Hydrogen's Electron, and Proton are Magnetic Monopoles

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Abstract The Hydrogen electron that orbits its proton, accelerates towards the proton, radiates power, and should spiral into the proton, leading to the atom's collapse. To prevent such collapse, the proton must orbit the Atom's center, and radiate the electron with the same power.

Then, the inertia moments of the electron orbiting at radius R, and the proton orbiting at radius ρ are equal,

$$M\rho^2 = mR^2 . \Rightarrow R = \rho\sqrt{\frac{M}{m}} \approx 42.85\rho .$$

The proton current $I_p = \frac{\mathrm{e}}{T_p}$ induces a magnetic field $B_{\mathrm{p}} = \frac{\mu_0}{2}\frac{1}{\rho}I_{\mathrm{p}}$.

The moving electron is a magnetic pole in the proton's field.

The electron current $I_{\mathrm{e}} = \frac{\mathrm{e}}{T_{\mathrm{e}}}$ induces a magnetic field $B_{\mathrm{e}} = \frac{\mu_0 I_{\mathrm{e}}}{2R}$.

The moving proton is a magnetic pole in the electron's field.

The electron is assumed to move at speed $v = \frac{\hbar}{mR}$ in the proton's electric field $\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}}{R^2}$, and magnetic field $\frac{\mu_0}{2}\frac{1}{\rho}I_{\mathrm{p}}$ under Lorentz force $\mathrm{e}(E + B_p v)$.

We compute the orbit radius, speeds, angular velocities, frequencies, periods, currents, and magnetic fields, magnetic pole charges, and magnetic forces and for the electron, and the proton:

	Hydrogen's Electron	Hydrogen's Proton
Orbit Radius	$R = (5.418690371)10^{-11}\mathrm{m}$	$\rho = (1.264561448)10^{-12}\mathrm{m}$
Speed	$v = (2.136450774)10^{6}\mathrm{m/s}$	$V = (3.263741033)10^{5}\mathrm{m/s}$
Ang Velocity	$\omega = (3.94274377)10^{16}\mathrm{rad/s}$	$\Omega = (2.580927191)10^{17}\mathrm{rad/s}$
Frequency	$f_{\mathrm{e}} = (6.275071603)10^{15}\mathrm{c/s}$	$f_{\mathrm{p}} = (4.107673202)10^{16}\mathrm{c/s}$
Period	$T_{\mathrm{e}} = (1.593607314)10^{-16}\mathrm{s}$	$T_{\mathrm{p}} = (2.43446825)10^{-17}\mathrm{s}$
Electric Charge	$\mathrm{e}^{-} = -(1.60217733)10^{-19}C$	$\mathrm{e}^{+} = (1.60217733)10^{-19}C$
Current	$I_{\mathrm{e}} = (1.005377746)\mathrm{m}A$	$I_{\mathrm{p}} = (6.581220889)\mathrm{m}A$
Magnetic Field	$B_{\mathrm{e}} = 11.65775169$ weber/m^2	$B_{\mathrm{p}} = 3269.9989802$ web/m^2
Magnetic Pole Charge	$\mathrm{e}v = (3.422972997)10^{-13}\mathrm{Am}$	$\mathrm{eV} = (5.229091894)10^{-14}\mathrm{Am}$
Magnetic Force on it	$B_p \mathrm{e}v = (1.789904033)10^{-9}\mathrm{N}$	$B_{\mathrm{e}}\mathrm{eV} = (6.095945487)10^{-13}\mathrm{N}$

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Contents

1.

The Hydrogen's Orbiting Proton

We consider the Hydrogen electron orbiting the proton along a circle of radius R so that the centrifugal force balances the electric attraction

$$m_e\omega^2 R = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}$$

Thus, the electron's accelerates towards the proton at

$$\omega^2 R = \frac{1}{m_e}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}.$$

The electron's velocity is far slower than light speed c.

To see that, note that a wave in the mid optical spectrum with length

$$\lambda \sim 6\times 10^{-7}\,\text{meter.}$$

corresponds to the optical frequency

$$\nu = \frac{c}{\lambda} \sim \frac{3\times 10^{8}}{6\times 10^{-7}} = 5\times 10^{14}\,\text{cycles/sec.}$$

and to the angular velocity

$$\omega = 2\pi\nu \sim 6\times 5\times 10^{14} = 3\times 10^{15}\,\text{radians/sec.}$$

If the electron orbit's radius is

$$R \sim 6\times 10^{-11}\,\text{meter}$$

The electron's velocity is

$$v = \omega R \sim 3\times 10^{15} \times 6\times 10^{-11} \sim 2\times 10^{5}\,\text{meter/sec}$$

Thus,

$$\frac{v}{c} \sim \frac{2\times 10^5}{3\times 10^8} \ll 1\,,$$

Then, Lorentz Factor

$$\gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1\,,$$

and the electron's velocity is $\gamma(v)v \approx v$.

The Electron Spiraling into the Proton

Assuming that

$$m\frac{v^2}{R} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\,,$$

the kinetic energy of the electron is

$$\tfrac{1}{2}mv^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R}\,.$$

The electric energy of the Hydrogen Atom is

$$-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R}\,.$$

Therefore, the total electron energy is

$$-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R}\,,$$

and its rate of change is

$$\frac{d}{dt}\left\{-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R}\right\} = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\frac{dR}{dt}.$$

The electron radiates energy as it accelerates towards the proton, at rate

$$\left[\frac{e^2}{6\pi\varepsilon_0 c^3}a^2\right]_{t-\frac{R}{c}},$$

where the retarded time is

$$t - \frac{R}{c} \approx t - \frac{6\times 10^{-11}}{3\times 10^{8}} = t - \frac{2}{10^{19}} \approx t,$$

and the acceleration of the electron is

$$a = \frac{v^2}{R}.$$

Therefore, the Electron's Radiation rate is

$$\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{v^2}{R}\right)^2 \approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\right)^2.$$

Since it equals the rate of change of the electron's energy,

$$\frac{\not{e}^2}{6\not{\pi}\not{\varepsilon}_0 c^3}\left(\frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\right)^2 \approx \frac{1}{2}\frac{1}{4\not{\pi}\not{\varepsilon}_0}\frac{\not{e}^2}{R^2}\frac{dR}{dt},$$

$$dt \approx \frac{3}{4}c^3\left(4\pi\varepsilon_0\frac{m}{e^2}\right)^2 R^2 dR$$

Therefore, the electron will spiral into the proton in

$$t\Big|_{R=0}^{R=6\times 10^{-11}} \approx \frac{1}{4}c^3\left(4\pi\varepsilon_0\frac{m}{e^2}\right)^2 R^3\Big|_{R=0}^{R=6\times 10^{-11}} \text{ sec}$$

$$\approx \underbrace{\frac{1}{4}c^3(\underbrace{\frac{4\pi}{\mu_0}}_{10^7}\underbrace{\mu_0\varepsilon_0}_{c^{-2}})^2}_{\frac{1}{4c}10^{14}}\left(\frac{m}{e^2}\right)^2 6^3 10^{-33}$$

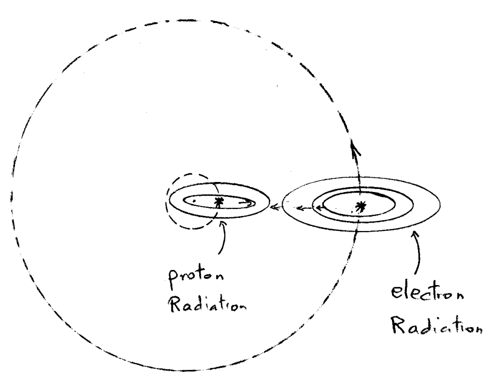
$$\approx \frac{1}{12\cdot 10^8}10^{14}\frac{(9.11\cdot 10^{-31})^2}{(1.6\cdot 10^{-19})^4}6^3\cdot 10^{-33}$$

$$\approx 18\cdot\frac{(9.11)^2}{(1.6)^4}10^{-13}$$

$$\approx 2.23\times 10^{-11}\,\text{sec.}$$

The Orbiting Proton

To prevent the Atom's collapse, the Proton must have its own orbits, in which it accelerates towards the electron, and radiates it. Then, the electron and the proton exchange equal amounts of energy between them, and balance each others loss. And the atom does not collapse.



$$\text{The Power} = \frac{d}{dt}\left\{\text{radiation energy}\right\}$$

radiated by the accelerating proton onto the electron equals the Power radiated by the electron onto the proton.

The Proton's Orbit Radius

Assume that the centrifugal force on the electron, $m\omega^2 R$, is balanced by the electric attraction on the electron $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}$,

$$m\omega^2 R = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2},$$

Then, the Electron Accelerates towards the proton at

$$a = \omega^2 R = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mR^2}$$

and radiates the proton at power

$$\frac{e^2}{6\pi\varepsilon_0 c^3}a^2 = \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{m_e}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\right)^2.$$

The proton sees the electron at the center of its orbit, at distance ρ, and accelerates towards it.

Assume that the centrifugal force on the proton, $M\Omega^2\rho$, equals the electric attraction, $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}$,

$$M\Omega^2\rho = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2},$$

Then, the Proton accelerates towards the electron at

$$A = \Omega^2 \rho = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{M\rho^2}$$

and radiates the electron with power

$$\frac{e^2}{6\pi\varepsilon_0 c^3} A^2 = \frac{e^2}{6\pi\varepsilon_0 c^3} \left(\frac{1}{M_p} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\rho^2} \right)^2.$$

At power equilibrium,

$$\frac{e^2}{6\pi\varepsilon_0 c^3} \left(\frac{1}{m_e} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{R^2} \right)^2 = \frac{e^2}{6\pi\varepsilon_0 c^3} \left(\frac{1}{M_p} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\rho^2} \right)^2.$$

Hence,

$$\boxed{m_e R^2 = M_p \rho^2}$$

That is, inertia moments are equal.

$$\boxed{\frac{R}{\rho} = \sqrt{\frac{M_p}{m_e}} \approx \sqrt{1836.152701} \approx 42.8503524}$$

The proton orbit radius, which is the nucleus radius of the Hydrogen is

$$\rho \approx R\sqrt{\frac{m_e}{M_p}}$$

$$= 5.29277249 \times 10^{-11} \;/\; 42.8503524$$

$$= 1.221173735 \times 10^{-12}.$$

2.

The Angular Momentum, and the Rotation Energy of the Electron

Bohr assumed that the Electron's Angular Momentum in its n-th orbit of radius R_n is quantized, so that

$$mvR_n = n\hbar,$$

Thus, at the Hydrogen Radius, $R_1 = R$

$$mvR = \hbar.$$

Multiplying by 2π,

$$2\pi R = \frac{h}{mv}.$$

De-Broglie claimed that the electron is associated with a wave of length $\lambda = 2\pi R$. Then,

$$\lambda = \frac{h}{mv}.$$

Multiplying $mvR = \hbar$ by the electron's angular speed ω,

$$mvR\omega = \hbar\omega,$$

$$m\omega RR\omega = \hbar\omega$$

$$\frac{1}{2}mR^2\omega^2 = \frac{1}{2}\hbar\omega.$$

The inertia moment of the electron is $I = \frac{1}{2}mR^2$, and its rotation energy is $\frac{1}{2}I\omega^2 = \frac{1}{2}mR^2\omega^2$.

While the rotation energy of the electron may be quantized, it is not clear why it should equal $\frac{1}{2}\hbar\omega$, and not, say, $\hbar\omega$.

Bohr used the assumption that $mvR = \hbar$ together with the balance of forces on the orbiting electron to compute R.

In the following we approximate R without using $mvR = \hbar$. But then v turns out to be very close to $\frac{\hbar}{mR}$.

Therefore, we accept $v = \frac{\hbar}{mR}$. And compute R using it. Our final value for R satisfies it.

3.

Bohr's Approximation of the Electron's Orbit, and Speed

Bohr assumed that the centrifugal force on the electron is balanced by the electric force that the proton applies to the electron. That is,

$$\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{R^2} = m\frac{v^2}{R},$$

$$\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{R} = mv^2.$$

Substituting $v = \frac{\hbar}{mR}$,

$$\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{R} = m\frac{\hbar^2}{m^2R^2},$$

$$R = \frac{4\pi\varepsilon_0\hbar^2}{m\mathrm{e}^2}$$

$$= \frac{4\pi(8.854187817)10^{-12}[(1.05457266)10^{-34}]^2}{(9.1093897)10^{-31}[(1.60217733)10^{-19}]^2}$$

$$= (5.29177249)10^{-11}\text{meter}$$

$$v = \frac{\hbar}{mR}$$

$$v = \frac{\hbar}{m\dfrac{4\pi\varepsilon_0\hbar^2}{m\mathrm{e}^2}}$$

$$= \frac{\mathrm{e}^2}{4\pi\varepsilon_0\hbar}$$

$$= \frac{[(1.60217733)10^{-19}]^2}{4\pi(8.854187817)10^{-12}(1.05457266)10^{-34}}$$

$$= (2.1876814)10^6\mathrm{m/s}$$

In fact, the electron moves in the Magnetic field of the orbiting proton, B_p, under the Lorentz force

$$\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{R^2} + B_\mathrm{p}\mathrm{e}v\,.$$

We proceed to compute the electron's orbit, and speed under the Lorentz force.

4.

The Electron's, and Proton's Periods, and Fields

The Electron's and Proton's Periods

Assuming

$$m\omega^2 R \approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2},$$

and

$$M\Omega^2 \rho \approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2},$$

It follows that

$$m\omega^2 R^3 \approx \frac{e^2}{4\pi\varepsilon_0} \approx M\Omega^2\rho^3$$

$$m\left(\frac{2\pi}{T_e}\right)^2 R^3 \approx M\left(\frac{2\pi}{T_p}\right)^2 \rho^3$$

$$\left(\frac{T_e}{T_p}\right)^2 \approx \frac{m}{M}\left(\frac{R}{\rho}\right)^3$$

$$= \frac{m}{M}\left(\sqrt{\frac{M}{m}}\right)^3$$

$$= \sqrt{\frac{M}{m}}$$

$$\boxed{\frac{T_e}{T_p} \approx \sqrt[4]{\frac{M}{m}}}$$

$$\boxed{\frac{\Omega_p}{\omega_e} \approx \frac{T_e}{T_p} = \sqrt[4]{\frac{M_p}{m_e}} = (1836.152701)^{\frac{1}{4}} = 6.546018057}$$

The Proton Period in Hydrogen is

$$\boxed{T_p \approx T_e \sqrt[4]{\frac{m_e}{M_p}}}$$

The Electron in the Proton's Field

Since the proton is moving, its distance from the electron changes between $R + \rho$, and $R - \rho$.

Assume that the proton charge is located at the center at distance R, from the electron. Then, the proton applies to the electron the Electric Force

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2},$$

and the Magnetic Force

$$B_p ev.$$

The Magnetic Field of the Proton is

$$B_{\text{p}} = \frac{\mu_0}{2}\frac{1}{\rho}I_{\text{p}}$$

Since $M\rho^2 \approx mR^2$,

$$\approx \frac{\mu_0}{2}\frac{1}{R\sqrt{\frac{m}{M}}}\frac{\mathrm{e}}{T_p}$$

$$= \frac{\mu_0}{2}\frac{1}{R}\sqrt{\frac{M}{m}}\frac{\mathrm{e}}{T_{\mathrm{e}}\sqrt[4]{\frac{m}{M}}}$$

$$= \frac{\mu_0}{2}\frac{1}{R}\left(\frac{M}{m}\right)^{\frac{3}{4}}\mathrm{e}\frac{\omega}{2\pi}$$

$$= \frac{\mu_0}{4\pi}\frac{1}{R}\left(\frac{M}{m}\right)^{\frac{3}{4}}\mathrm{e}\frac{v}{R}$$

Therefore, the magnetic force on the electron is

$$B_p \mathrm{e}v = \frac{\mu_0}{4\pi}\frac{1}{R}\left(\frac{M}{m}\right)^{\frac{3}{4}}\mathrm{e}\frac{v}{R}\mathrm{e}v$$

$$= \frac{1}{4\pi\varepsilon_0}\varepsilon_0\mu_0\frac{\mathrm{e}^2}{R^2}\left(\frac{M}{m}\right)^{\frac{3}{4}}v^2$$

$$= \frac{1}{4\pi\varepsilon_0}\varepsilon_0\mu_0\frac{\mathrm{e}^2}{R^2}\left(\frac{M}{m}\right)^{\frac{3}{4}}v^2$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{R^2}\left(\frac{M}{m}\right)^{\frac{3}{4}}\frac{v^2}{c^2}$$

The Electromagnetic Force on the electron is

$$\underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}}_{\text{electric}} - \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\left(\frac{M}{m}\right)^{\frac{3}{4}}\frac{v^2}{c^2}}_{\text{magnetic}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\left(1-\left(\frac{M}{m}\right)^{\frac{3}{4}}\frac{v^2}{c^2}\right)$$

Therefore,

$$m\omega^2 R = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\left(1-\left(\frac{M}{m}\right)^{\frac{3}{4}}\frac{v^2}{c^2}\right).$$

Then, the electron's acceleration towards the proton is

$$a = \omega^2 R \approx \frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\left(1-\left(\frac{M}{m}\right)^{\frac{3}{4}}\frac{v^2}{c^2}\right).$$

The electron radiates the proton with power

$$\frac{e^2}{6\pi\varepsilon_0 c^3}a^2 \approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\right)^2\left[1-\left(\frac{M}{m}\right)^{\frac{3}{4}}\frac{v^2}{c^2}\right]^2$$

with Bohr's $v = (2.1876814)10^6\text{m/s}$,

$$\left[1-\left(\frac{M}{m}\right)^{\frac{3}{4}}\frac{v^2}{c^2}\right]^2 = \left(1-\underbrace{\left(1836.132701\right)^{\frac{3}{4}}\left(\frac{(2.1876914)10^6}{(2.99792458)10^8}\right)^2}_{0.014936973}\right)^2$$

$$\approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}\right)^2\left[1-2(0.015)\right]$$

$$\approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{M}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\right)^2$$

The Proton in the Electron's Field

Assume that the electron charge is located at the center at distance ρ, from the proton. Then, the electron applies to the proton the Electric Force

$$E_{\mathrm{e}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2},$$

and the Magnetic Force

$$B_{\mathrm{e}}\mathrm{eV}.$$

The Magnetic Field of the Electron is

$$B_{\mathrm{e}} = \frac{\mu_0}{2}\frac{1}{R}I_{\mathrm{e}}$$

$$= \frac{\mu_0}{2}\frac{1}{R}\frac{\mathrm{e}}{T_{\mathrm{e}}}$$

$$= \frac{\mu_0}{2}\frac{1}{R}\mathrm{e}\frac{\omega}{2\pi}$$

$$= \frac{\mu_0}{4\pi}\frac{1}{R}\mathrm{e}\frac{v}{R}$$

Therefore, the magnetic force on the proton is

$$B_{\mathrm{e}}\mathrm{eV} = \frac{\mu_0}{4\pi}\frac{1}{R}\mathrm{e}\frac{v}{R}\mathrm{eV}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{R^2}\varepsilon_0\mu_0 v\mathrm{V}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{R^2}\frac{v}{c^2}\mathrm{V}$$

Since $V = \Omega\rho = v\dfrac{\Omega\rho}{\omega R} = v\sqrt[4]{\dfrac{M}{m}}\sqrt{\dfrac{m}{M}} = v\left(\dfrac{m}{M}\right)^{\frac{1}{4}}$,

$$B_{\mathrm{e}}\mathrm{eV} = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{\rho^2\dfrac{M}{m}}\frac{v^2}{c^2}\left(\frac{m}{M}\right)^{\frac{1}{4}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{\rho^2}\left(\frac{m}{M}\right)^{\frac{5}{4}}\frac{v^2}{c^2}$$

Applying $v = (2.1876914)10^6$,

$$B_{\mathrm{e}}\mathrm{e}V = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{\rho^2}\left(\frac{1}{1836.152701}\right)^{\frac{5}{4}}\left(\frac{(2.1876914)10^6}{(2.99792458)10^8}\right)^2$$

$$= (4.430418159)10^{-9}$$

Therefore,

$$M\Omega^2\rho = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\left(1 - (4.430418159)10^{-9}\right).$$

Then, the proton's acceleration is

$$A = \Omega^2\rho = \frac{1}{M}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\left(1 - (4.430418159)10^{-9}\right)$$

The proton radiates the electron with power

$$\frac{\mathrm{e}^2}{6\pi\varepsilon_0 c^3}A^2 \approx \frac{\mathrm{e}^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{M}\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{\rho^2}\right)^2\left[1 - (4.430418159)10^{-9}\right]^2$$

$$\approx \frac{\mathrm{e}^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{M}\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{\rho^2}\right)^2\left[1 - 2(4.430418159)10^{-9}\right]$$

$$\approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{M}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\right)^2$$

The radiation power balance leads to the equality of the inertial moments of the Electron, and the Proton,

$$mR^2 \approx M\rho^2$$

5.

The Electron's Orbit, and Speed Avoiding Quantized Angular Momentum

Without assuming $mvR = \hbar$, we approximate the electron's orbit, and speed from the balance of forces on the electron in its orbit,

$$\underbrace{m\frac{v^2}{R}}_{\text{centrifugal}} = \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}}_{\text{electric}} - \underbrace{\frac{\mu_0}{4\pi}\frac{1}{R^2}\left(\frac{M}{m}\right)^{\frac{3}{4}} e^2 v^2}_{\text{magnetic}}$$

$$\frac{1}{e^2}mv^2R = \frac{1}{4\pi\varepsilon_0} - \frac{\mu_0}{4\pi}\left(\frac{M}{m}\right)^{\frac{3}{4}} v^2$$

$$v^2\left[\frac{1}{e^2}mR + \frac{\mu_0}{4\pi}\left(\frac{M}{m}\right)^{\frac{3}{4}}\right]4\pi\varepsilon_0 = 1$$

$$v^2\left[\frac{4\pi\varepsilon_0}{e^2}mR + \varepsilon_0\mu_0\left(\frac{M}{m}\right)^{\frac{3}{4}}\right] = 1$$

$$v = \frac{1}{\sqrt{\frac{4\pi\varepsilon_0}{e^2}mR + \varepsilon_0\mu_0\left(\frac{M}{m}\right)^{\frac{3}{4}}}}$$

$$v = \frac{1}{\sqrt{\frac{4\pi\varepsilon_0}{e^2} mR + \frac{1}{c^2}\left(\frac{M}{m}\right)^{\frac{3}{4}}}}$$

$$\frac{4\pi\varepsilon_0}{e^2} m = \frac{4\pi(8.854187817)10^{-12}}{[(1.60217733)10^{-19}]^2}(9.1093897)10^{-31}$$

$$= (3.948450619)10^{-3}$$

$$\frac{1}{c^2}\left(\frac{M}{m}\right)^{\frac{3}{4}} = \frac{1}{[(2.99792458)10^8]^2}\left(1836.152701\right)^{\frac{3}{4}}$$

$$= (0.11265005)10^{-16}(280.4991806)$$

$$= (3.159824672)10^{-15}$$

$$\boxed{v = \frac{1}{\sqrt{(3.948450619)10^{-3} R + (3.159824672)10^{-15}}}}$$

$\boxed{v_1}$

Using Bohr's $R_1 = (5.29177249)10^{-11}$,

$$v_1 = \frac{1}{\sqrt{(3.948450619)10^{-3}(5.29177249)10^{-11} + (3.159824672)10^{-15}}}$$

$$v_1 = (2.171334584)10^6 \text{m/s}$$

For R, we obtain

$$R = \frac{1}{\frac{4\pi\varepsilon_0}{e^2}m}\left(\frac{1}{v^2} - \frac{1}{c^2}\left(\frac{M}{m}\right)^{\frac{3}{4}}\right)$$

$$\boxed{R = \frac{1}{(3.948450619)10^{-3}}\left(\frac{1}{v^2} - (3.159824672)10^{-15}\right)}$$

$\boxed{R_2}$

Using $v_1 = (2.171334584)10^6 \text{m/s}$,

$$R_2 = \frac{1}{(3.948450619)10^{-3}}\left(\frac{1}{[(2.171334584)10^6]^2} - (3.159824672)10^{-15}\right)$$

$$= \frac{1}{(3.948450619)}10^3\left((212.1028483)10^{-15} - (3.159824672)10^{-15}\right)$$

$$R_2 = (5.291772546)10^{-11}\text{m}.$$

$\boxed{v_2}$

$$v = \frac{1}{\sqrt{(3.948450619)10^{-3}R + (3.159824672)10^{-15}}}$$

Using $R_2 = (5.291772546)10^{-11}\text{m}$

$$v_2 = \frac{1}{\sqrt{(3.948450619)10^{-3}(5.291772546)10^{-11} + (3.159824672)10^{-15}}}$$

$$v_2 = (2.171334573)10^6 \text{m/s}$$

$\boxed{R_3}$

$$R = \frac{1}{(3.948450619)10^{-3}}\left(\frac{1}{v^2} - (3.159824672)10^{-15}\right)$$

Using $v_2 = (2.171334573)10^6 \text{m/s}$

$$R_3 = \frac{1}{(3.948450619)10^{-3}}\left(\frac{1}{[(2.171334573)10^6]^2} - (3.159824672)10^{-15}\right)$$

$$= \frac{1}{(3.948450619)}10^3\left((212.1028505)10^{-15} - (3.159824672)10^{-15}\right)$$

$$R_3 = (5.291772546)10^{-11}\text{m}.$$

$\boxed{v_3}$

$$v = \frac{1}{\sqrt{(3.948450619)10^{-3}R + (3.159824672)10^{-15}}}$$

Using $R_3 = (5.291772546)10^{-11}\text{m}$

$$v_3 = \frac{1}{\sqrt{(3.948450619)10^{-3}(5.291772546)10^{-11} + (3.159824672)10^{-15}}}$$

$$v_3 = (2.171334575)10^6 \text{m/s}$$

$\boxed{R_4}$

$$R = \frac{1}{(3.948450619)10^{-3}}\left(\frac{1}{v^2} - (3.159824672)10^{-15}\right)$$

Using $v_3 = (2.171334575)10^6 \text{m/s}$

$$R_4 = \frac{1}{(3.948450619)10^{-3}}\left(\frac{1}{[(2.171334575)10^{6}]^2} - (3.159824672)10^{-15}\right)$$

$$= \frac{1}{(3.948450619)}10^{3}\left((212.10285)10^{-15} - (3.159824672)10^{-15}\right)$$

$$R_4 = (5.291772533)10^{-11}\text{m}$$

$\boxed{v_4}$

$$v = \frac{1}{\sqrt{(3.948450619)10^{-3}R + (3.159824672)10^{-15}}}$$

Using $R_4 = (5.291772533)10^{-11}\text{m}$

$$v_4 = \frac{1}{\sqrt{(3.948450619)10^{-3}(5.291772533)10^{-11} + (3.159824672)10^{-15}}}$$

$$v_4 = (2.17334575)10^{6}\text{m/s}$$

Therefore,

$$\boxed{R = (5.291772533)10^{-11}\text{m}}$$

Compare with Bohr's $R_1 = (5.29177249)10^{-11}$

$$\frac{(5.291772533)10^{-11} - (5.29177249)10^{-11}}{(5.291772533)10^{-11}} = \frac{(4.3)10^{-8}}{5.291772533}$$

$$= (8.125821685)10^{-9}.$$

And

$$\boxed{v = (2.17334575)10^{6}\text{m/s}}$$

Compare with

$$v = \frac{\hbar}{mR}$$

$$= \frac{(1.05457266)10^{-34}}{(9.1093897)10^{-31}(5.291772533)10^{-11}}$$

$$= (2.187691395)10^{6}\text{m/s}$$

$$\frac{(2.187691395)10^{6} - (2.17334575)10^{6}}{(2.187691395)10^{6}} = \frac{0.014345645}{2.187691395}$$

$$= 0.006557435401$$

Consequently,

$$v \approx \frac{\hbar}{mR},$$

Therefore, We will use $v = \dfrac{\hbar}{mR}$ to compute R, and v.

6.

The Electron's Orbit, and Speed Assuming Quantized Angular Momentum

The Electron's Orbit

$$m\frac{v^2}{R} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2} - \frac{\mu_0}{4\pi}\frac{1}{R^2}\left(\frac{M}{m}\right)^{\frac{3}{4}} e^2 v^2$$

$$\frac{1}{e^2} m v^2 R = \frac{1}{4\pi\varepsilon_0} - 10^{-7}\left(\frac{M}{m}\right)^{\frac{3}{4}} v^2$$

$$\left(\frac{1}{e^2} mR + 10^{-7}\left(\frac{M}{m}\right)^{\frac{3}{4}}\right) v^2 = \frac{1}{4\pi\varepsilon_0}$$

Assuming that $v = \dfrac{\hbar}{mR}$,

$$\left(\frac{1}{e^2} mR + 10^{-7}\left(\frac{M}{m}\right)^{\frac{3}{4}}\right)\frac{\hbar^2}{m^2 R^2} = \frac{1}{4\pi\varepsilon_0}$$

$$\frac{1}{e^2} mR + 10^{-7}\left(\frac{M}{m}\right)^{\frac{3}{4}} = \frac{1}{4\pi\varepsilon_0}\frac{m^2}{\hbar^2} R^2$$

$$\underbrace{\frac{m}{4\pi\varepsilon_0\hbar^2}}_{a}R^2 \underbrace{-\frac{1}{\mathrm{e}^2}}_{b}R \underbrace{-10^{-7}\left(\frac{M}{m}\right)^{\frac{3}{4}}\frac{1}{m}}_{c} = 0$$

$$a = \frac{m}{4\pi\varepsilon_0\hbar^2}$$

$$= \frac{(9.1093897)10^{-31}}{4\pi(8.854187817)10^{-12}[(1.05457266)10^{-34}]^2}$$

$$= (7.3616924)10^{47}$$

$$b = -\frac{1}{\mathrm{e}^2}$$

$$= -\frac{1}{[(1.60217733)10^{-19}]^2}$$

$$= -(3.89564016)10^{37}$$

$$c = -10^{-7}\left(\frac{M}{m}\right)^{\frac{3}{4}}\frac{1}{m}$$

$$= -10^{-7}\left(1836.152701\right)^{\frac{3}{4}}\frac{1}{(9.1093897)10^{-31}}$$

$$= -(3.079231319)10^{25}$$

$$(7.3616924)10^{47}R^2 - (3.89564016)10^{37}R - (3.079231319)10^{25} = 0$$

$$\underbrace{(7.3616924)10^{22}}_{A}R^2 \underbrace{-(3.89564016)10^{12}}_{B}R \underbrace{-(3.079231319)}_{C} = 0$$

$$\sqrt{B^2 - 4AC} = \sqrt{[(3.89564016)10^{12}]^2 + 4(7.3616924)10^{22}(3.079231319)}$$

$$= (4.082506185)10^{12}$$

$$-B + \sqrt{B^2 - 4AC} = (3.89564016)10^{12} + (4.082506185)10^{12}$$

$$= (7.978146345)10^{12}$$

$$R = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{(7.978146345)10^{12}}{2(7.3616924)10^{22}}$$

$$\boxed{R = (5.418690371)10^{-11}\text{meter}}$$

Compare with Bohr's approximation

$$(5.29177249)10^{-11}\text{meter}$$

The Electron's Speed

$$v = \frac{\hbar}{mR}$$

$$= \frac{(1.05457266)10^{-34}}{(9.1093897)10^{-31}(5.418690371)10^{-11}}$$

$$\boxed{v = (2.136450774)10^{6}\text{m/s}}$$

Compare with Bohr's approximation

$$(2.1876814)10^{6}\text{m/s}$$

7.

The Electron's Angular Velocity, Frequency, Period, Current, and Magnetic Field

The Electron's Angular Velocity

$$\omega = \frac{v}{R}$$

$$= \frac{(2.136450774)10^{6}}{(5.418690371)10^{-11}}$$

$$\boxed{\omega = (3.94274377)10^{16}\text{rad/sec}}$$

The Electron's Frequency

$$f_{\text{e}} = \frac{1}{2\pi}\omega$$

$$= \frac{1}{2\pi}(3.94274377)10^{16}\text{rad/sec}$$

$$\boxed{f_{\text{e}} = (6.275071603)10^{15} c\ /\ s}$$

The Electron's Period

$$T_e = \frac{1}{f_e}$$

$$= \frac{1}{(6.275071603)10^{15}}$$

$$\boxed{T_e = (1.593607314)10^{-16}}$$

The Electron's Current

The electron's moving charge e over a period T_e seconds generates the current

$$I_e = \frac{e}{T_e}$$

$$\approx \frac{(1.60217733)10^{-19}C}{(1.593607314)10^{-16}\,\text{sec}}$$

$$= (1.005377746)\text{m}A$$

The Electron's Magnetic Field

The electron's magnetic Field is

$$B_e = \frac{\mu_0}{2R} I_e$$

$$\approx \frac{(4\pi)10^{-7}}{2(5.418690371)10^{-11}}(1.005377746)10^{-3}$$

$$= 11.65775169 \text{ weber/m}^2$$

8.

The Proton's Orbit, Speed, Angular Velocity, Frequency, Period, Current, and Magnetic Field

The Proton Orbit Radius

$$\rho = R\sqrt{\frac{m}{M}}$$

$$= (5.418690371)10^{-11}\sqrt{(5.44617013)10^{-4}}$$

$$= (1.264561448)10^{-12}\text{m}.$$

The Proton's Speed

$$V = v\sqrt[4]{\frac{m}{M}}$$

$$= (2.136450774)10^{6}\sqrt[4]{(5.44617013)10^{-4}}$$

$$= (3.263741033)10^{5}\text{m/s}$$

The Proton Angular Velocity

$$\Omega = \omega \sqrt[4]{\frac{M}{m}}$$

$$= (3.94274377)10^{16} \sqrt[4]{1836.152701}$$

$$= (2.580927191)10^{17} \text{rad/sec}$$

The Proton's Frequency

$$f_p = \frac{1}{2\pi}\Omega$$

$$= \frac{1}{2\pi}(2.580927191)10^{17}$$

$$= (4.107673202)10^{16} \text{c/s}$$

The Proton's Period

$$T_p = \frac{1}{f_p}$$

$$= \frac{1}{(4.107673202)10^{16}}$$

$$= (2.43446825)10^{-17} \text{ sec}$$

The Proton's Current

The proton's moving charge e over a period T_p seconds generates the current

$$I_p = \frac{e}{T_p}$$

$$\approx \frac{(1.60217733)10^{-19}C}{(2.43446825)10^{-17}\sec}$$

$$= (6.581220889)\text{m}A$$

The Proton's Magnetic Field

The proton's magnetic field is

$$B_p = \frac{\mu_0}{2\rho} I_p$$

$$\sim \frac{(4\pi)10^{-7}}{2(1.264561448)10^{-12}}(6.581220889)10^{-3}$$

$$\sim 5229.091887 \text{ weber/m}^2$$

Magnetic Fields of the Proton and Electron

$$\frac{B_p}{B_e} = \frac{\frac{\mu_0}{2\rho} I_p}{\frac{\mu_0}{2R} I_e}$$

$$= \frac{R}{\rho}\frac{I_p}{I_e}$$

$$= \sqrt{\frac{M}{m}}\frac{T_e}{T_p}$$

$$= \sqrt{\frac{M}{m}}\sqrt[4]{\frac{M}{m}}$$

$$= \left(\sqrt[4]{\frac{M}{m}}\right)^3$$

$$= \left(\sqrt[4]{1836}\right)^3$$

$$\sim 280$$

9.

Magnetic Monopoles, and Forces

Magnetic Pole Charge of the Electron

$$\mathrm{e}v = (1.60217733)10^{-19}(2.136450774)10^{6}$$

$$= (3.422972997)10^{-13}\,\text{(Ampere)(meter)}$$

Proton Magnetic Field Force on the Electron

$$B_p \mathrm{e}v \sim (5229.091887)(3.422972997)10^{-13}$$

$$= (1.789904033)10^{-9}\ \text{Newton}$$

Magnetic Pole Charge of the Proton

$$\mathrm{eV} = (1.60217733)10^{-19}(3.263741033)10^{5}$$

$$= (5.229091894)10^{-14}\,\text{(Ampere)(meter)}$$

Electron Magnetic Field Force on the Proton

$$B_{\mathrm{e}}\mathrm{eV} \sim (11.65775169)(5.229091894)10^{-14}$$

$$= (6.095945487)10^{-13}\ \text{Newton}$$

References

[Dannon] H. Vic Dannon, "Radiation Equilibrium, Inertia Moments, and the Nucleus Radius in the Electron-Proton Atom", Gauge Institute Journal, Vol. 10, No. 3, August 2013
http://www.gauge-institute.org/Atom/RadiationEquilibrium.pdf

[Poole] Charles P. Poole, "*The Physics Handbook Fundamentals and Key Equations*", Wiley, 1998.

[Woan] Graham Woan, "*The Cambridge Handbook of Physics Formulas*", Cambridge, 2000.