

$$= \frac{(2.99792458)10^{8}}{(2.761878585)10^{15}}$$

$$\boxed{\Lambda_{7\to 5} = (108.5465739)\text{nm}}$$

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