The Electron and Proton Orbits in the Hydrogen Atom

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Abstract In the Bohr Atom, the electron with mass m_e, and charge e encircles the proton with mass $M_p \approx 1836 m_e$, and charge $-e$. The radius of the circular orbit is r.

The electric attraction between the electron and the proton

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2},$$

is balanced by the centrifugal repulsion of the electron from the center. The electron speed is v, and the centrifugal force is

$$m_e\frac{v^2}{r}.$$

As the electron accelerates towards the proton, the electron radiates photons, and looses energy to the proton. That loss is so great that the electron should spiral into the proton in a fraction of a second. And yet, it does not.

Clearly, the electron energy in its orbit does not change. But why?

Bohr proposed that the electron orbits can defy Electrodynamics

because they have angular momentums that are discrete integer multiples of $\hbar$.

Consequently, the orbits are occupied by standing waves, and no radiation takes place in them.

However, while the standing wave argument is pure hypothesis, the radiation by an accelerating charge is a fundamental fact of electrodynamics.

In 2014, we submitted[1] that the electron's lost radiation is returned by the proton. The proton must be orbiting the center to be accelerating, and to shower the electron with photons.

At the center, the proton "sees" a static electron towards which it is electrically attracted, and from which it is centrifugally repulsed, due to the circular motion.

Clearly, the Proton has its own orbits, in which it accelerates towards the electron, and showers the electron with radiation.

If ρ is the radius of the proton orbit, and V is the proton speed in that orbit, the electric attraction between the proton and the electron imagined at the center

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2},$$

is balanced by the centrifugal repulsion of the proton from the

[1] H. Vic Dannon, Radiation Equilibrium, Inertia Moments, and the Nucleus Radius in the Electron-Proton Atom Gauge Institute Journal, Vol.10 No 3, August 2014,

center

$$M_p \frac{V^2}{\rho}.$$

Assuming that the power of the energy radiated by the proton into the electron field equals the power of the energy radiated by the electron into the proton field, we showed that

The electron-proton Atom is <u>*electrodynamically stable*</u> *if and only if the electron and proton inertia moments are equal*

$$\boxed{m_e r^2 \approx M_p \rho^2}$$

This equality determines the radius of the proton's orbit, which is the Nucleus Radius of the Hydrogen Atom:

$$\boxed{\rho \approx r\sqrt{\frac{m_e}{M_p}} \approx (1.221173735)10^{-12}}$$

The Proton's Period is

$$\boxed{T_p \approx T_e \sqrt[4]{\frac{m_e}{M_p}} \sim (2)10^{-15}\frac{1}{6.546} \sim (3)10^{-16}}$$

The Proton's Angular Velocity is

$$\boxed{\Omega_{\mathrm{p}} \approx \omega_e \sqrt[4]{\frac{M_p}{m_e}} \sim (3)10^{15}(6.546) \sim (2)10^{16}}$$

The Proton's Quantized Angular Momentums are

$$\boxed{M\Omega_n \rho_n^2 \sim n\hbar \sqrt[4]{\frac{M_p}{m_e}} \approx n\hbar(6.546)}$$

The Hydrogen Atom includes the proton orbits. These satisfy the force balance

$$M_p \Omega_n^2 \rho_n \approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_n^2}.$$

The Magnetic Energy of the Hydrogen Atom in its n^{th} Orbit

$$\boxed{U_{\text{magnetic},r_n} = \frac{1}{(4\pi)^2}\mu_0 v_n^2 e^2 \frac{1}{r_n} = \frac{1}{(4\pi)^2}\mu_0 V_n^2 e^2 \frac{1}{\rho_n}}$$

is negligible compared with its electric energy $U_{\text{electric},r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}$

$$\boxed{\frac{U_{\text{magnetic},r_n}}{U_{\text{electric},r_n}} = \frac{1}{n^2}\frac{1}{2\pi}\left(\frac{1}{137}\right)^2 \approx \frac{1}{n^2}(4.239835449)10^{-6}}.$$

The Electron's Kinetic Energy in its n^{th} Orbit equals Electric Energy at r_n

$$\boxed{\frac{1}{2}m_e v_n^2 \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}}.$$

The Proton's Kinetic Energy in its n^{th} Orbit equals Electric Energy at ρ_n

$$\boxed{\frac{1}{2}M_p V_n^2 \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_n}}.$$

The Electron's Kinetic Energy in its n^{th} Orbit equals the Zero Point Energy of n Photons with ω_n

$$\boxed{n\frac{1}{2}\hbar\omega_n \approx \underbrace{\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}}_{\frac{1}{2}m_e v_n^2} \approx \frac{1}{n^2}(13.61902314)\text{eV}}$$

$\left(\frac{m_e}{M_p}\right)^{\frac{1}{4}} (\approx \frac{1}{6.546})$ of the Proton's Kinetic Energy in its n^{th} Orbit equals the Zero Point Energy of n Photons with Ω_n

$$\boxed{n\frac{1}{2}\hbar\Omega_n \approx \frac{1}{6.546}\underbrace{\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_n}}_{\frac{1}{2}M_p V_n^2} \approx \frac{1}{n^2}(89.1498413)\text{eV}}.$$

Keywords: Electromagnetic Radiation of Accelerated Charge, Radiation Loss, Quantized Angular Momentum, Atomic Orbits, Electron, Proton, photon, Atom, Nucleus Radius, Zero Point Energy, Bohr's Atom, Fine Structure Constant, Magnetic Energy, Electric Energy,

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03.50.Kk, 03.65.-w. 41.60-m; 32; 32.10-f; 31.15.B-;

Contents

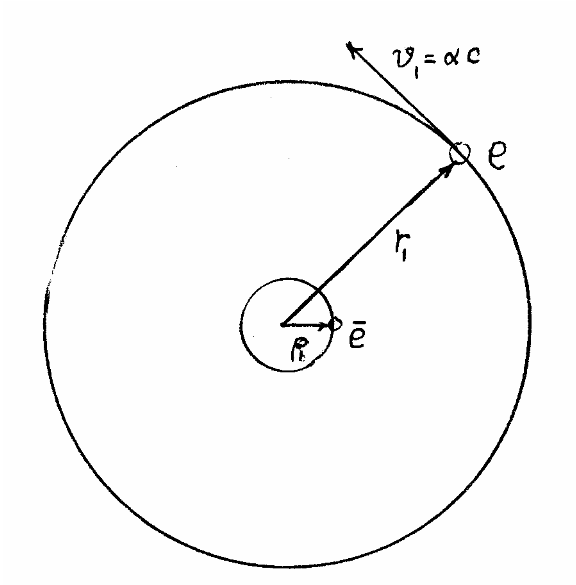
1.

The Electron, and the Proton in their First Orbits

Consider an electron with mass m_e, and charge e, in the first orbit of radius r_1, at frequency ν_1, and speed

$$v_1 = \omega_1 r_1 = 2\pi\nu_1 r_1,$$

And a proton with charge $-e$, in orbit of radius ρ_1, frequency Ω_1, and speed V_1



An electron with charge e, orbit radius r_1, frequency ν_1, and speed $v_1 = \alpha c = \omega_1 r_1 = 2\pi\nu_1 r_1$, and a proton with charge $-e$, orbit radius ρ_1, frequency Ω_1, and speed V_1.

The centrifugal acceleration of the electron is

$$\frac{v_1^2}{r_1} = \omega_1^2 r_1 .$$

The centrifugal acceleration of the proton is

$$\frac{V_1^2}{\rho_1} = \Omega_1^2 \rho_1 .$$

1.1 $$\boxed{\omega_1^2 r_1 \approx \Omega_1^2 \rho_1}$$

<u>*Proof*</u>: From [Dan1],

$$\frac{\rho_1}{r_1} \approx \sqrt{\frac{m_e}{M_p}}$$

$$\frac{\Omega_1^2}{\omega_1^2} \approx \sqrt{\frac{M_p}{m_e}}$$

Therefore,

$$\frac{\Omega_1^2 \rho_1}{\omega_1^2 r_1} \approx \sqrt{\frac{M_p}{m_e}} \sqrt{\frac{m_e}{M_p}} = 1 . \square$$

The centrifugal repulsion on the electron is

$$m_e \frac{v_1^2}{r_1} = m_e \omega_1^2 r_1$$

And the centrifugal repulsion on the proton is

$$M_p \frac{V_1^2}{\rho_1} = M_p \Omega_1^2 \rho_1 .$$

1.2 $$\boxed{M_p \Omega_1^2 \rho_1 \approx 1836 m_e \omega_1^2 r_1}$$

Proof: $\dfrac{M_p \Omega_1^2 \rho_1}{m_e \omega_1^2 r_1} \approx \dfrac{M_p}{m_e} \approx 1836 . \square$

Ignoring the magnetic forces,

1.3e $$\boxed{m_e \omega_1^2 r_1 \approx \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_1^2}}$$

1.3p $$\boxed{M_{\mathrm{p}} \Omega_1^2 \rho_1 \approx \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\rho_1^2}}.$$

1.3 1.3e, and 1.3p are equivalent to each other

Proof: Since $M_p \Omega_1^2 \rho_1 \approx 1836 m_e \omega_1^2 r_1$, and $\rho_1^2 \approx r_1^2 \dfrac{1}{1836} . \square$

To find v_1

$$m_e (\omega_1 r_1) r_1 (\omega_1 r_1) \approx \frac{1}{4\pi\varepsilon_0} e^2 .$$

Substituting $\hbar$ for the electron's angular momentum $m_e v_1 r_1$,

$$\underbrace{m_e(\omega_1 r_1)r_1}_{\hbar}\underbrace{(\omega_1 r_1)}_{v_1} \approx \frac{1}{4\pi\varepsilon_0}e^2$$

$$\hbar v_1 \approx \frac{1}{4\pi\varepsilon_0}e^2$$

Hence,

$$v_1 \approx \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}}_{\approx 1/137}c$$

$$\approx \frac{1}{137}3\cdot 10^8 \approx 2{,}189{,}789\text{m/sec}$$

1.4e $$\boxed{v_1 \approx \frac{1}{137}3\cdot 10^8 \approx 2{,}189{,}789\text{m/sec}}$$

To find V_1,

$$M_p(\Omega_1\rho_1)\rho_1(\Omega_1\rho_1) \approx \frac{1}{4\pi\varepsilon_0}e^2.$$

The Proton's angular momentum is

$$M_p V_1\rho_1 = M_p(\Omega_1\rho_1)\rho_1$$

$$= \left(\frac{M_p}{m_e}m_e\right)\left(\omega_1\left(\frac{M_p}{m_e}\right)^{\frac{1}{4}}\right)r_1^2\frac{m_e}{M_p}$$

$$= \left(\frac{M_p}{m_e}m_e\right)\left(\omega_1\left(\frac{M_p}{m_e}\right)^{\frac{1}{4}}\right)r_1^2\frac{m_e}{M_p}$$

$$= \underbrace{m_{\mathrm{e}}(\omega_1 r_1) r_1}_{\hbar} \left(1836\right)^{\frac{1}{4}} \approx 6.546\hbar$$

Therefore,

$$(6.546\hbar)V_1 \approx \frac{1}{4\pi\varepsilon_0} e^2$$

Hence,

$$V_1 \approx \frac{1}{6.546} v_1$$

$$\approx \frac{1}{6.546} 2{,}189{,}789 \mathrm{m/sec}$$

$$\approx 334{,}522 \mathrm{m/sec}$$

That is,

1.4p $$\boxed{V_1 \approx \frac{1}{6.546} v_1 \approx 334{,}522 \mathrm{m/sec}}$$

In the Bohr Atom

$$\cancel{V_1 = 0}$$

To find r_1, from **1.3e**,

$$\hbar \underbrace{(m_e v_1 r_1)}_{\hbar} \approx m_e r_1 \frac{1}{4\pi\varepsilon_0} e^2$$

Hence,

$$r_1 \approx \frac{\hbar^2}{e^2} \frac{4\pi\varepsilon_0}{m_e}$$

$$\approx \frac{1}{\underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}}_{\approx 137}} \frac{\hbar}{m_e c}$$

$$= 137 \frac{(1.05457266)10^{-34}}{(9.1093897)10^{-31} 3 \cdot 10^{8}}$$

$$= (5.286722791)10^{-11}\text{m}$$

That is,

1.5e $$\boxed{r_1 \approx \frac{\hbar^2}{e^2} \frac{4\pi\varepsilon_0}{m_e} \approx (5.286722791)10^{-11}\text{m}}$$

To find ρ_1,

$$\rho_1 \approx r_1 \sqrt{\frac{m_\text{e}}{M_\text{p}}}$$

$$\approx (5.286722791)10^{-11} \frac{1}{42.85}$$

$$\approx (1.23377428)10^{-12}\text{m}$$

That is,

1.5p $$\boxed{\rho_1 \approx \frac{1}{42.85} r_1 \approx (1.23377428)10^{-12}\text{m}}$$

In the Bohr Atom,

$$\cancel{\rho_1 = 0}$$

To find ω_1,

$$\omega_1 = \frac{v_1}{r_1} \approx \frac{2{,}189{,}789\text{m/sec}}{(5.286722791)10^{-11}\text{m}}$$

$$\approx (4.142053757)10^{16}\text{radians/sec}$$

That is,

1.6e $$\boxed{\omega_1 \approx (4.142053757)10^{16}\text{radians/sec}}$$

To find $\Omega_1 = \dfrac{V_1}{\rho_1}$,

$$\Omega_1 \approx \omega_1 \left(\frac{M_\text{p}}{m_\text{e}}\right)^{\frac{1}{4}}$$

$$\approx (4.142053757)10^{16}(6.546)$$

$$\approx (2.711388389)10^{17}\text{radians/sec}$$

That is,

1.6p $$\boxed{\Omega_1 \approx \omega_1 \left(\frac{M_\text{p}}{m_\text{e}}\right)^{\frac{1}{4}} \approx (2.711388389)10^{17}\text{radians/sec}}$$

In the Bohr Atom,

$$\cancel{\Omega_1 = 0}$$

To find the frequency of the electron motion $\nu_1 = \dfrac{\omega_1}{2\pi}$

$$\nu_1 = \frac{\omega_1}{2\pi} \approx \frac{(4.142053757)10^{16}\text{radians/sec}}{2\pi}$$

$$\approx (6.5922833)10^{15}\,\text{cycles/sec}$$

That is,

1.7e $$\boxed{\nu_1 \approx (6.5922833)10^{15}\,\text{cycles/sec}}$$

The frequency of the proton motion in its first orbit is

$$\frac{\Omega_1}{2\pi} \approx \frac{(2.711388389)10^{17}\,\text{radians/sec}}{2\pi}$$

$$\approx (4.31530864)10^{16}\,\text{radians/sec}$$

That is,

1.7p $$\boxed{\frac{\Omega_1}{2\pi} \approx (4.31530864)10^{16}\,\text{radians/sec}}$$

In the Bohr Atom,

$$\cancel{\frac{\Omega_1}{2\pi} = 0}$$

The time period of the electron's in a first orbit cycle is

$$t_1 = \frac{1}{\nu_1} \approx \frac{1}{(6.5922833)10^{15}\,\text{cycles/sec}}$$

$$\approx (1.516925099)10^{-16}\,\text{sec/cycle}$$

That is,

1.8e $$\boxed{t_1 = \frac{1}{\nu_1} \approx (1.516925099)10^{-16}\,\text{sec/cycle}}$$

The time period of the proton in a first orbit cycle is

$$T_1 \approx t_1 \left(\frac{m_e}{M_p} \right)^{\frac{1}{4}}$$

$$\approx (1.516925099)10^{-16} \frac{1}{6.546}$$

$$\approx (2.31733134)10^{-17} \text{sec/cycle}$$

That is,

1.8p $$\boxed{T_1 \approx t_1 \left(\frac{m_e}{M_p} \right)^{\frac{1}{4}} \approx (2.31733134)10^{-17} \text{sec/cycle}}$$

In the Bohr Atom,

$$\cancel{T_1 = \infty}$$

2.

The Electron, and the Proton in their n^th Orbit

In the n^{th} orbit, the centrifugal acceleration of the electron is

$$\frac{v_n^2}{r_n} = \omega_n^2 r_n .$$

The centrifugal acceleration of the proton is

$$\frac{V_n^2}{\rho_n} = \Omega_n^2 \rho_n .$$

2.1 $$\boxed{\omega_n^2 r_n \approx \Omega_n^2 \rho_n}$$

Proof: From [Dan],

$$\frac{\rho_n}{r_n} \approx \sqrt{\frac{m_e}{M_p}}$$

$$\frac{\Omega_n^2}{\omega_n^2} \approx \sqrt{\frac{M_p}{m_e}}$$

Therefore,

$$\frac{\Omega_n^2\rho_n}{\omega_n^2 r_n} \approx \sqrt{\frac{M_p}{m_e}}\sqrt{\frac{m_e}{M_p}} = 1.\square$$

The centrifugal repulsion on the electron is

$$m_e\frac{v_n^2}{r_n} = m_e\omega_n^2 r_n$$

And the centrifugal repulsion on the proton is

$$M_p\frac{V_n^2}{\rho_n} = M_p\Omega_n^2\rho_n.$$

2.2 $$\boxed{M_p\Omega_n^2\rho_n \approx 1836 m_e\omega_n^2 r_n}$$

Proof: $$\frac{M_p\Omega_n^2\rho_n}{m_e\omega_n^2 r_n} \approx \frac{M_p}{m_e} \approx 1836.\square$$

Ignoring the magnetic forces,

2.3e $$\boxed{m_e\omega_n^2 r_n \approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2}}$$

2.3p $$\boxed{M_{\mathrm{p}}\Omega_n^2\rho_n \approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_n^2}}.$$

where $M_p\Omega_n^2\rho_n \approx 1836 m_e\omega_n^2 r_n$,

and $\rho_n^2 \approx r_n^2\frac{1}{1836}$,

2.3 2.3e, and 2.3p are equivalent to each other

To find v_n, from **2.3e**,

$$m_e(\omega_n r_n)r_n(\omega_n r_n) \approx \frac{1}{4\pi\varepsilon_0}e^2.$$

Substituting $n\hbar$ for the electron's angular momentum $m_e v_n r_n$,

$$\underbrace{m_e(\omega_n r_n)r_n}_{n\hbar}\underbrace{(\omega_n r_n)}_{v_n} \approx \frac{1}{4\pi\varepsilon_0}e^2$$

$$n\hbar v_n \approx \frac{1}{4\pi\varepsilon_0}e^2$$

$$v_n \approx \frac{1}{n}\underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}c}_{\approx v_1}$$

$$\approx \frac{1}{n}2{,}189{,}789\text{m/sec}$$

That is,

2.4e
$$\boxed{v_n \approx \frac{1}{n}v_1 \approx \frac{1}{n}2{,}189{,}789\text{m/sec}}$$

To find V_n, from **2.3p**,

$$M_{\text{p}}(\Omega_n \rho_n)\rho_n(\Omega_n \rho_n) \approx \frac{1}{4\pi\varepsilon_0}e^2.$$

The Proton's angular momentum is

$$M_{\mathrm{p}} V_n \rho_n = M_{\mathrm{p}} (\Omega_n \rho_n) \rho_n$$

$$= \left(\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}} m_{\mathrm{e}} \right) \left(\omega_n \left(\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}} \right)^{\frac{1}{4}} \right) r_n^2 \frac{m_{\mathrm{e}}}{M_{\mathrm{p}}}$$

$$= \left(\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}} m_{\mathrm{e}} \right) \left(\omega_n \left(\frac{M_{\mathrm{p}}}{m_{\mathrm{e}}} \right)^{\frac{1}{4}} \right) r_n^2 \frac{m_{\mathrm{e}}}{M_{\mathrm{p}}}$$

$$= \underbrace{m_{\mathrm{e}} (\omega_n r_n) r_n}_{n\hbar} \left(1836\right)^{\frac{1}{4}} \approx 6.546 n\hbar$$

Therefore,

$$(6.546 n\hbar) V_n \approx \frac{1}{4\pi\varepsilon_0} e^2$$

$$V_n \approx \frac{1}{n} \underbrace{\frac{1}{6.546} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\hbar c} c}_{\approx V_1}$$

$$\approx \frac{1}{n} 334{,}522 \mathrm{m/sec}$$

That is,

2.4p

$$\boxed{V_n \approx \frac{1}{n} V_1 \approx \frac{1}{n} 334{,}522 \mathrm{m/sec}}$$

In the Bohr Atom,

$$\cancel{V_n = 0}$$

To find r_n, from **2.3e**,

$$n\hbar\underbrace{(m_e v_n r_n)}_{n\hbar} \approx m_e r_n \frac{1}{4\pi\varepsilon_0} e^2$$

$$r_n \approx n^2 \underbrace{\frac{\hbar^2}{e^2}\frac{4\pi\varepsilon_0}{m_e}}_{r_1}$$

$$\approx n^2(5.286722791)10^{-11}\mathrm{m}\,.$$

That is,

2.5e $$\boxed{r_n \approx n^2 r_1 \approx n^2(5.286722791)10^{-11}\mathrm{m}}$$

To find ρ_n,

$$\rho_n \approx r_n\sqrt{\frac{m_\mathrm{e}}{M_\mathrm{p}}}$$

$$\approx n^2 r_1\sqrt{\frac{m_\mathrm{e}}{M_\mathrm{p}}}$$

$$\approx n^2\rho_1$$

$$\approx n^2(1.23377428)10^{-12}\mathrm{m}$$

That is,

2.5p $$\boxed{\rho_n \approx n^2\rho_1 \approx n^2(1.23377428)10^{-12}\mathrm{m}}$$

In the Bohr Atom,

$$\cancel{\rho_n = 0}$$

To find ω_n,

$$\omega_n = \frac{v_n}{r_n} \approx \frac{\frac{1}{n}v_1}{n^2 r_1}$$

$$\approx \frac{1}{n^3}\omega_1$$

$$\approx \frac{1}{n^3}(4.142053757)10^{16}\text{radians/sec}$$

That is,

2.6e $$\boxed{\omega_n \approx \frac{1}{n^3}(4.142053757)10^{16}\text{radians/sec}}$$

To find $\Omega_n = \dfrac{V_n}{\rho_n}$,

$$\Omega_n \approx \omega_n \left(\frac{M_{\text{p}}}{m_{\text{e}}}\right)^{\frac{1}{4}}$$

$$\approx \frac{1}{n^3}\omega_1 \left(\frac{M_{\text{p}}}{m_{\text{e}}}\right)^{\frac{1}{4}}$$

$$\approx \frac{1}{n^3}\Omega_1$$

$$\approx \frac{1}{n^3}(2.711388389)10^{17}\text{radians/sec}$$

That is,

2.6p
$$\boxed{\Omega_n \approx \frac{1}{n^3}\Omega_1 \approx \frac{1}{n^3}(2.711388389)10^{17}\text{radians/sec}}$$

In Bohr Atom,

$$\cancel{\Omega_n = 0}$$

To find the frequency of the electron motion in its nth orbit is

$$\nu_n = \frac{\omega_n}{2\pi}$$

$$\approx \frac{1}{n^3}\frac{\omega_1}{2\pi}$$

$$\approx \frac{1}{n^3}\nu_1$$

$$\approx \frac{1}{n^3}\frac{(4.142053757)10^{16}\text{radians/sec}}{2\pi}$$

$$\approx \frac{1}{n^3}(6.5922833)10^{15}\text{cycles/sec}$$

That is,

2.7e
$$\boxed{\nu_n \approx \frac{1}{n^3}(6.5922833)10^{15}\text{cycles/sec}}$$

The frequency of the proton motion in its nth orbit is

$$\frac{\Omega_n}{2\pi} \approx \frac{1}{n^3}\frac{\Omega_1}{2\pi}$$

$$\approx \frac{1}{n^3}\frac{(2.711388389)10^{17}\text{radians/sec}}{2\pi}$$

$$\approx \frac{1}{n^3}(4.31530864)10^{16}\text{radians/sec}$$

That is,

2.7p $$\boxed{\frac{\Omega_n}{2\pi} \approx \frac{1}{n^3}(4.31530864)10^{16}\text{radians/sec}}$$

In Bohr's Atom,

$$\cancel{\frac{\Omega_n}{2\pi} = 0}$$

The time period of the electron in an n$^{\text{th}}$ orbit cycle is

$$t_n = \frac{1}{\nu_n} \approx n^3 \frac{1}{(6.5922833)10^{15}\text{cycles/sec}}$$

$$\approx n^3(1.51692509)10^{-16}\text{sec/cycle}$$

That is,

2.8e $$\boxed{t_n = \frac{1}{\nu_n} \approx n^3(1.51692509)10^{-16}\text{sec/cycle}}$$

The time period of the proton in an n$^{\text{th}}$ orbit cycle is

$$T_n \approx t_n\left(\frac{m_{\text{e}}}{M_{\text{p}}}\right)^{\frac{1}{4}}$$

$$\approx n^3 t_1 \left(\frac{m_e}{M_p} \right)^{\frac{1}{4}}$$

$$\approx n^3 T_1$$

$$\approx n^3 (2.31733134) 10^{-17} \text{sec/cycle}$$

That is,

2.8p $$\boxed{T_n \approx n^3 T_1 \approx n^3 (2.31733134) 10^{-17} \text{sec/cycle}}$$

In the Bohr Atom,

$$\cancel{T_n = \infty}$$

3.

The Magnetic Energy in the First Orbits

3.1e The Magnetic Energy of the Electron Current in its First Orbit

$$U_{\text{e-magnetic}} = \tfrac{1}{4}\mu_0 r_1 e^2 \nu_1^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 v_1^2 e^2 \frac{1}{r_1}$$

<u>*Proof*</u>: In its first orbit, the electron turns ν_1 cycles/second, and constitutes a current

$$I_e = e\nu_1$$

The Magnetic Energy of this current is [Benson, p.486]

$$\tfrac{1}{2} L_e I_e^2 .$$

By [Fischer, p.97]

$$L_e = \mu_0 \frac{\pi r_1^{\;2}}{2\pi r_1} = \tfrac{1}{2}\mu_0 r_1 .$$

Thus, the magnetic energy due to the electron motion is

$$\tfrac{1}{2}\tfrac{1}{2}\mu_0 r_1 (e\nu_1)^2 = \tfrac{1}{4}\mu_0 r_1 e^2 \nu_1^{\;2}$$

$$= \tfrac{1}{4}\mu_0 e^2 r_1 \underbrace{\nu_1^2}_{\frac{1}{4\pi^2}\omega_1^2}$$

$$= \frac{1}{(4\pi)^2}\mu_0 v_1^2 e^2 \frac{1}{r_1}.\square$$

3.1p The Magnetic Energy of the Proton Current in its First Orbit

$$U_{\text{p-magnetic}} = \tfrac{1}{4}\mu_0 \rho_1 e^2 \left(\frac{\Omega_1}{2\pi}\right)^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 V_1^2 e^2 \frac{1}{\rho_1}$$

Proof: In its first orbit, the proton turns $\frac{\Omega_1}{2\pi}$ cycles/second, and constitutes a current

$$I_p = -e\frac{\Omega_1}{2\pi}$$

The Magnetic Energy of this current is [Benson, p.486]

$$\tfrac{1}{2}L_p I_p^2.$$

By [Fischer, p.97]

$$L_p = \mu_0 \frac{\pi {\rho_1}^2}{2\pi\rho_1} = \tfrac{1}{2}\mu_0\rho_1.$$

Thus, the magnetic energy due to the proton's motion is

$$\tfrac{1}{2}\tfrac{1}{2}\mu_0\rho_1\left(e\frac{\Omega_1}{2\pi}\right)^2 = \tfrac{1}{4}\mu_0\rho_1 e^2\left(\frac{\Omega_1}{2\pi}\right)^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 V_1^2 e^2 \frac{1}{\rho_1}.\square$$

3.2 The Magnetic Energy of the Hydrogen Atom is

$$\boxed{\frac{1}{(4\pi)^2}\mu_0 v_1^2 e^2 \frac{1}{r_1} = \frac{1}{(4\pi)^2}\mu_0 V_1^2 e^2 \frac{1}{\rho_1}}$$

<u>*Proof*</u>: By **1.1,** $v_1^2 \frac{1}{r_1} = V_1^2 \frac{1}{\rho_1}.\square$

3.3 In the First Orbits, the Magnetic Energy is Negligible Compared with the Electric Energy

<u>*Proof*</u>:

$$\frac{U_{\text{magnetic}}}{U_{\text{electric}}} = \frac{\frac{1}{(4\pi)^2}\mu_0 v_1^2 e^2 \frac{1}{r_1}}{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_1}}$$

$$= \frac{1}{4\pi}\underbrace{\mu_0\varepsilon_0}_{1/c^2}\frac{v_1^2\frac{1}{r_1}}{\frac{1}{r_1}}$$

$$= \frac{1}{4\pi}\frac{1}{c^2}v_1^2$$

$$= \frac{1}{4\pi}\frac{1}{c^2}\left(\frac{1}{137}c\right)^2$$

$$= \frac{1}{4\pi}\left(\frac{1}{137}\right)^2$$

$$\approx (4.239835449)10^{-6}. \square$$

4.

The Magnetic Energy in the nth Orbits

4.1e The Magnetic Energy of the Electron's Current in its nth Orbit

$$U_{\text{e-magnetic}} = \tfrac{1}{4}\mu_0 r_n e^2 \nu_n^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 v_n^2 e^2 \frac{1}{r_n}$$

__Proof__: In its nth orbit, the electron turns ν_n cycles/second, and constitutes a current

$$I_e = e\nu_n$$

The Magnetic Energy of this current is [Benson, p.486]

$$\tfrac{1}{2} L_e I_e^2 .$$

By [Fischer, p.97]

$$L_e = \mu_0 \frac{\pi r_n{}^2}{2\pi r_n} = \tfrac{1}{2}\mu_0 r_n .$$

Thus, the magnetic energy due to the electron motion is

$$\tfrac{1}{2}\tfrac{1}{2}\mu_0 r_n (e\nu_n)^2 = \tfrac{1}{4}\mu_0 r_n e^2 \nu_n{}^2$$

$$= \tfrac{1}{4}\mu_0 e^2 r_n \underbrace{\nu_n^2}_{\frac{1}{4\pi^2}\omega_n^2}$$

$$= \frac{1}{(4\pi)^2}\mu_0 v_n^2 e^2 \frac{1}{r_n}.\square$$

4.1p The Magnetic Energy of the Proton's Current in its n^th Orbit

$$U_{\text{p-magnetic}} = \tfrac{1}{4}\mu_0 \rho_n e^2 \left(\frac{\Omega_n}{2\pi}\right)^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 V_n^2 e^2 \frac{1}{\rho_n}$$

Proof: In its n^th orbit, the proton turns $\frac{\Omega_n}{2\pi}$ cycles/second, and constitutes a current

$$I_p = -e\frac{\Omega_n}{2\pi}$$

The Magnetic Energy of this current is [Benson, p.486]

$$\tfrac{1}{2}L_p I_p^2.$$

By [Fischer, p.97]

$$L_p = \mu_0 \frac{\pi \rho_n{}^2}{2\pi\rho_n} = \tfrac{1}{2}\mu_0\rho_n.$$

Thus, the magnetic energy due to the proton's motion is

$$\tfrac{1}{2}\tfrac{1}{2}\mu_0\rho_n\left(e\frac{\Omega_n}{2\pi}\right)^2 = \tfrac{1}{4}\mu_0\rho_n e^2\left(\frac{\Omega_n}{2\pi}\right)^2$$

$$= \frac{1}{(4\pi)^2}\mu_0 V_n^2 e^2 \frac{1}{\rho_n}.\square$$

4.2 The Magnetic Energy of the Hydrogen Atom in its n^{th} Orbit

$$\frac{1}{(4\pi)^2}\mu_0 v_n^2 e^2 \frac{1}{r_n} = \frac{1}{(4\pi)^2}\mu_0 V_n^2 e^2 \frac{1}{\rho_n}$$

Proof: By **2.1,** $v_n^2 \dfrac{1}{r_n} = V_n^2 \dfrac{1}{\rho_n}.\square$

4.3 The Magnetic Energy of the Hydrogen Atom in its n^th Orbits is Negligible Compared with the Electric Energy

Proof:

$$\frac{U_{\text{magnetic}}}{U_{\text{electric}}} = \frac{\frac{1}{(4\pi)^2}\mu_0 v_n^2 e^2 \frac{1}{r_n}}{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}}$$

$$= \frac{1}{4\pi}\underbrace{\mu_0\varepsilon_0}_{1/c^2}\frac{v_n^2\frac{1}{r_n}}{\frac{1}{r_n}}$$

$$= \frac{1}{4\pi}\frac{1}{c^2}v_n^2$$

$$= \frac{1}{4\pi}\frac{1}{c^2}\left(\frac{1}{n}v_1\right)^2$$

$$= \frac{1}{2\pi}\frac{1}{c^2}\left(\frac{1}{n^2}\frac{1}{137}c\right)^2$$

$$= \frac{1}{n^2}\frac{1}{2\pi}\left(\frac{1}{137}\right)^2$$

$$\approx \frac{1}{n^2}(4.239835449)10^{-6}\,.\square$$

5.

Kinetic and Electric Energies

5.1e The Electron's Kinetic Energy in its First Orbit is the Electric Energy at r_1

$$\boxed{\frac{1}{2}m_e v_1^2 \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_1}}$$

<u>*Proof*</u>: By **1.3e**,

$$m_e \omega_1^2 r_1 \approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_1^2}$$

$$\frac{1}{2}m_e \underbrace{\omega_1^2 r_1^2}_{v_1^2} \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_1}$$

$$\frac{1}{2}m_e v_1^2 \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_1}.\square$$

5. 2e The Electron's Kinetic Energy in its n^{th} Orbit is the Electric Energy

$$\boxed{\frac{1}{2}m_e v_n^2 \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}}$$

<u>*Proof*</u>: By **2.3e**,

$$m_e \omega_1^2 r_n \approx \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_n^2}$$

$$\frac{1}{2} m_e \underbrace{\omega_n^2 r_n^2}_{v_n^2} \approx \frac{1}{2} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_n}$$

$$\frac{1}{2} m_e v_n^2 \approx \frac{1}{2} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_n}. \square$$

5.1p The Proton's Kinetic Energy in its First Orbit is the Electric Energy

$$\boxed{\frac{1}{2} M_p V_1^2 \approx \frac{1}{2} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\rho_1}}$$

Proof: By **1.3p**,

$$M_p \Omega_1^2 \rho_1 \approx \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\rho_1^2}$$

$$\frac{1}{2} M_p \underbrace{\Omega_1^2 \rho_1^2}_{V_1^2} \approx \frac{1}{2} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\rho_1}$$

$$\frac{1}{2} M_p V_1^2 \approx \frac{1}{2} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\rho_1}. \square$$

5.2p The Proton's Kinetic Energy in its n^{th} Orbit is Electric Energy at ρ_n

$$\boxed{\frac{1}{2}M_p V_n^2 \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_n}}$$

Proof: By **2.3p**,

$$M_p \Omega_n^2 \rho_n \approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_n^2}$$

$$\frac{1}{2}M_p \underbrace{\Omega_n^2 \rho_n^2}_{V_n^2} \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_n}$$

$$\frac{1}{2}M_p V_n^2 \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_n}.\square$$

6.

Kinetic and Zero Point Energies

6.1e The Electron's Kinetic Energy in its First Orbit is the Zero Point Energy of a Photon with ω_1

$$\boxed{\frac{1}{2}\hbar\omega_1 \approx \underbrace{\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_1}}_{\frac{1}{2}m_e v_1^2} \approx (13.61902314)\mathrm{eV}}$$

<u>*Proof*</u>: By **1.4e**,

$$v_1 \approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}c\,.$$

Substituting $v_1 = \omega_1 r_1$

$$\frac{1}{2}\hbar\omega_1 \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_1}\left(=\frac{1}{2}m_e v_1^2\right)$$

$$\approx \frac{1}{2}\frac{1}{4\pi\frac{1}{\mu_0 c^2}}\frac{e^2}{r_1}$$

$$\approx \frac{1}{2}\underbrace{\frac{\mu_0}{4\pi}}_{1/10^7}\left[(3)10^8\right]^2\frac{\left[(1.6)10^{-19}\right]^2}{(5.286722791)10^{-11}\mathrm{m}}$$

$$\approx (2179.043702)10^{-21}\mathrm{Joule}$$

$$\approx \frac{2179.043702}{160}\text{eV}$$

$$\approx (13.61902314)\text{eV} .\square$$

6.2e The Electron's Kinetic Energy in its n^{th} Orbit is the Zero Point Energy of n Photons with ω_n

$$\boxed{n\frac{1}{2}\hbar\omega_n \approx \underbrace{\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}}_{\frac{1}{2}m_e v_n^2} \approx \frac{1}{n^2}(13.61902314)\text{eV}}$$

Proof: By **2.4e**,

$$v_n \approx \frac{1}{n}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}c .$$

Substituting $v_n = \omega_n r_n$

$$n\frac{1}{2}\hbar\omega_n \approx \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}\left(= \frac{1}{2}m_e v_n^2\right)$$

$$\approx \frac{1}{2}\frac{1}{4\pi\frac{1}{\mu_0 c^2}}\frac{e^2}{n^2 r_1}$$

$$\approx \frac{1}{n^2}\frac{1}{2}\underbrace{\frac{\mu_0}{4\pi}}_{1/10^7}\left[(3)10^8\right]^2\frac{\left[(1.6)10^{-19}\right]^2}{(5.286722791)10^{-11}\text{m}}$$

$$\approx \frac{1}{n^2}(2179.043702)10^{-21}\text{Joule}$$

$$\approx \frac{1}{n^2}\frac{2179.043702}{160}\,\text{eV}$$

$$\approx \frac{1}{n^2}(13.61902314)\text{eV}\,.\square$$

6.1e $\left(\frac{m_e}{M_p}\right)^{\frac{1}{4}}$ ($\approx \frac{1}{6.546}$) of the Proton's Kinetic Energy in its First Orbit is the Zero Point Energy of a Photon with Ω_1

$$\boxed{\frac{1}{2}\hbar\Omega_1 \approx \frac{1}{6.546}\underbrace{\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_1}}_{\frac{1}{2}M_pV_1^2} \approx (89.1498413)\text{eV}}$$

Proof: By **1.4p**,

$$V_1 \approx \frac{1}{6.546}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}c\,.$$

Substituting $V_1 = \Omega_1\rho_1$

$$\frac{1}{2}\hbar\Omega_1 \approx \frac{1}{6.546}\underbrace{\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_1}}_{\frac{1}{2}M_pV_p^2}$$

$$\approx \frac{1}{6.546}\frac{1}{2}\frac{1}{4\pi\frac{1}{\mu_0c^2}}\frac{e^2}{\rho_1}$$

$$\approx \frac{1}{6.546}\frac{1}{2}\underbrace{\frac{\mu_0}{4\pi}}_{1/10^7}\left[(3)10^8\right]^2\frac{\left[(1.6)10^{-19}\right]^2}{(1.23377428)10^{-12}\,\mathrm{m}}$$

$$\approx (14263.98146)10^{-21}\,\mathrm{Joule}$$

$$\approx \frac{14263.98146}{160}\,\mathrm{eV}$$

$$\approx (89.1498413)\mathrm{eV}\,.\square$$

6.2p $\left(\dfrac{m_e}{M_p}\right)^{\frac{1}{4}}$ $(\approx \dfrac{1}{6.546})$ **of the Proton's Kinetic Energy in its n^{th} Orbit is the Zero Point Energy of n Photons with Ω_n**

$$\boxed{n\frac{1}{2}\hbar\Omega_n \approx \frac{1}{6.546}\underbrace{\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_n}}_{\frac{1}{2}M_pV_n^2} \approx \frac{1}{n^2}(89.1498413)\mathrm{eV}}$$

Proof: By **2.4p**,

$$V_n \approx \frac{1}{n}\frac{1}{6.546}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}c\,.$$

Substituting $V_n = \Omega_n\rho_n$

$$n\frac{1}{2}\hbar\Omega_n \approx \frac{1}{6.546}\underbrace{\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho_n}}_{\frac{1}{2}M_pV_p^2}$$

$$\approx \frac{1}{6.546}\frac{1}{2}\frac{1}{4\pi\dfrac{1}{\mu_0 c^2}}\frac{e^2}{n^2\rho_1}$$

$$\approx \frac{1}{n^2}\frac{1}{6.546}\frac{1}{2}\underbrace{\frac{\mu_0}{4\pi}}_{1/10^7}\left[(3)10^8\right]^2\frac{\left[(1.6)10^{-19}\right]^2}{(1.23377428)10^{-12}}$$

$$\approx \frac{1}{n^2}(14263.98146)10^{-21}\text{Joule}$$

$$\approx \frac{1}{n^2}\frac{14263.98146}{160}\text{eV}$$

$$\approx \frac{1}{n^2}(89.1498413)\text{eV}\,.\square$$

References

[Benson], Benson Walter, Harris John, Stocker Horst, Lutz Holger, "*Handbook of Physics*", Springer, 2002.

[Born] Max Born, "The Mechanics of the Atom", Ungar Publishing, 1924

[Dan] H. Vic Dannon, Radiation Equilibrium, Inertia Moments, and the Nucleus Radius in the Electron-Proton Atom Gauge Institute Journal, Vol.10 No 3, August 2014,

[Fischer] Fischer-Cripps, A., C., "*The Physics Companion*", IoP, 2003.

[Polyanin] Andrei D. Polyanin, Alexi I. Chernoutsan, "*A Concise Handbook of Mathematics, Physics, and Engineering Sciences*" CRC, 2011.

[Woan] Graham Woan, "*The Cambridge Handbook of Physics Formulas*", Cambridge 2000.